CHAPTER 15

NONNEWTONIAN FLUIDS

This topic was introduced in Sec. 1.5C. In this brief chapter some comparisons are made between the behavior of newtonian and nonnewtonian fluids in pipe flow, and references are given for the student who wishes to pursue the subject further.

15.1 THE ROLE OF STRUCTURE IN NONNEWTONIAN BEHAVIOR

Almost all nonnewtonian fluids contain suspended particles or dissolved molecules which are large compared with the size of typical fluid molecules (a typical polymer molecule may be many thousand times as large as a water molecule). Most nonnewtonian behavior is believed to be associated with the "long-range structure" due to such larger constituents, where "long-range" implies long compared with the diameter of a small molecule such as water. For example, a Bingham fluid is assumed to have a three-dimensional elastic structure, which will resist small shearing stresses but which comes apart when subjected to a stress higher than its yield strength. Pseudoplastic fluids (by far the most common type of nonnewtonian fluid) mostly have dissolved or dispersed particles (e.g., dissolved long-chain molecules), which have a random orientation in the fluid at rest but which line up when the fluid is sheared. They offer more resistance to deformation in the random position, so the viscosity drops as they become aligned. Dilatant fluids are almost all slurries of solid

particles in which there is barely enough liquid to keep the solid particles from touching each other. Their behavior is explained by assuming that at low shear rates the fluid between the particles is able to lubricate the sliding of one particle past another but that at high shear rates this lubrication breaks down. Thixotropic fluids are assumed to have alignable particles, as pseudoplastic fluids do (most thixotropic fluids are pseudoplastic), but with a finite time required for the particles to become aligned with the flow. An additional factor in thixotropic behavior is probably the existence of weak bonds between molecules (e.g., hydrogen bonds or entanglements of polymer chains). The bonds are gradually destroyed by shearing (some authors suggest that ordinary pseudoplastic fluids are really thixotropic fluids whose particles align or whose bonds break much faster than can be observed on currently available viscometers). Rheopectic fluids are rare and generally only show rheopectic behavior under very mild shearing. It has been suggested that mild shearing may help particles in the fluid to fit together better, thus forming a tighter structure and increasing the viscosity. Viscoelastic fluids normally contain long-chain molecules, which can exist in coiled or extended forms and which can connect one to another. When stretched, these molecules straighten out, but when the flow stops, they tend to revert to their coiled position, causing the elastic behavior.

These descriptions are in accord with most observed behavior of these fluids and thus offer a mental picture of what may be going on within the fluid. However, they are by no means rigorous descriptions of the microscopic internal behavior of such fluids, and they may be modified by further studies of nonnewtonian fluids.

15.2 MEASUREMENT AND DESCRIPTION OF NONNEWTONIAN FLUIDS

Much of the past and present research in nonnewtonian fluids has consisted of measuring their stress-rate-of-strain curves (such as Fig. 1.5) and trying to find mathematical descriptions of these curves. The study of the flow behavior of materials is called *rheology* (from Greek words meaning "the study of flow"), and diagrams like Fig. 1.5 are often called *rheograms*.

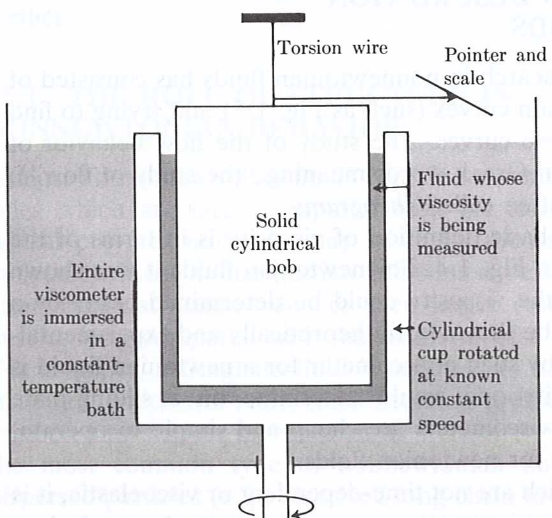
As shown in Sec. 1.5, the basic definition of viscosity is in terms of the sliding-plate experiment shown in Fig. 1.4. For newtonian fluids it was shown in Sec. 6.3 (Example 6.2) that the viscosity could be determined easily by a capillary-tube viscometer. It can be shown both theoretically and experimentally that the viscosity determined by such a viscometer for a newtonian liquid is exactly the same as the viscosity one would determine on a sliding-plate viscometer. Since capillary-tube viscometers are cheap and simple to operate, they are widely used in industry for newtonian fluids.

