Chemical Engineering 374

Fluid Mechanics

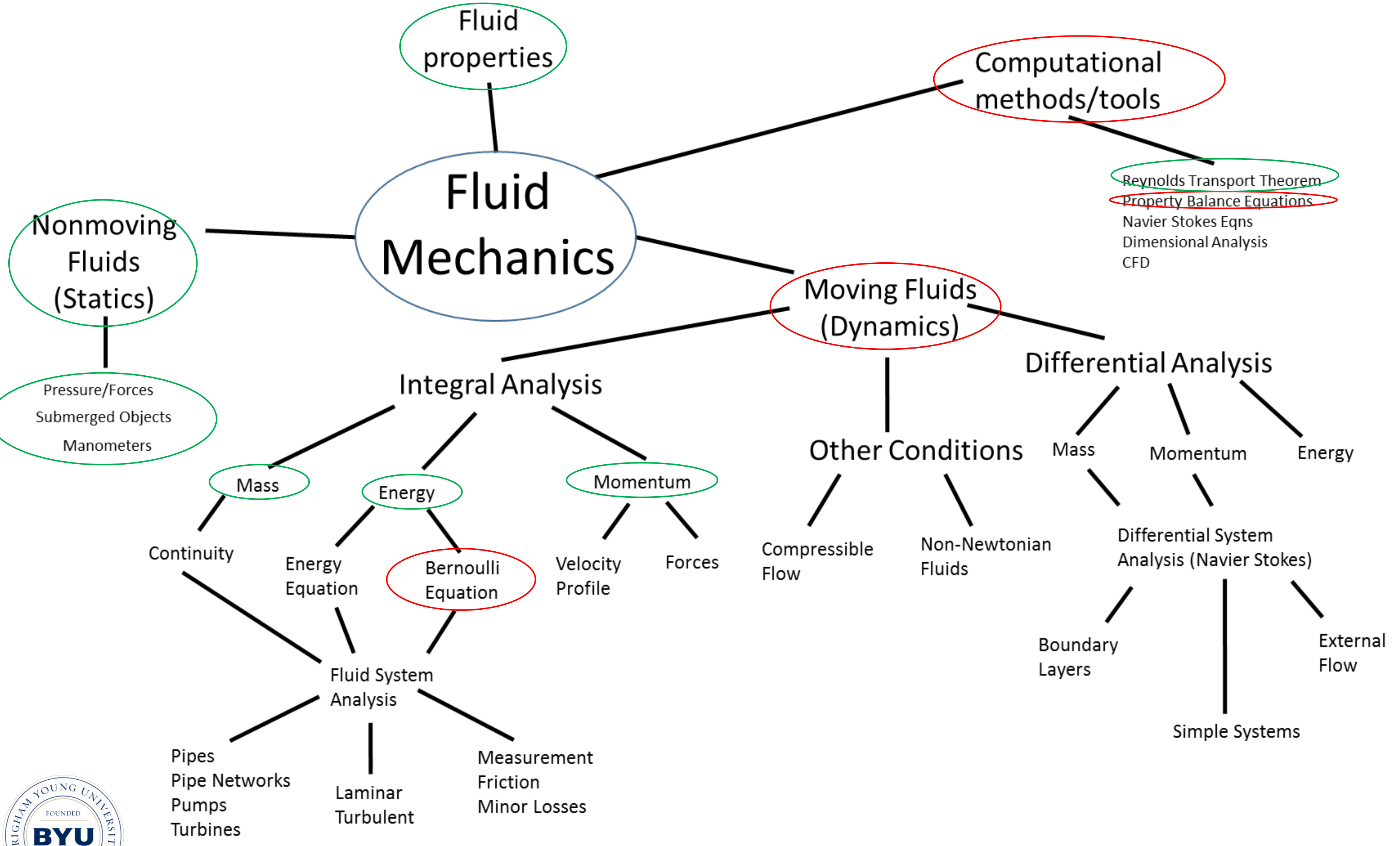
Bernoulli Equation

Spiritual Thought

Exam

- Take Home Exam (9/30/24)
- Monday to Friday (turned in at start of class)
- 4 hour exam (only need 2) – ONE SITTING!
- Closed book
- You can use 1 sheet (one side) of handwritten notes – stapled to back of exam
- Required info (like tables, units, properties) are provided.

