ChEn 374
Fluid Mechanics

Momentum Balance

Spiritual Thought

“It was meant to be that life would be a challenge. To suffer some anxiety, some depression, some disappointment, even some failure is normal. Teach our members that if they have a good, miserable day once in a while, or several in a row, to stand steady and face them. Things will straighten out. There is great purpose in our struggle in life”

Elder Boyd K. Packer

OEP 3 Clip

OEP 3

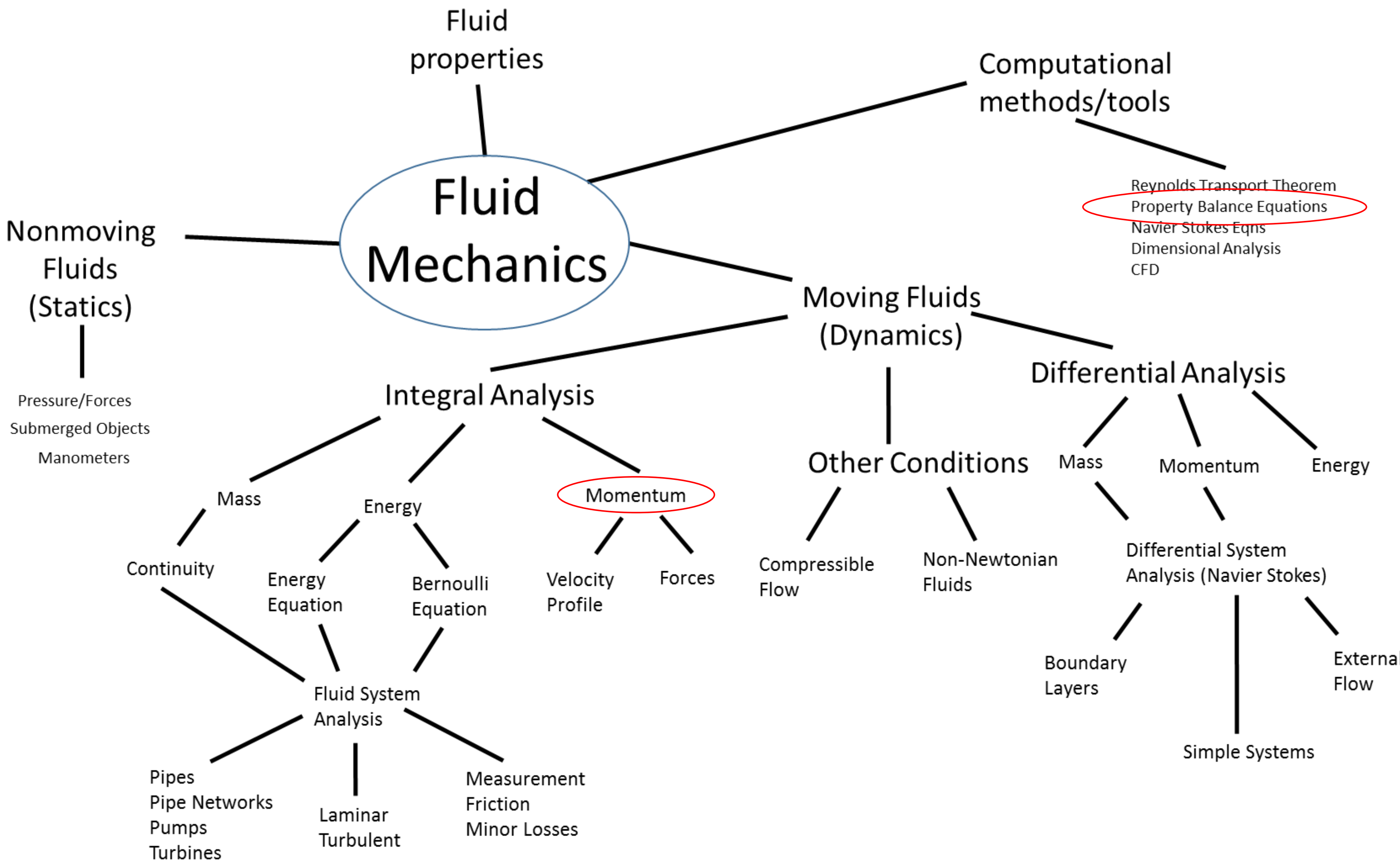
Open Ended Problem #3

The Little Rascals

INDIVIDUAL WORK ONLY, Due 9/21/16 at beginning of class

I can't even begin to describe the amazing technologies and physics that exist in their world to make this scene possible. However, I'll be a good movie watcher and suspend my disbelief. You, however, need to do some calculating. Based upon the angle and location of the hose, how much water must be flowing through the hose to lift Spanky in the air at the speed seen in the movie?