For nonnewtonial fluids which are not time-dependent or viscoelastic, it is possible to convert capillary-tube viscometer measurements to the equivalent sliding-plate measurements, but this involves some mathematical manipulations. For time-dependent (e.g., thixotropic) fluids, this does not seem to be possible. Thus, most studies of the behavior of nonnewtonian fluids use some variant of the sliding-plate viscometer. The most common is the concentric-cylinder viscometer; see Fig. 15.1. (Cone-and-plate viscometers are also widely used, but they are not discussed here [1, p. 517].)

In such a device a motor-driven cylindrical cup is rotated at a constant speed. The fluid being tested is in the thin, annular region between the cup and the bob. The shear stress generated by the fluid on the wall of the bob tends to turn the bob, but this turning motion is resisted by the torsion wire which supports the bob. The bob takes up a position where the torque exerted by the torsion wire is equal and opposite to the torque supplied by the fluid shear on its surface; from its position, as indicated by a pointer and scale and the calibration of the torsion wire, one can readily compute the shear stress at the wall.

This device is really the sliding-plate device wrapped around a cylinder. Mathematical corrections are needed to make the readings of this viscometer correspond exactly to those of the sliding-plate viscometer [2], but these are generally small; see Prob. 15.11. This type of viscometer is suited to newtonian or nonnewtonian fluids with or without time dependence. Several other comparable viscometer types are known [2].

The experimental data from a viscometer like that shown in Fig. 15.1 are normally represented on a plot such as Fig. 1.5. For newtonian fluids the stress-rate-of-strain behavior is described by Newton's law of viscosity, Eq. 1.5. In reading the nonnewtonian literature, observe that most authors use μ as the symbol for viscosity only of newtonian fluids and use η as the symbol for viscosity of nonnewtonian fluids.



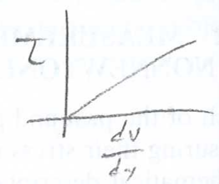


FIGURE 15.1
Concentric cylinder or "cup and bob" viscometer.

The data on a plot such as Fig. 1.5 can be used more easily if they can be represented by an equation. The Bingham fluid can be easily represented by

$$T \le \tau_{\text{yield}} \qquad \frac{dV}{dy} = 0 \qquad \tau \ge \tau_{\text{yield}} \qquad \tau = \tau_{\text{yield}} + \mu_0 \frac{dV}{dy} \tag{15.1}$$

where μ_0 is the slope of the curve on Fig. 1.5.

In many cases the experimental curves for both dilatant and pseudoplastic fluids can be reasonably well represented by the *power law*, also called the *Ostwald-de Waele equation*:

$$\tau = K\left(\frac{dV}{dy}\right)^n \tag{15.2}$$

Here K and n are constants whose values are determined by fitting experimental data. For newtonian fluids $n = 1$ and $K = \mu$. For pseudoplastic fluids n is less than 1, and for dilatant fluids it is greater than 1. The power law has little theoretical basis; its virtues are that it represents a considerable amount of experimental data with reasonable accuracy and that it leads to relatively simple mathematics. Many other equations have been used to represent these stress-strain rate curves. Some of the simpler ones are those of Ellis [3]

$$\tau = \frac{dV/dy}{A + B\tau^C} \tag{15.3}$$

Reiner-Phillipoff [3]

$$\tau = \left[A + \frac{B - A}{1 + (\tau/C)^2}\right]\frac{dV}{dy} \tag{15.4}$$

and Powell-Eyring [4, p. 372]

$$\tau = A\frac{dV}{dy} + \frac{1}{B}\sinh^{-1}\left(\frac{1}{C}\frac{dV}{dy}\right) \tag{15.5}$$

In each of these three equations A, B, and C are constants determined from experimental data. The Ellis and Reiner-Phillipoff equations are based simply on looking at experimental-data plots and deciding what form of equation would give the best fit (Probs. 15.1 and 15.2); the Powell-Eyring equation results from Eyring's theories of the structure of the liquid state. Since these three equations each contain three adjustable constants, compared with the two adjustable constants of the power-law equation, they can fit experimental data somewhat better than the power law but at the expense of greater mathematical complexity in their use. These three equations are about equal in ability to represent wide ranges of experimental data accurately and in extrapolating experimental data. Such equations are frequently called *constitutive equations* or *rheological equations of state*.