Fluids Roadmap



Fluid properties

Computational methods/tools

Reynolds Transport Theorem
Property Balance Equations
Navier Stokes Eqns
Dimensional Analysis
CFD

Fluid Mechanics

Nonmoving Fluids (Statics)

Pressure/Forces
Submerged Objects
Manometers

Moving Fluids (Dynamics)

Integral Analysis

Mass

Continuity

Energy

Energy Equation

Bernoulli Equation

Momentum

Velocity Profile

Forces

Fluid System Analysis

Pipes
Pipe Networks
Pumps
Turbines

Laminar
Turbulent

Measurement
Friction
Minor Losses

Other Conditions

Compressible Flow

Non-Newtonian Fluids

Differential Analysis

Mass

Momentum

Energy

Differential System Analysis (Navier Stokes)

Boundary Layers

Simple Systems

External Flow

“Generation” Accumulation Out In

$$\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{d}{dt}\left[\rho(\underbrace{u + \frac{1}{2}v^2 + gz}_{e})V\right] + \left[\rho v A(u + \underbrace{\frac{P}{\rho} + \frac{1}{2}v^2 + gz)}_{e_{mech}}\right]_{out} - []_{in}$$

Can rearrange to familiar **(Accumulation) = (In) – (Out) + (“Generation”)**

<u>Simplify</u>

- Steady State
- Ws = 0
- Q = 0
- No friction (viscous effects)
    - This and no Q give const. u
- Incompressible → constant density

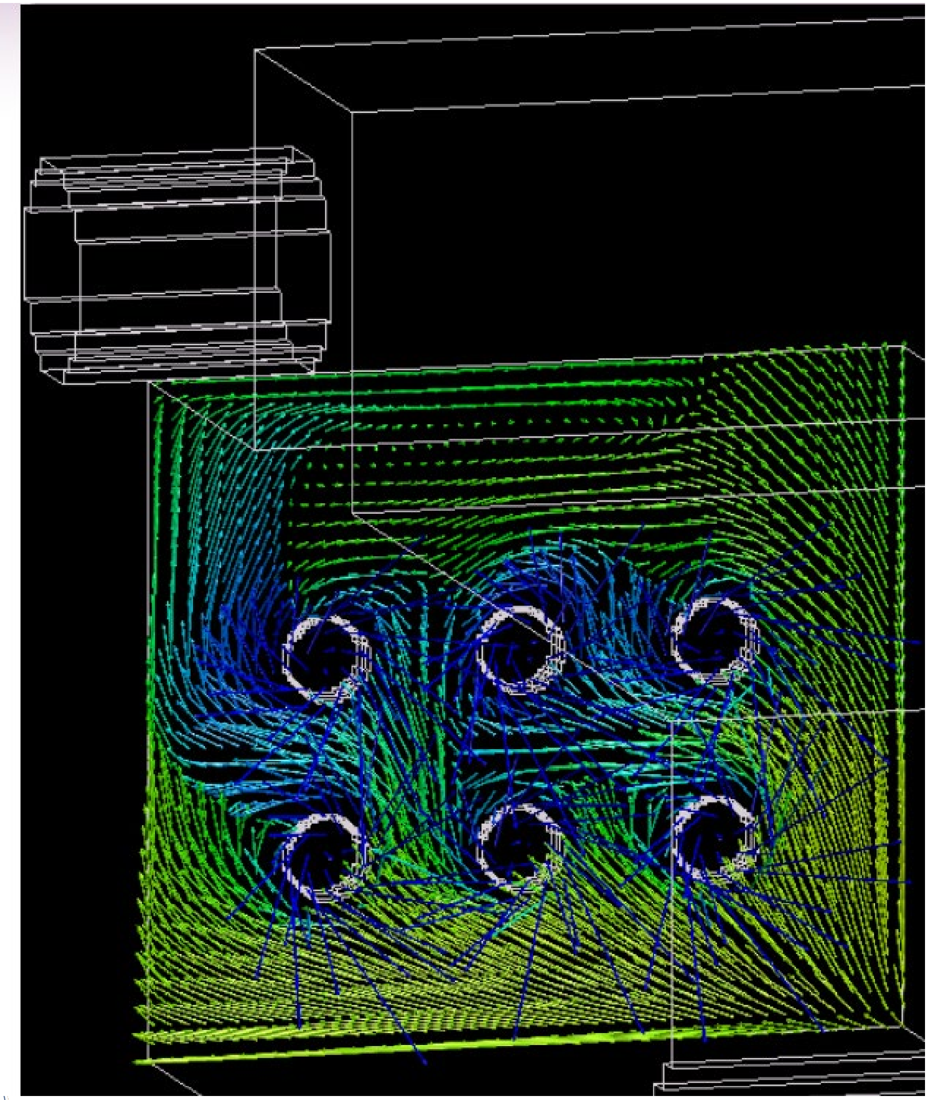
$$\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_{in} = \left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_{out}$$