Fluids Roadmap



Key Concepts

- Momentum Balance
  - Force meaning w/respect to momentum
  - RTT simplification
- Control Volume Selection
- Forces
  - Body, Surface, Other
- Examples

Differential Balances

Mass

$$\nabla \cdot \vec{v} = 0$$

Momentum

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \cdot \nabla \vec{v} = -\nabla P + \rho \vec{g} - \nabla \cdot \tau$$

Mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

X-Mom

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial P}{\partial x} + \rho g_x + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

Y-Mom

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial P}{\partial y} + \rho g_y + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

Z-Mom

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial P}{\partial z} + \rho g_z + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

RTT Momentum Balance

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, dV + \int_{CS} \rho b \vec{v} \cdot \vec{n} \, dA$$

B = $m\vec{v}$

b = $\vec{v}$

m_{sys} = constant

$$\frac{dB_{sys}}{dt} = \frac{dm\vec{v}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} = \sum \vec{F}$$

Uniform Properties:

$$\sum \vec{F} = \frac{dm\vec{v}}{dt} + \sum (\dot{m}\vec{v})_{out} - \sum (\dot{m}\vec{v})_{in}$$

external

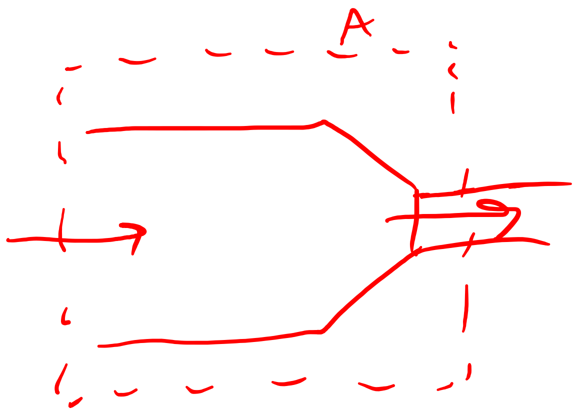
Internal ⇓ control volume

Vector equation!

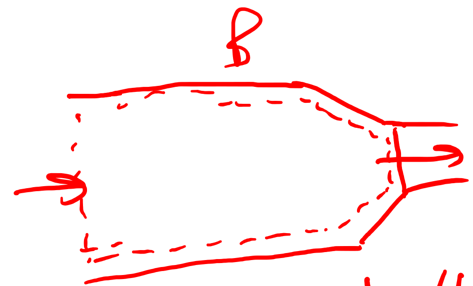
Fluid stream momentums

Control Volume

- Velocities are RELATIVE to CV ($v_{rel} = v - v_{CV}$)



1) simpler
2) Ignore details inside



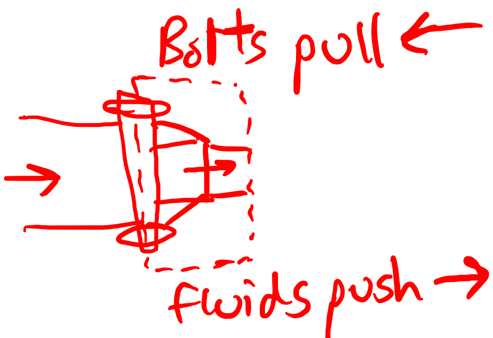
1) More complex/harder
2) Must account for all forces on nozzle, walls, fluid, etc.

- Choose CV to be perpendicular to inlet/outlet
- CV not limited to fluid!

– Choose CV ⊥ to inlet/outlet

Forces

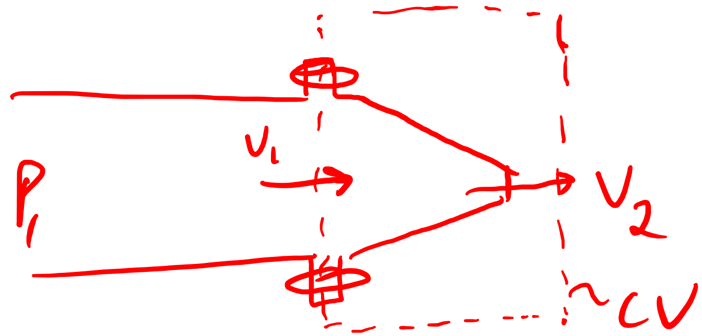
- EXTERNAL forces
  - Body forces
    - Gravity
    - Acceleration
  - Surface forces (fluid)
    - Pressure → surface P_{atm} usually cancels, just use
    - Viscous – usually ignore
  - Other
    - Bolts, ground, anchoring forces →
    - Force ON the CV
- Sign convention is KING here!!!!!