For time-dependent fluids (thixotropic or rheopectic) there are no simple relations now available for showing the stress-strain-rate-time dependence. Figure 15.2 is a typical stress-time curve for a thixotropic fluid, showing lines of

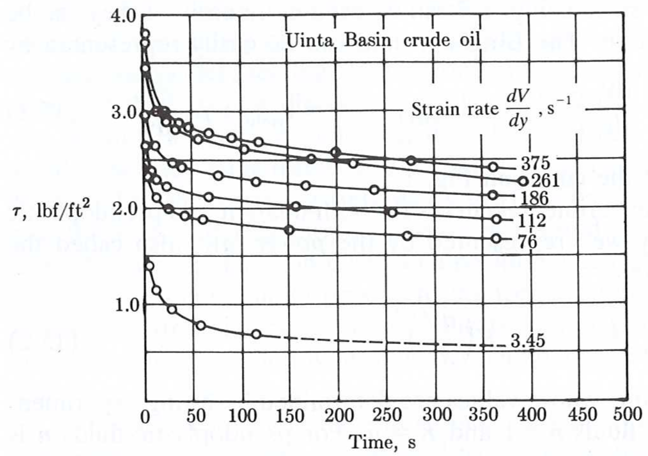


FIGURE 15.2
Stress-time curve for various strain rates for a typical thixotropic fluid, obtained in an apparatus like that shown in Fig. 15.1. [Courtesy of the late E. B. Christiansen.]

constant strain rate. The change with time occurs mostly in the first 60 s, after which the change with time is minor. For most engineering applications it would be safe to treat this fluid as a simple pseudoplastic fluid with properties corresponding to those of the right-hand side of Fig. 15.2.

For viscoelastic fluids no simple relations are known at all, and current thought is that it may never be possible to describe these fluids adequately by simple scalar equations, only by tensorial equations [5].

15.3 LAMINAR FLOW OF NONNEWTONIAN FLUIDS IN CIRCULAR TUBES

Most fluids with pronounced nonnewtonian behavior have such high viscosities that their flow is laminar in most industrially interesting situations. We saw in Sec. 6.3 that for any fluid the shear stress at any point in a horizontal circular pipe is given by

$$\tau = \frac{-r(P_1 - P_2)}{2\,\Delta x} = \frac{r}{2}\frac{dP}{dx} \tag{6.3}$$

For laminar flow of newtonian fluids we substituted Newton's law of viscosity for the shear stress and integrated twice to find Poiseuille's equation.

For nonnewtonian fluids we can experimentally determine a plot like Fig. 1.5 of the shear stress as a function of dV/dy (which equals dV/dr for circular pipe flow). From this plot we can find the equivalent of Poiseuille's equation by two graphical integrations. This is tedious and has prompted much of the work of trying to find equations that will represent the data in curves such as Fig. 1.5. If the data can be fit by the power law (Eq. 15.2), then the two

integrations can be easily performed (Prob. 15.4), yielding

$$V = \left(\frac{1}{2K}\frac{-dP}{dx}\right)^{1/n} \cdot \frac{n}{n+1} \cdot \left(r_W^{(n+1)/n} - r^{(n+1)/n}\right) \tag{15.6}$$

$$Q = \frac{n\pi}{3n+1}\left(\frac{1}{2K}\frac{-dP}{dx}\right)^{1/n} r_W^{(3n+1)/n} = \frac{n\pi D^3}{8(3n+1)}\left(\frac{D}{4K}\frac{-dP}{dx}\right)^{1/n} \tag{15.7}$$

where r_W is the radius of the tube or pipe. It is also possible to integrate several other of the shear-stress-strain-rate equations to find analytical solutions for laminar flow in a circular tube [4, p. 377]. Closed-form solutions for the flow of power-law fluids in a variety of other geometries are shown by Bird et al. [1, p. 176].