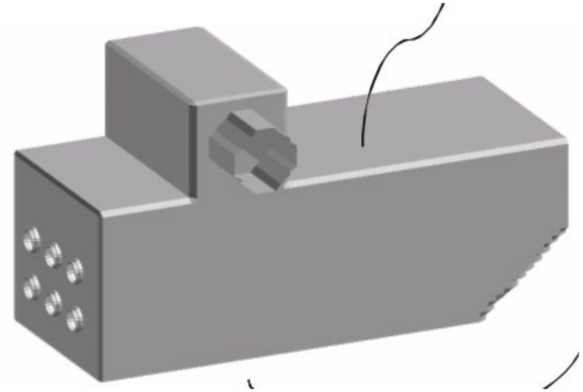
Or

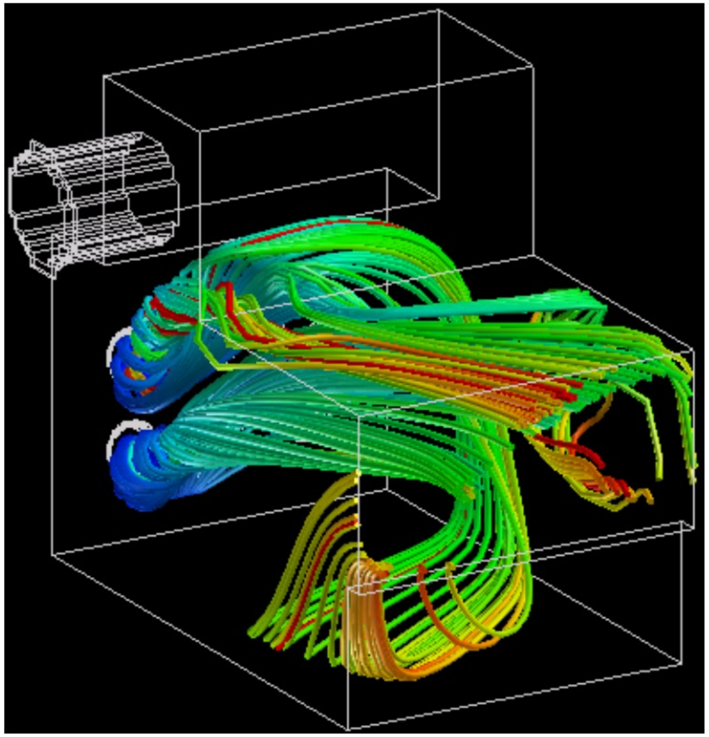
$$\Delta\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right) = 0$$