Example 1

- Flow through Nozzle
- SS
- Find Force on Bolts?

  Given P_1, V_1, A_1, A_2

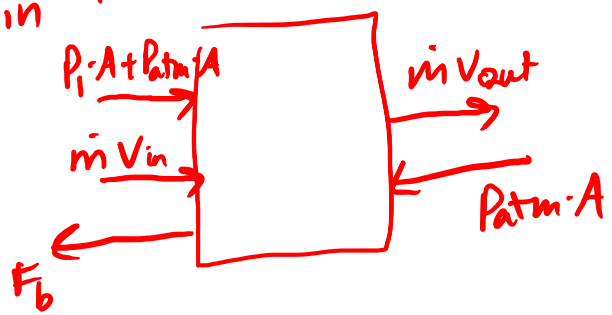


– RTT!

$$\Sigma \vec{F} = \left(\frac{dm\vec{v}}{dt}\right) + (\dot{m}\vec{v})_{out} - (\dot{m}\vec{v})_{in} \Rightarrow x\text{-direction}$$

- $\dot{m} = \rho A |v|$

– Forces from P & Bolts



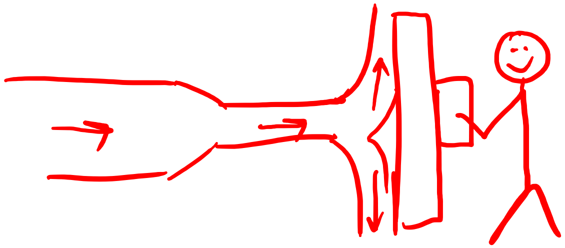
$$-F_b + A(P_{1g} + \cancel{P_{atm}} - \cancel{P_{atm}}) = (0) + \rho A_2 v_2^2 - \rho A_1 v_1^2$$

continuity → → → $v_1 A_1 = v_2 A_2$

$$F_b = P_1 \cdot A_1 + \rho A_1 v_1^2 - \rho A_2 \left(\frac{v_1 A_1}{A_2}\right)^2$$

$$\boxed{F_b = A_1\left[P_1 + \rho v_1^2\left(1 - \frac{A_1}{A_2}\right)\right]}$$

Example 2



- Flow splits & turns (½ up, ½ down)
- SS

y-direction?

- cancels

x-direction

- P_{atm} cancels

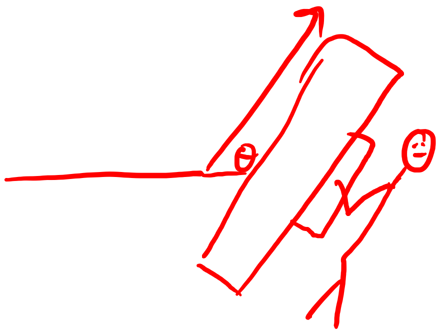
Force is left = negative!

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt}\xrightarrow{}0 + \dot{m}\vec{V}_{out}\xrightarrow{}0 - \dot{m}\vec{V}_{in}$$

$$-\sum \vec{F} = -F_{arm} = -\dot{m}\vec{V}_{in}$$

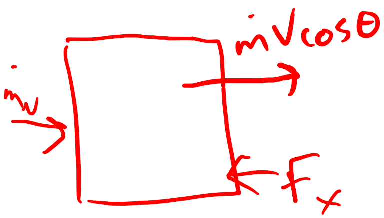
$$F_{arm} = \dot{m}V_{in}$$

Example 3



- Now hold at incline
- SS
- Force to hold?

$$\sum \vec{F} = \frac{d m\vec{v}}{dt} + (\dot{m}\vec{v})_{out} - (\dot{m}\vec{v})_{in}$$



x-momentum: $-F_x = \dot{m}V\cos\theta - \dot{m}V$

y-momentum: $F_y = \dot{m}V\sin\theta$

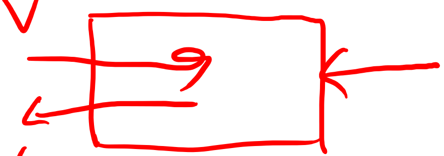
★
$$\begin{cases} F_x = \dot{m}V(1-\cos\theta) \\ F_y = \dot{m}V\sin\theta \\ \vec{F} = -F_x\vec{i} + F_y\vec{j} \end{cases}$$

Example 4

S.S.

x-only

$$\Sigma \vec{F} = \frac{d m\vec{v}}{dt} + (\dot{m}V)_{out} - (\dot{m}V)_{in}$$



$$-F_{arm} = (-\dot{m}V) - (\dot{m}V)$$

$$F_{arm} = 2\dot{m}V \,!$$

2x the force of  case