Chemical Engineering 374

Fluid Mechanics

Mass Balance (Examples)

Spiritual Thought

Sooner or later, I believe that all of us experience times when the very fabric of our world tears at the seams, leaving us feeling alone, frustrated, and adrift. It can happen to anyone. No one is immune. Everyone's situation is different, and the details of each life are unique. Nevertheless, I have learned that there is something that would take away the bitterness that may come into our lives. There is one thing we can do to make life sweeter, more joyful, even glorious.

We can be grateful!

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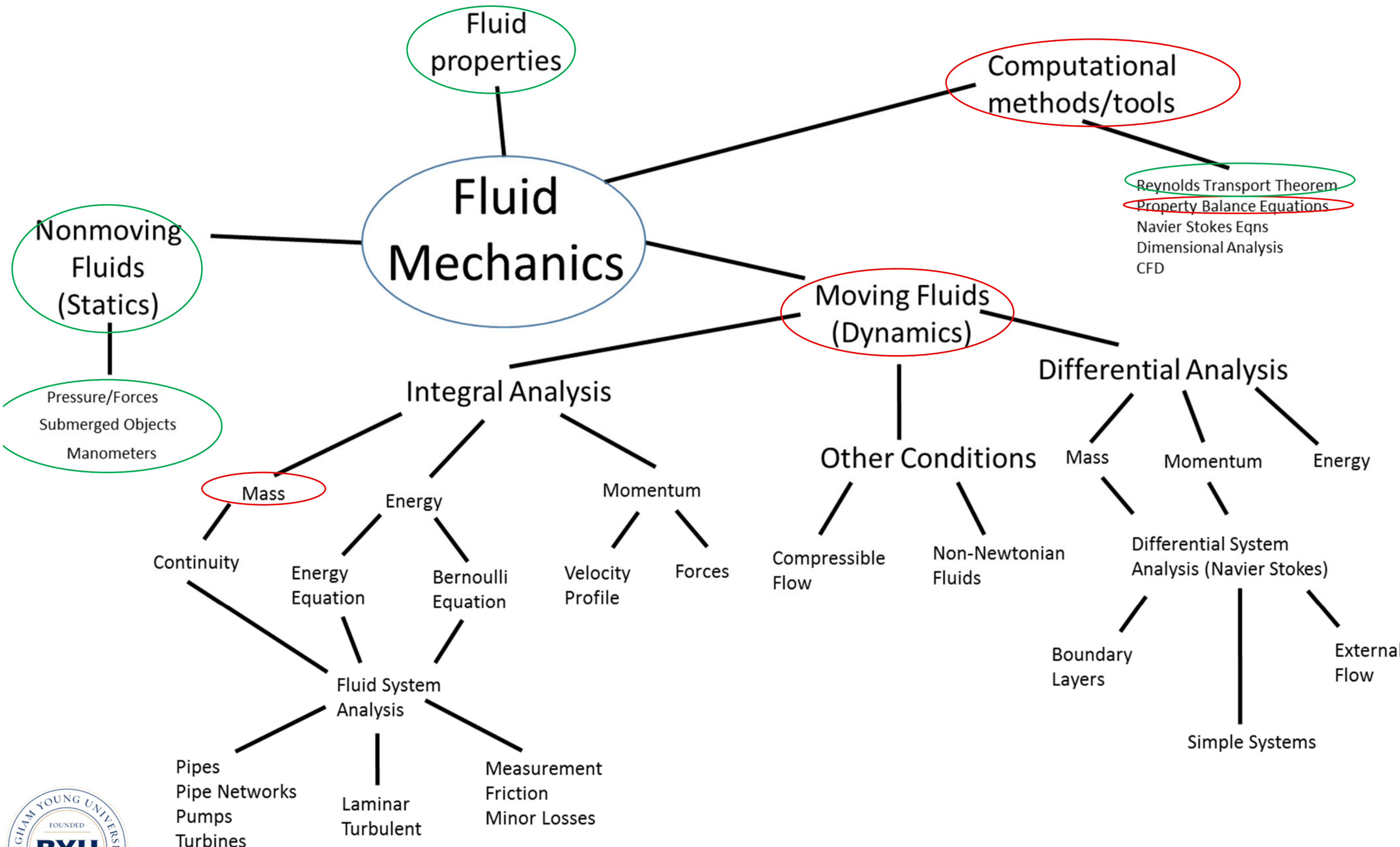
Fluids Roadmap



