Chemical Engineering 374

Fluid Mechanics

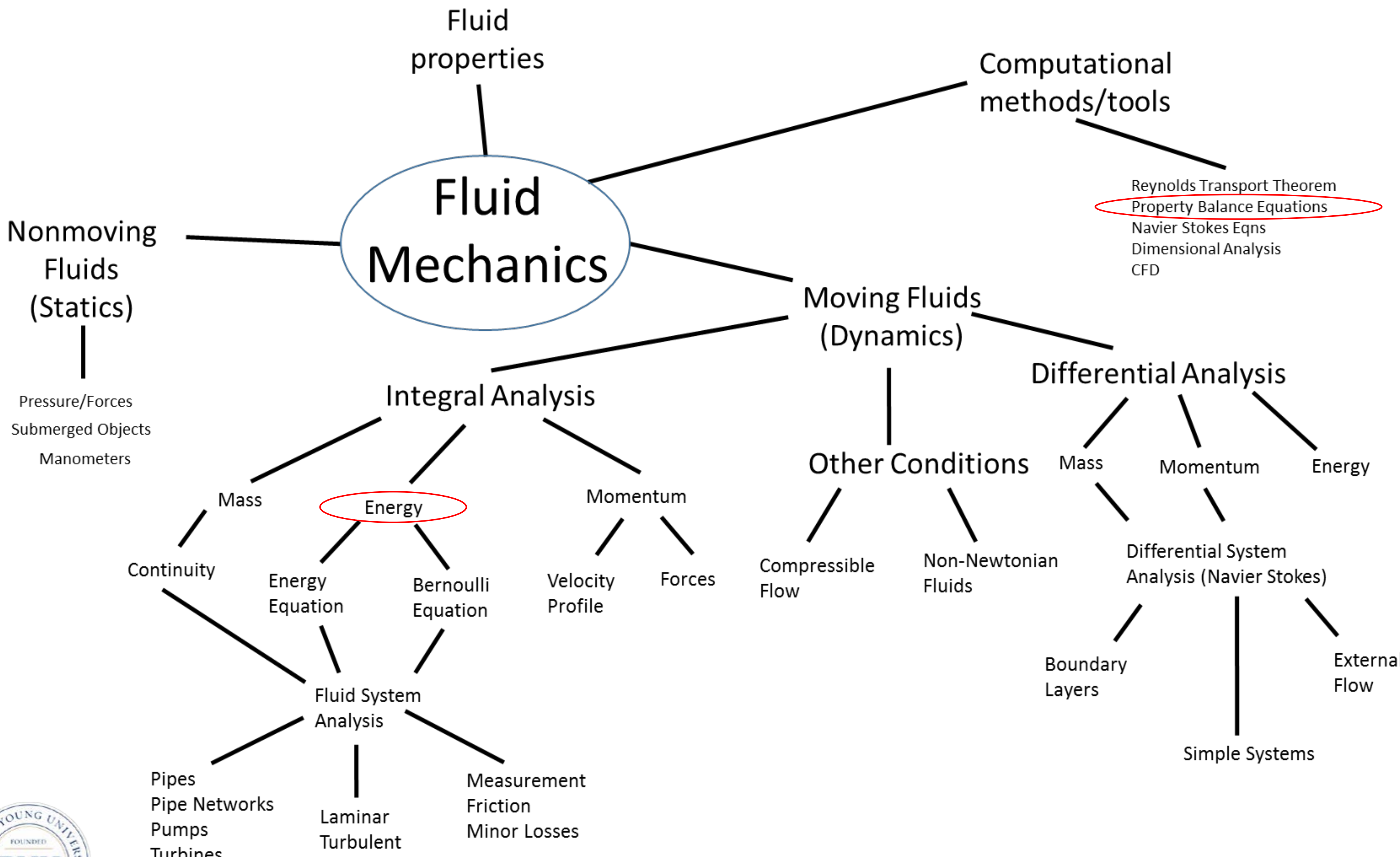
Integral Energy Balance

Spiritual Thought

3 Nephi 11:15

And it came to pass that the multitude went forth, and thrust their hands into his side, and did feel the prints of the nails in his hands and in his feet; and this they did do, going forth one by one until they had all gone forth, and did see with their eyes and did feel with their hands, and did know of a surety and did bear record, that it was he, of whom it was written by the prophets, that should come.