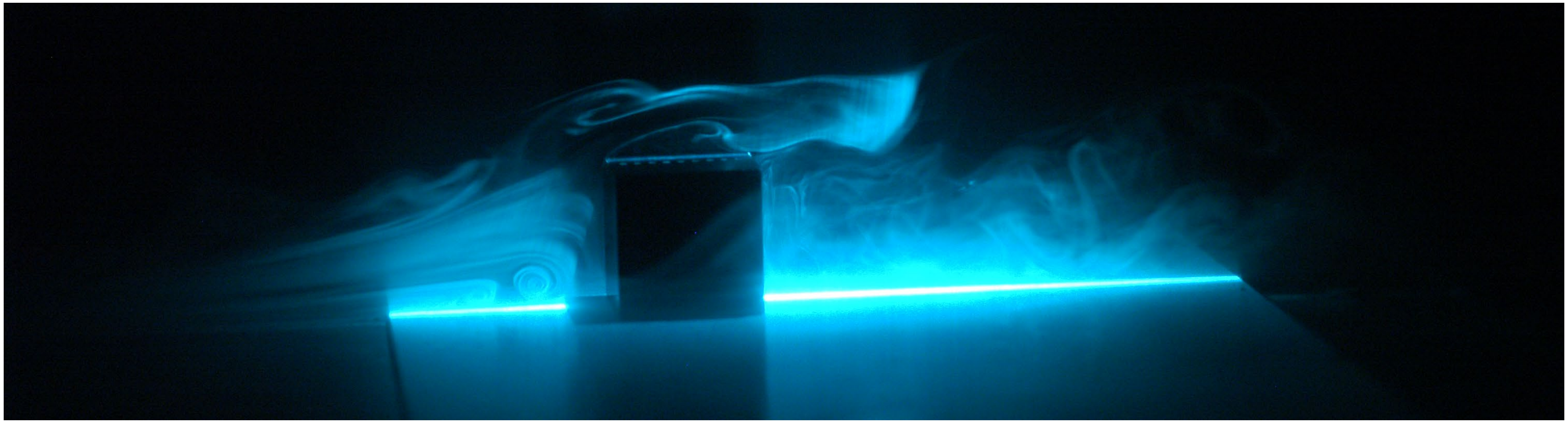
e_{mech} is conserved

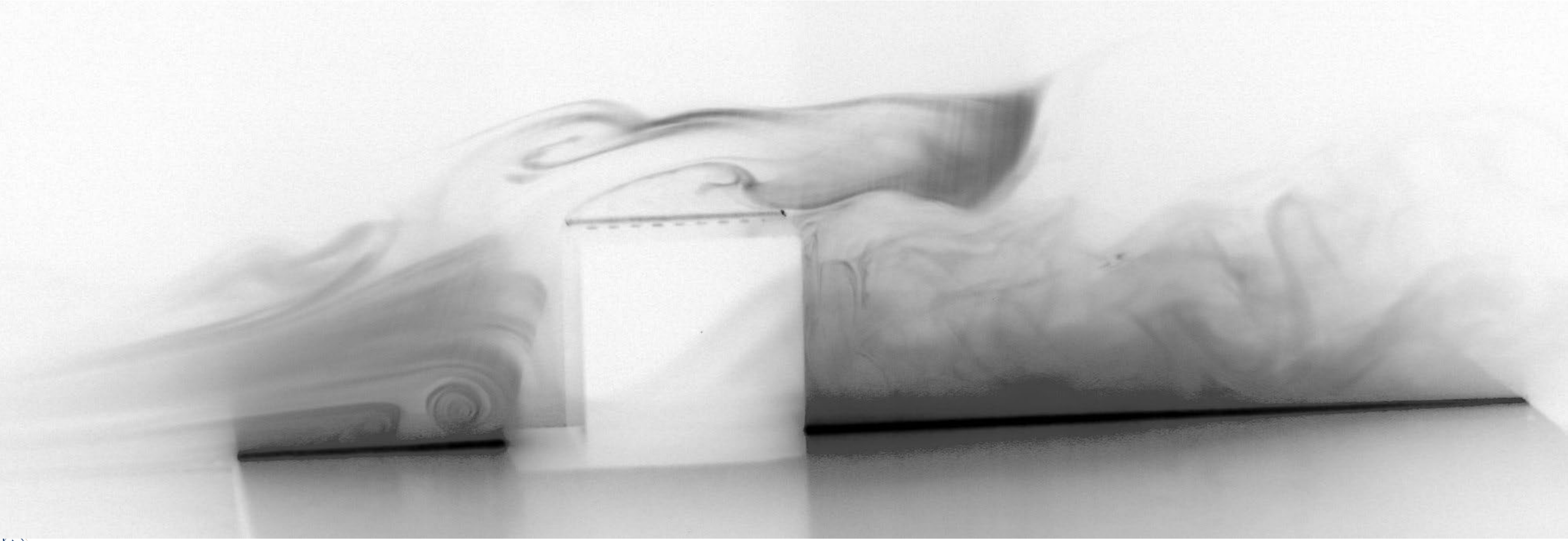
Streamlines