The laminar flow of various kinds of fluids in circular pipes can be easily compared by plotting $(D/4)(-dP/dx)$ versus $32Q/(\pi D^3) = 8V_{av}/D$, as shown in Fig. 15.3. This plot (or its equivalent on logarithmic paper) is very widely used in nonnewtonian flow calculations and publications. Its merit can be seen by rewriting Poiseuille's equation (Eq. 6.8) in the form

$$\frac{D}{4}\frac{-dP}{dx} = \mu\frac{32Q}{\pi D^3} = \mu\frac{8V_{av}}{D} \tag{15.8}$$

From Eq. 15.8 we see that for newtonian fluids this plot must be a straight line through the origin with a slope equal to the viscosity, as in Fig. 1.5. The left-hand side of Eq. 15.8 (and the ordinate of Fig. 15.3) is exactly equal to the shear stress at the wall of the pipe, as may be seen by comparison with Eq. 6.3. The abscissa is related to the shear rate at the wall by

$$\left(\frac{dV}{dr}\right)_w = \frac{8V_{av}}{D}\left[\frac{3}{4} + \frac{1}{4}\frac{d\ln(8V_{av}/D)}{d\ln(D/4)(-dP/dx)}\right] \tag{15.9}$$

This equation, due to Rabinowitsch and Mooney [4, p. 377; 6] is derivable for

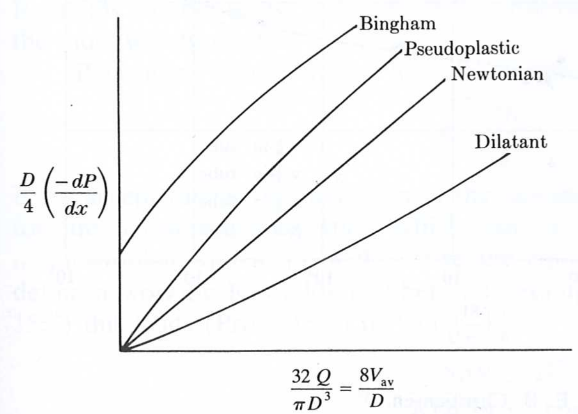


FIGURE 15.3

the laminar flow of any homogeneous, non-time-dependent fluid in a circular pipe. It has been shown experimentally to work quite well for slurries, which are not homogeneous, so its applicability is quite broad. For newtonian fluids the term in brackets on the right in Eq. 15.9 is equal to 1, so the abscissa is exactly equal to the shear rate at the wall (Prob. 15.5). For more complicated fluids the term in brackets is either a constant (for power-law fluids) or some relatively simple function of the shear rate. Thus, Fig. 15.3 is the same kind of figure as Fig. 1.5, except for a scale factor or some scale-changing function.

Just as the curve for a specific fluid at a given temperature must be the same in Fig. 1.5, independent of the kind or size of viscometer used, so also the curve for a given fluid at a given temperature in laminar flow must be the same in Fig. 15.3, independent of the size of tube in which the fluid is flowing. Figure 15.4 shows a set of experimental data for a lime-water slurry flowing in four tubes of different diameters. As indicated above, the laminar-flow data all lie on one curve. The steeply rising parts at the upper right of the curve are for turbulent flow, discussed in Sec. 15.4.

Example 15.1. It is desired to pump 15 gal/min of 23 percent lime slurry in a 1-in pipe. What is the required pressure gradient?

From App. A.3 we have

$$V_{av} = \frac{15 \text{ gal/min}}{2.69 \text{ (gal/min)/(ft/s)}} = 5.6 \frac{\text{ft}}{\text{s}} = 1.7 \frac{\text{m}}{\text{s}}$$

so that

$$\frac{1}{4}\left(\frac{8V_{av}}{\pi D}\right) = \frac{1}{4}\left(\frac{8}{\pi}\right)\left(\frac{5.6 \text{ ft/s}}{1.049 \text{ ft}/12}\right) = \frac{42.7}{\text{s}}$$

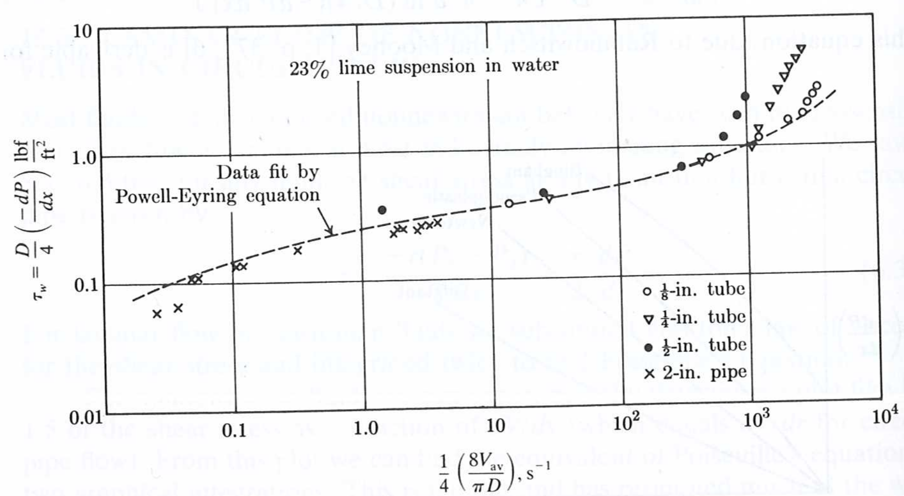


FIGURE 15.4
Data of Alves et al. [6], replotted by E. B. Christiansen.