Fluids Roadmap



Integral Energy Balance

- We are writing balance equations using RTT
  - Fluid Statics (no flow)
  - Mass Balance (Mon)
  - Momentum Balance (Wed)
  - Energy Balance (today)

Reynolds Transport Theorem

$$\underbrace{\frac{dB_{sys}}{dt}}_{\text{System of fixed Mass (closed system)}} = \underbrace{\frac{d}{dt}\int_{CV} \rho b dV + \int_{CS} \rho b \vec{v}\cdot\vec{n} dA}_{\text{Control Volume: Some (usually fixed) region of space}}$$

Energy of the system: Internal, Kinetic, Potential

$$E = U + \frac{1}{2}mv^2 + mgz$$

$$e = u + \frac{1}{2}v^2 + gz$$

Conservation law for our system? 1st Law of thermodynamics

$$\frac{dE}{dt} = \frac{dQ}{dt} + \frac{dW}{dt}$$

RTT → B_{sys} = E, b = e

$$\frac{dQ}{dt} + \frac{dW}{dt} = \frac{d}{dt}\int_{CV} \underbrace{\rho(u + \frac{1}{2}v^2 + gz)dV}_{\rho e dV} + \int_{CS} \underbrace{\rho(u + \frac{1}{2}v^2 + gz)\vec{v}\cdot\vec{n}dA}_{\rho e \vec{v}\cdot\vec{n}dA}$$

Volumetric energy

Energy flux

Assume Uniform Properties within a control volume, and Uniform Velocities

$$\frac{dQ}{dt} + \frac{dW}{dt} = \frac{d}{dt}\left[\rho(u + \frac{1}{2}v^2 + gz)V\right] + \left[\underbrace{\rho v A}_{\dot{m}}(u + \frac{1}{2}v^2 + gz)\right]_{out} - \left[\rho v A(u + \frac{1}{2}v^2 + gz)\right]_{in}$$

Pressure is buried in the work term

$$\frac{dW}{dt} : \quad dW = \vec{F} \cdot d\vec{x}$$

Forces at the Surface

W_p – Pressure forces (stress)

W_v – Viscous stresses (usually ignore as small)

Forces internal to the system

W_s – Shaft work (pump, turbine)

W_o – Other (electric, magnetic, surface tension)

W_s is left as is → either specified directly, or computed
Positive when work is done on the system
Negative when system does work on surroundings

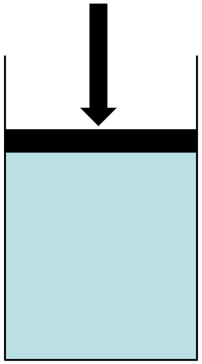
W_p is pressure work, or work to deform the boundary of the SYSTEM

$$dW = Fdx = PAdx$$

$$\frac{dW}{dt} = PA\frac{dx}{dt} = PAv$$

$$\frac{dW}{dt} = -\int_{CS} P\vec{v}\cdot\vec{n}dA = -\int_{CS}\frac{P}{\rho}\rho\vec{v}\cdot\vec{n}dA$$

Consider piston compression



General control volume

- P/ρ (=) energy per mass
- Note the negative sign
- This work is the rate of energy flux across the system surface
- associated with pressure work (deformation).
- This is the rate of energy needed to move the fluid (to move the system)

$$\frac{dQ}{dt} + \frac{dW_s}{dt} - \int_{CS} \frac{P}{\rho} \rho \vec{v} \cdot \vec{n} dA = \frac{d}{dt} \int_{CV} \rho(u + \frac{1}{2}v^2 + gz) dV + \int_{CS} \rho(u + \frac{1}{2}v^2 + gz) \vec{v} \cdot \vec{n} dA$$

- Move term to RHS
- Assume uniform properties

$$\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{d}{dt}\left[\rho(u + \frac{1}{2}v^2 + gz)V\right] + \left[\rho v A(u + \frac{P}{\rho} + \frac{1}{2}v^2 + gz)\right]_{out} - []_{in}$$