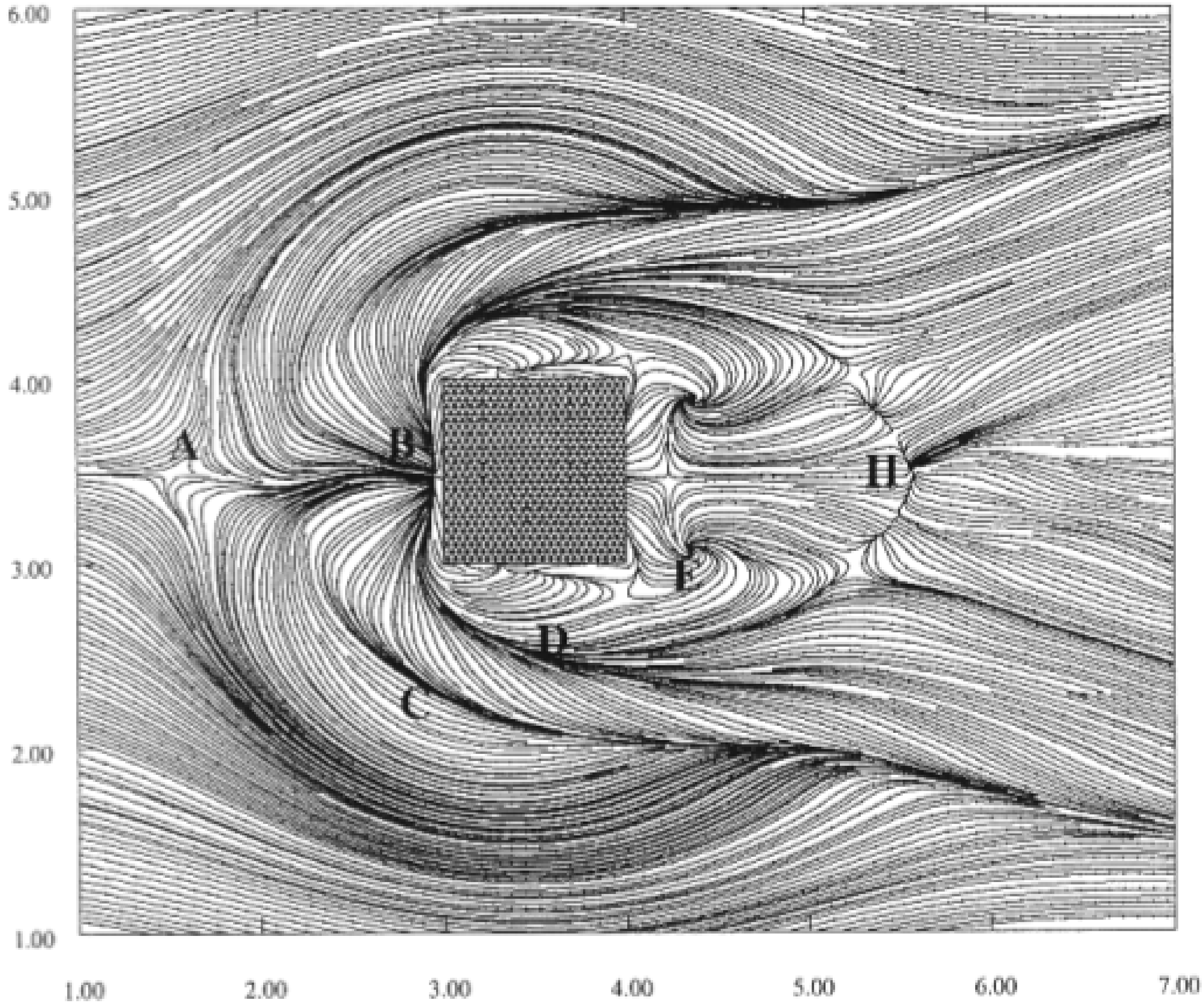








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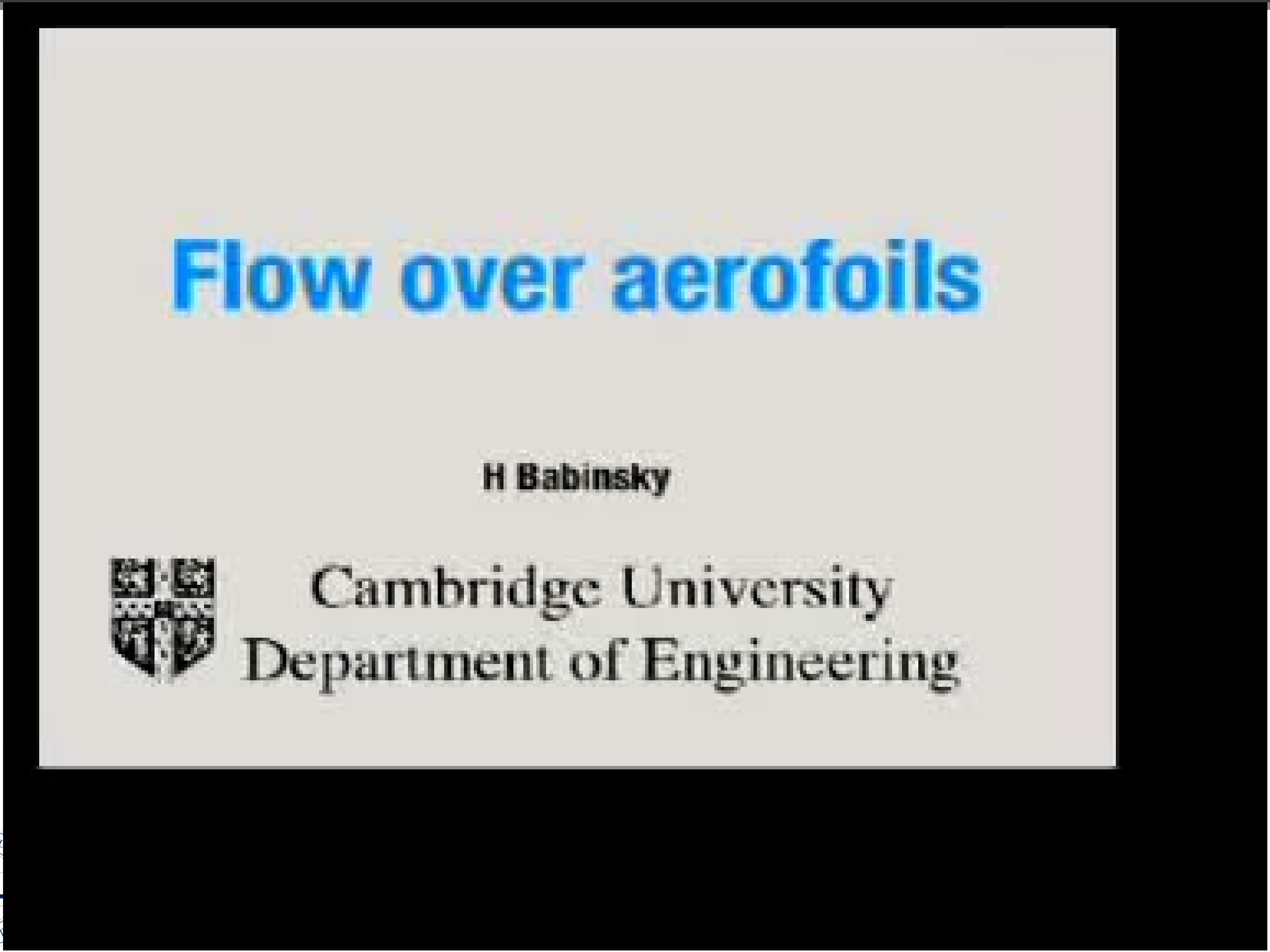