From Fig. 15.4 we have

$$\frac{D}{4}\left(\frac{-dP}{dx}\right) = 0.45 \frac{\text{lbf}}{\text{ft}^2} = 21.6 \text{ Pa}$$

$$\frac{-dP}{dx} = 0.45 \frac{\text{lbf}}{\text{ft}^2} \cdot \frac{4}{1.049 \text{ ft}/12} \cdot \frac{\text{ft}^2}{144 \text{ in}^2} = 0.143 \frac{\text{psi}}{\text{ft}} = 3.23 \frac{\text{kPa}}{\text{m}}$$ ■

Example 15.2. We wish to double the flow rate in Example 15.1. How much must we increase the pressure gradient? How much would we have to increase it if the fluid were newtonian?

From Poiseuille's equation we know that if the fluid were newtonian (in laminar flow), we would have to double the pressure gradient to double the flow rate. From Fig. 15.4 we can measure the slope of the curve at $\frac{1}{4}[8V_{av}/(\pi D)] = 42.7$ and find that it is approximately 0.13; so we must multiply the pressure gradient by only $2^{0.13} = 1.094$ to double the flow rate. ■

15.4 TURBULENT FLOW OF NONNEWTONIAN FLUIDS IN PIPES

For turbulent flow of newtonian fluids in pipes, the experimental pressure-gradient data are represented by a friction factor–Reynolds number plot (Fig. 6.10). It seems logical to do the same for nonnewtonian fluids, but in so doing we must redefine the Reynolds number.

For newtonian fluids the viscosity is independent of the shear stress, so there is no ambiguity as to which value of the viscosity to use in the Reynolds number. However, for a nonnewtonian fluid the viscosity is a strong function of the shear stress; and from Eq. 6.3 we see that the shear stress decreases linearly with the distance from the wall, becoming zero at the tube center. Thus, there is no obvious choice of the correct viscosity to use in calculating the Reynolds number. Numerous theories and methods have been proposed for determining the critierion for laminar-turbulent transition and the proper Reynolds number to use in Fig. 6.10. The following method is the simplest and the most widely used.

Poiseuille's equation, Eq. 6.8, can be rewritten as Eq. 6.19:

$$\mathscr{R} = \frac{16}{f} \tag{6.19}$$

If we accept this as the definition of the laminar-flow Reynolds number, then for any constitutive equation which can be integrated twice to give the nonnewtonian equivalent of the Poiseuille equation, Eq. 15.9 can be used to define a working Reynolds number. For example, for power-law fluids (Eq. 15.7) this leads (Prob. 15.7) to

$$\mathscr{R} = \frac{8\rho V_{av}^{2-n} D^n}{K[2(3n+1)/n]^n} \tag{15.10}$$

Figure 15.5 shows a plot of the friction factor versus the Reynolds number as defined in Eq. 15.10. Because the Reynolds number has been defined by Eq. 15.10, the laminar-flow data must fall on the line shown. For flow at Reynolds numbers greater than 2000, two possible kinds of behavior are known. All slurries and many polymer solutions are represented by the solid curves in Fig 15.5. These do not seem to significantly suppress the turbulent behavior of the fluid. However, some polymer solutions and polymer melts, particularly those which show distinct viscoelastic behavior (such as rubber cement) obey the curves shown dotted at the right in Fig. 15.5. Visual observation [7] indicates that for these fluids the turbulence in the fluid is much less than it would be for a newtonian fluid at the same Reynolds number.

The decrease in the friction factor for polymer solutions compared with newtonian fluids can be quite startling. Dissolving as little as 5 ppm of some polymers in water produces a solution with only 60 percent of the friction factor of water at high Reynolds numbers [1, p. 88]. Such pressure-loss-reduction additives are in current large-scale industrial use [8, 9].