- Multiple streams need multiple terms
- u+P/ρ = h = u+P*v*

$$\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{d}{dt}\left[\rho(u + \frac{1}{2}v^2 + gz)V\right] + \left[\rho v A(u + \frac{P}{\rho} + \frac{1}{2}v^2 + gz)\right]_{out} - []_{in}$$

Simplify

•Steady State
•Q=0 (no heat transfer)
•Constant mass flow
•Constant internal energy (no friction, ΔT, Q)

$$\frac{dW_s}{dt} = \left[\rho v A(\frac{P}{\rho} + \frac{1}{2}v^2 + gz)\right]_{out} - \left[\rho v A\underbrace{(\frac{P}{\rho} + \frac{1}{2}v^2 + gz)}_{e_{mech}}\right]_{in}$$

$$= \dot{m}\Delta e_{mech} = \Delta \dot{E}_{mech}$$

•Shaft work converted to mechanical energy.
•Mechanical energy is the energy that can be directly converted to mechanical work.
•Ideal, no losses (friction/heat)
•Real systems have losses
•Convenient to consider the ideal case with some efficiency: known or compute.

$$\eta = \frac{E_{mech,\,real}}{E_{mech,\,ideal}}$$

$$\eta_{pump} = \frac{\Delta E_{mech}}{W_{shaft}}$$

$$\eta_{turbine} = \frac{W_{shaft}}{\Delta E_{mech}}$$

- Efficiency is positive, so use absolute values if needed.
- Pump/motor, turbine/motor → product of efficiencies

Example 1

Frictionless, Steady, Vertical Pipe Flow

$$\underbrace{\cancel{\frac{dQ}{dt}}}_{0} + \underbrace{\cancel{\frac{dW_s}{dt}}}_{0} = \underbrace{\cancel{\frac{d}{dt}\left[\rho(u + \frac{1}{2}v^2 + gz)V\right]}}_{0} + \left[\rho v A(u + \frac{P}{\rho} + \frac{1}{2}v^2 + gz)\right]_{out} - [\,]_{in}$$

Δu?

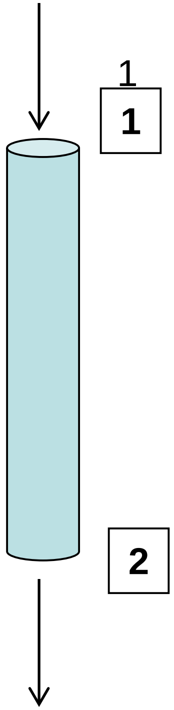
Δv?

Δm?

$$\left(\frac{P}{\rho} + gz\right)_{out} = \left(\frac{P}{\rho} + gz\right)_{in}$$

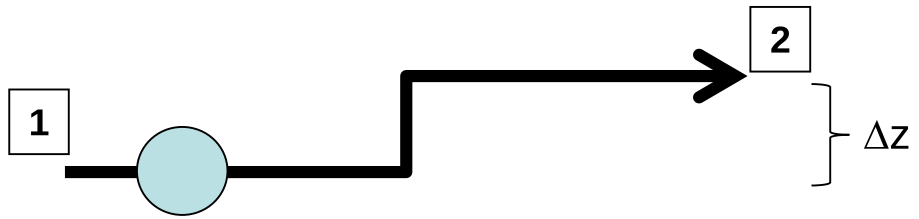
$$(P_2 - P_1) = -\rho g(z_2 - z_1) = \rho g h$$

Our old friend!



Example 2

- Pump liquid through a steady, frictionless nozzle
  - Nozzle, so A_1 not equal to A_2
- Not open to the atmosphere (pipe continues in both directions)



$$\underbrace{\cancel{\frac{dQ}{dt}}}_{0} + \frac{dW_s}{dt} = \underbrace{\cancel{\frac{d}{dt}\left[\rho(u + \frac{1}{2}v^2 + gz)V\right]}}_{0} + \left[\rho v A(u + \frac{P}{\rho} + \frac{1}{2}v^2 + gz)\right]_{out} - []_{in}$$

Δu?
Δv?
Δm?

$$\frac{\dot{W}_p}{\dot{m}} = \frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1)$$

What if ends open to the atmosphere?
What if the inlet and outlet pipes are the same size?