Flow over aerofoils

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- For streamlines, mechanical energy on a streamline is constant.
- Can derive the Bernoulli equation by making the same set of assumptions and “dot” the momentum equation (force balance equation) with displacement along a streamline.
- Cengel and Boles give a simpler derivation in terms of Newton’s Second Law (force balance), again along a streamline.
- Other forms of Bernoulli’s equation exist
  - Unsteady
  - Compressible
  - As usual, back up in the derivation when making assumptions.

Bernoulli Equation and Pressure

$$\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)=C$$

Units

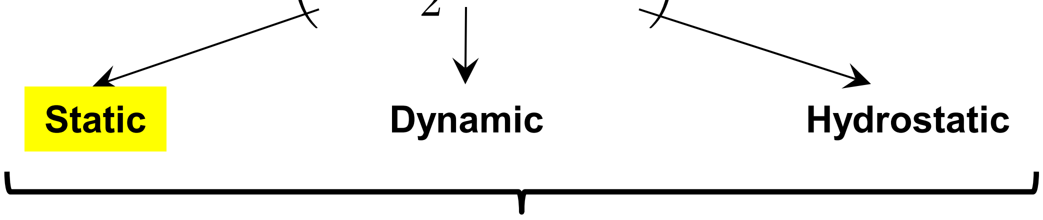
$$\frac{P}{\rho}(=)\frac{N}{m^2\cdot kg/m^3}(=)\frac{kg\cdot m}{s^2\cdot m^2\cdot kg/m^3}(=)\frac{m^2}{s^2}$$

$$(=)\frac{J}{kg}(=)\frac{kg\cdot m^2}{kg\cdot s^2}$$

B.E. units are energy per unit mass

But since the mechanical energy is constant, can multiply through by density to give units of pressure.