Key Points

- Reynolds Transport Theorem
- Mass Balance Application – continuity eq.
  - 4 cases:
    - Steady State
    - Constant Density
    - Uniform properties
      - inside CV
      - inlet/outlets
    - Fixed C.V. or variable C.V.
- Examples

Mass Balance

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho \cancelto{1}{b}\, dV + \int_{CS} \rho \cancelto{1}{b}\, \vec{v} \cdot \vec{n}\, dA$$

$$B = \text{mass} \Rightarrow b = \frac{\text{mass}}{\text{mass}} = 1$$

$$\frac{dB}{dt} = \frac{dM}{dt} = 0$$

$$0 = \overbrace{\frac{d}{dt}\int_{CV} \rho\, dV}^{\text{accum}} + \overbrace{\int_{CS} \rho \vec{v} \cdot \vec{n}\, dA}^{\text{in-out}}$$

Consideration 1: Steady State

$$0 = \cancel{\frac{d}{dt}\int_{CV}}^{\,0} \rho \, dV + \int_{CS} \rho \vec{v} \cdot \vec{n} \, dA$$

$$0 = \int_{CS} \rho \vec{v} \cdot \vec{n} \, dA$$

Consideration 2: Constant Density

$$0 = \frac{d}{dt}\int_{CV} \cancel{\rho}\, dV + \int_{CS} \cancel{\rho}\vec{v}\cdot\vec{n}\, dA$$

$$\frac{}{\rho} \qquad \frac{}{\rho} \qquad \frac{}{\rho}$$

$$0 = \frac{d}{dt}\int_{cv} dV + \int_{cs} \vec{v}\cdot\vec{n}\, dA$$

fixed cv: $\frac{dV}{dt} = 0 \Rightarrow 0 = \int_{cs} \vec{v}\cdot\vec{n}\, dA$

– Liquid: ρ = const

– Gases: ρ=MP/RT → P, T, M may vary

– High speed flow → High ΔP

– Reactions → ΔT or Δmoles (ΔM)

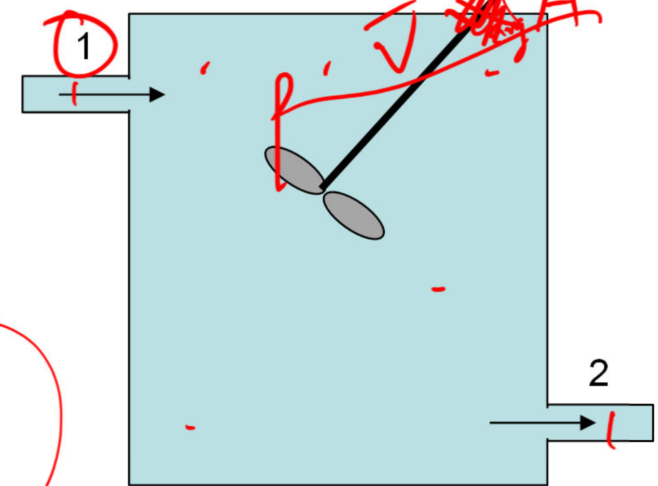
Consideration 3 – Uniform Properties

$\bar{V}_{avg}$

$$0 = \frac{d}{dt}\int_{CV}\rho\, dV + \int_{CS}\rho\vec{v}\cdot\vec{n}\, dA$$

$\int_{CS} \cdot dA$

ρ $\vec{V}$ A



$$0 = \frac{d}{dt}\int_{CV}\rho\, dV + \int_{CS}\rho\vec{v}\cdot\vec{n}\, dA$$

$$\Downarrow$$

$$\frac{d\rho}{dt}\int_{CV} dV \Rightarrow \rho\frac{d(\cancel{\rho}V)}{dt} \Rightarrow \frac{dm}{dt}$$