In this brief chapter we have not discussed elastic effects in fluid flow, which occur in many polymer melts and solutions of polymers. Some of these

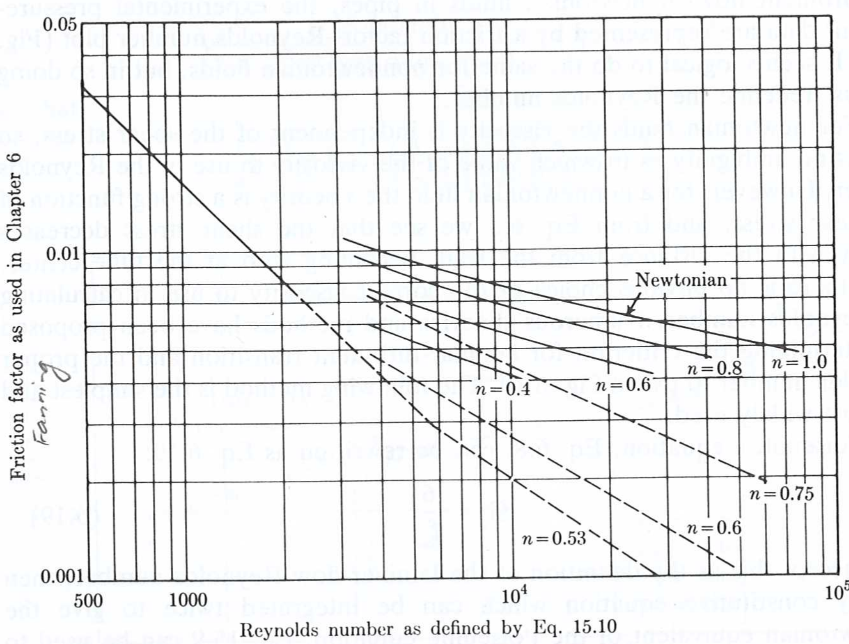


FIGURE 15.5
Friction factor plot for power-law nonnewtonian fluids. The line at the left is the laminar-flow curve, which is the same for newtonian and nonnewtonian fluids. The upper solid curve at the right is the turbulent, smooth-tubes line for newtonian fluids from Fig. 6.10. The other solid curves at the right are based on turbulent-flow data of Dodge and Metzner for nonnewtonian fluids which do not suppress turbulence. The dotted curves at the right are based on the data of Shaver and Merrill

effects are quite bizarre and startling; their theoretical explanation is one of the current major challenges in nonnewtonian fluid mechanics [1, chap. 2].

15.5 SUMMARY

1. Although intuitive explanations and numerous equations are available to describe the behavior of nonnewtonian fluids, no general, universally applicable theory or equation has been developed yet. For time-dependent and viscoelastic fluids, our knowledge consists mostly of descriptions of observed behavior.
2. For laminar flow of nonnewtonian fluids in circular pipes, we can readily calculate the behavior from pipe flow data in pipes of other sizes or from data from any kind of viscometer.
3. For turbulent flow the friction factors for nonnewtonian fluids are generally less than those for newtonian fluids. Some polymer solutions have surprisingly low friction factors.
4. The behavior of nonnewtonian fluids is currently a very active research topic. More detailed summaries of results to date can be found [1, 10].

PROBLEMS

See the Common Units and Values for Problems and Examples inside the back cover.

15.1. For pseudoplastic fluids (the most common types of nonnewtonian fluid) the fluid frequently appears to be a newtonian fluid with very high viscosity μ_0 at low shear rates and then again to be a newtonian fluid of lower viscosity μ_∞ at higher shear rates, with a transition between, as sketched in Fig. 15.6. Show that the Reiner-Phillipoff equation corresponds to this behavior, and show what constants in that equation corresponds to μ_0 and μ_∞.

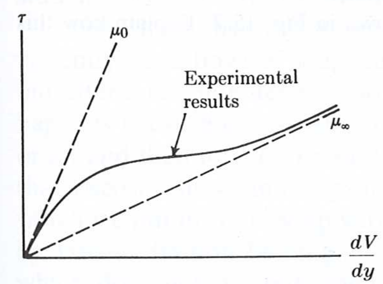


FIGURE 15.6

15.2. Show that the Ellis equation corresponds, practically, to a newtonian fluid at low shear rates and a power-law fluid at high shear rates. Show what constant or combination of constants in the Ellis equation corresponds to μ_0 in Fig. 15.6.

15.3. The data for 200 s in Fig. 15.2 can be reasonably represented by a power-law expression. Find the constants in that expression. *Hint:* The power law can be represented as a straight line with slope n on log paper.