$$\left(P+\frac{1}{2}\rho v^2+\rho gz\right)=C$$



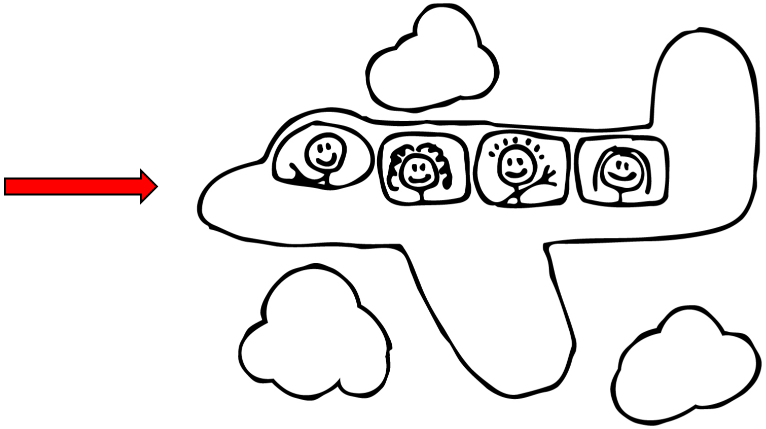
Static

Dynamic

Hydrostatic

Total pressure

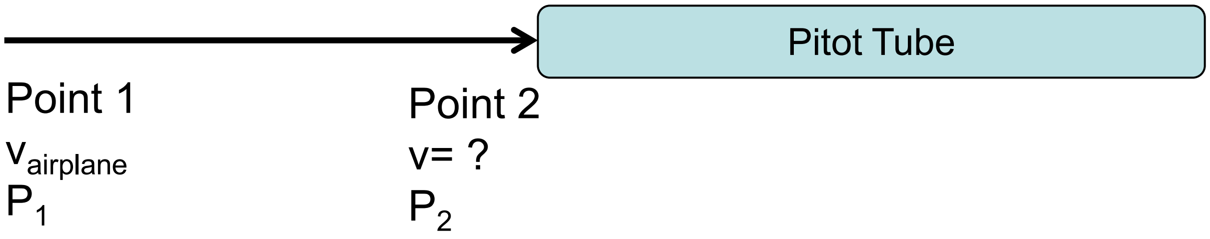
Application



- You are an airplane.
- Measure your velocity.
- How?
- Apply Bernoulli Equation $$\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)_1=\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)_2$$
- You have a bunch of variables
  - One is unknown.
  - The rest are either: *known*, *measured*, *controlled*
  - You have a constraint, what is it?

Pitot Tube

Pitot Tube



Pitot Tube

Point 1
$v_{airplane}$
P_1

Point 2
v= ?
P_2

- Note the correlation between points and the device.
- Note the streamline.
- Note the control over v_2
- What is the principle: how does it work?

Velocity Measurement

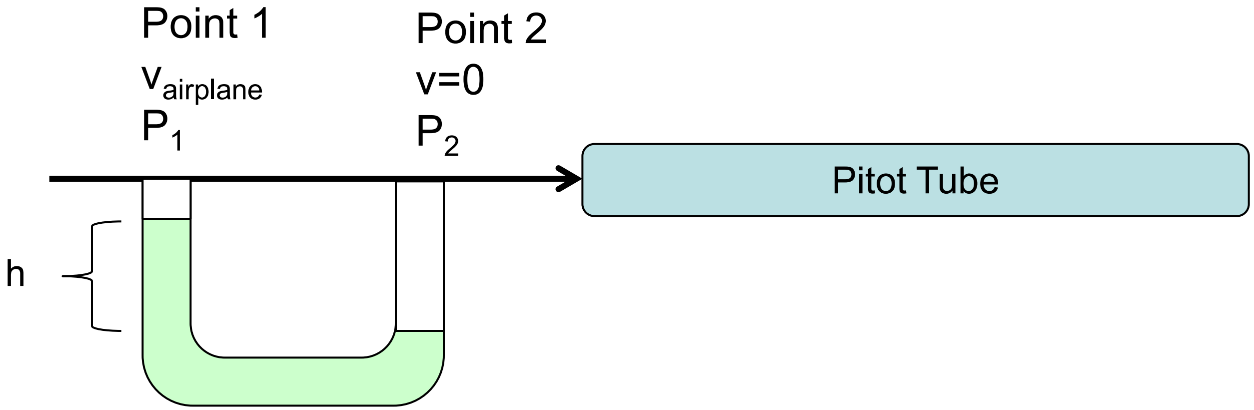
- Velocity measurement
- Total pressure is constant along a streamline
- Measure pressure at two points on the same streamline
  - Where the velocity is desired
  - At a point where the velocity has stagnated
- $P_{stagnation} = P_{static} + P_{dynamic}$
- Stagnation pressure is the pressure to bring the fluid to zero velocity without friction.

$$\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_1 = \left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_2$$

$$\Downarrow$$

$$\left(P + \frac{1}{2}\rho v^2\right)_1 = \left(P + \frac{1}{2}\rho v^2\right)_2 \quad \Rightarrow \quad v = \sqrt{\frac{2}{\rho}(P_2 - P_1)}$$