$$-\rho_1 V_1 A_1 + \rho_2 V_2 A_2$$

$\dot{m}_{out}$ $\dot{m}_{in}$

$$\frac{dM}{dt} = (\rho_1 vA)_{out} + (\rho_2 vA)_{in}$$

Consideration 4: CV fixed/variable

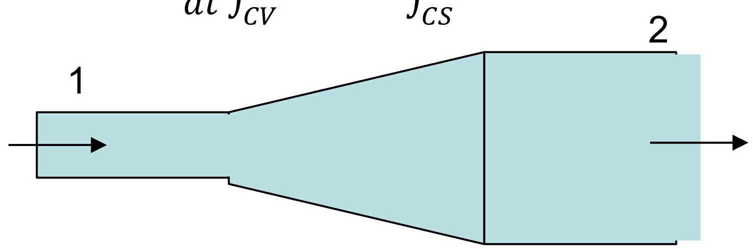
- If variable, need to figure out how it changes w/time
- If moving, find relative velocity

$$\vec{v} = \overrightarrow{v_{sys}} - \overrightarrow{v_{CV}}$$

- If fixed, we can essentially move the d/dt inside integral

Example 1

$$0 = \frac{d}{dt}\int_{CV} \rho \, dV + \int_{CS} \rho \vec{v} \cdot \vec{n} \, dA$$



Water, Steady State

$$0 = \frac{d}{dt}\int_{CV} \rho \, dV$$

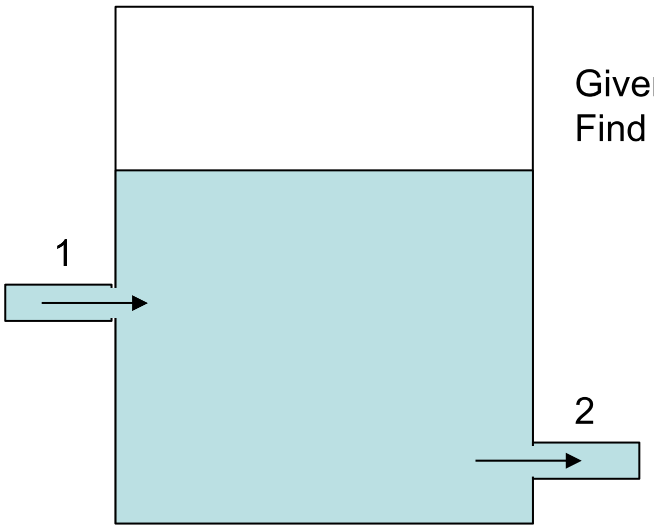
$$0 = \int_{CS} \rho \vec{v} \cdot \vec{n} \, dA \rightarrow 0 = \int_{CS} \vec{v} \cdot \vec{n} \, dA$$

$$v_2 A_2 - v_1 A_1 = 0$$

Q: Fixed CV?
-yes
Q: SS?
-yes
Q: constant ρ?
-yes
Q: Uniform in/out?
-yes

Example 2

$$0 = \frac{d}{dt}\int_{CV} \rho \, dV + \int_{CS} \rho \vec{v} \cdot \vec{n} \, dA$$



Given v_1, v_2, A_1, A_2, A_t
Find h(t)

$$0 = \frac{dV}{dt} + \int_A \vec{v} \cdot \vec{n} \, dA$$

$$0 = \frac{dV}{dt} + v_2 A_2 - v_1 A_1$$

Q: Fixed CV?
-no
What CV?
Tank Water!
Q: SS?
-no
Q: constant ρ?
-yes
Q: Uniform in/out?
-yes

Example 2 (cont)

- Is tank volume known?
  - Break it down!
    - $V_t = A_t h$

$$\frac{dh}{dt} = \left(\frac{V_1 A_1 - V_2 A_2}{A_t}\right)$$

$$h = h_0 + \left(\frac{V_1 A_1 - V_2 A_2}{A_t}\right) t$$