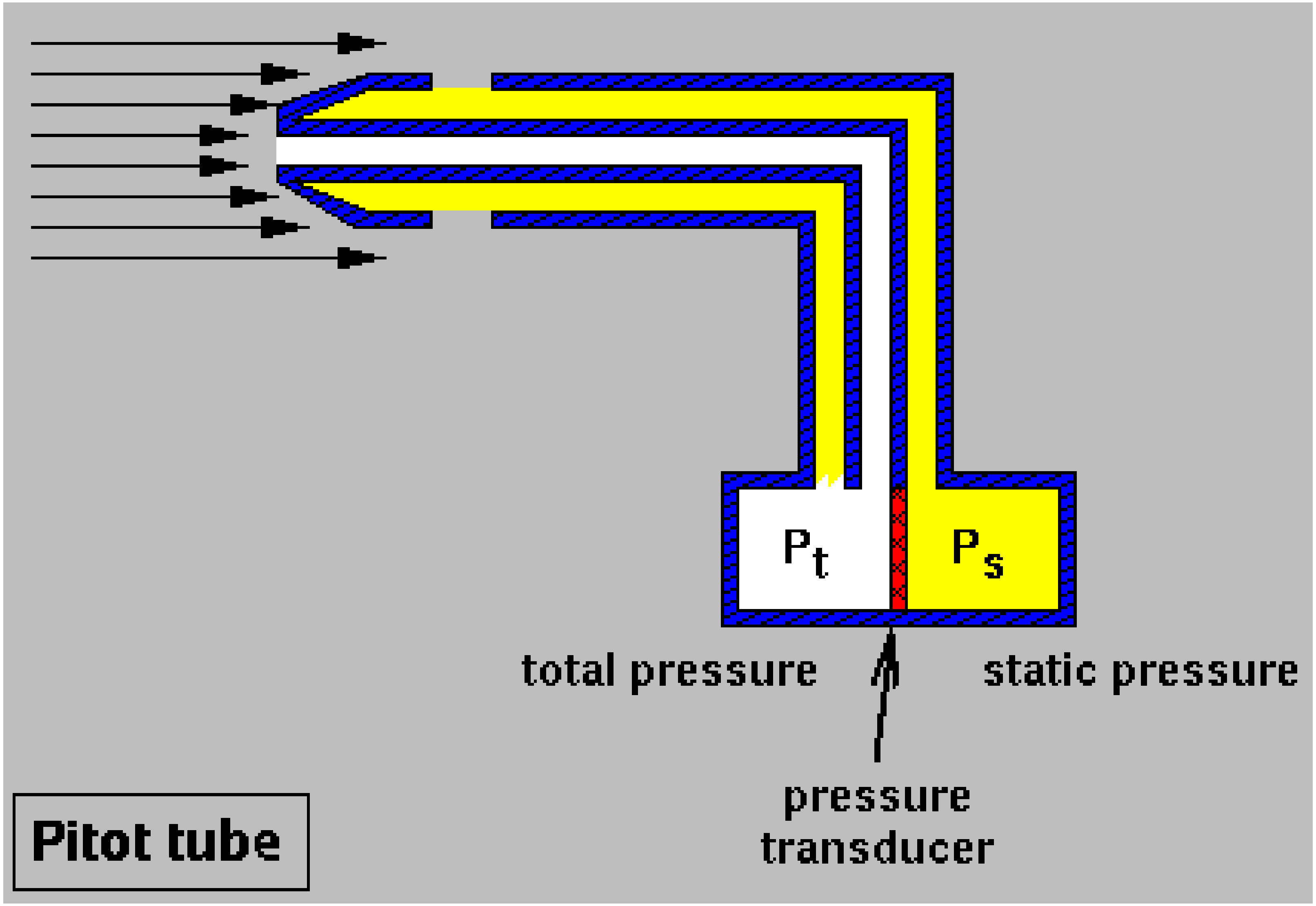
How to measure P_2-P_1



$$P_2 - P_1 = \rho g h$$

- Use a manometer,
- Or a pressure transducer, etc.
- Note, the real device is not laid out like this, but is analysed like this.

Pitot Tube


P_t
P_s
total pressure
static pressure
pressure transducer
Pitot tube

Velocity Measurement

- Problem solving with the Bernoulli equation amounts to:
  - Splitting configuration into points, evaluating P,v,z at one point and two of P,v,z at the other, and solving for the unknown with B.E.
  - Countless examples, all boil down to this.
  - Often involve multiple applications → two B.E. in two unknowns.

- Real flows are not ideal, and have friction losses.
- Friction results in a variation in internal energy (u).
- Rather than include Δu, include a friction loss term *F*
- For constant height and velocity, friction causes pressure drop.
  - Bigger fans, pumps, turbines needed for the same flow!
  - Minimize the pressure drop (friction).

$$\Delta\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)=0 \quad\longrightarrow\quad \Delta\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)=-F$$