of the $\mathscr{F}$'s from one reservoir to another with the $g \, \Delta z$ term between the same two reservoirs. When the proper set of flows has been chosen, these will all agree. For this three-branch, one-node example, the trial-and-error method is quite easy (Prob. 6.60). For more complex examples, it is not. A widely used systematic procedure for solving this type of system was developed by Cross [13]. Computer programs are available to carry out that solution [14].

6.13   ECONOMIC PIPE DIAMETER

From the foregoing we can easily calculate the flow rate, given the pipe diameter and pressure drop, or calculate the pipe diameter, given the flow rate and pressure drop, and so forth. A much more interesting question is, Given the flow rate, what size of pipe should we select? It is possible that the choice is dictated by aesthetics; e.g., the pipe goes through a lobby, and we want it to be the same size as other exposed pipes in the lobby. Or the choice may be dictated by the supply; e.g., we have on hand a large amount of surplus 4-in pipe which we want to use. Most often the choice is based on economics; the engineer is asked to make the most economical selections, all things considered.

For economic analysis we must consider two possibilities:

1. The fluid is available at a high pressure and eventually will be throttled to a low pressure, so the energy needed to overcome friction losses may come from the available pressure drop.
2. The fluid is not available at a high pressure, so a pump or compressor is needed to overcome the effects of fluid friction.

The first is simple: We select the smallest size of pipe which will carry the required flow with the available pressure drop. Example 6.5 is that case.

If the effects of friction must be overcome by a pump or compressor, then the total annual costs of the pump pipeline system are the following:

1. Power to run the pump
2. Maintenance charges on pump and line
3. Capital-cost charges for both line and pump

How these change with increasing line size is sketched in Fig. 6.20. The figure indicates the following:

1. The larger the pipe diameter, the greater the capital charges. The cost of pipeline is roughly proportional to the pipe diameter; bigger pipes cost more to buy, require more expensive supports, take longer to install, etc. The cost of the pump is proportional to the cost of the pipe and is included in it.
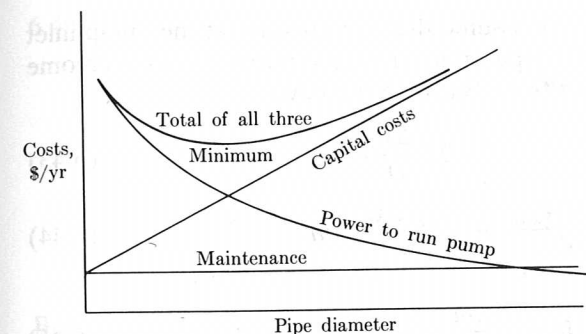2. The maintenance cost is not affected much by pipe size.

FIGURE 6.20

3. The pumping cost goes down rapidly as the pipe size goes up. The pumping cost is proportional to the pressure drop (see Example 6.3), which for turbulent flow is proportional to the velocity to the 1.8 to 2.0 power divided by the diameter. The velocity (for constant flow rate) is proportional to the reciprocal of the square of the diameter, so the pumping cost is proportional to the reciprocal of the diameter to the 4.6 to 5 power.

As Fig. 6.20 shows, the sum of these has a rather sharp minimum. This minimum occurs at the economic pipe diameter. Recognize here that we are taking the sum of a power cost during some finite period, e.g., a year, and the annual charge for owning the pipeline and the pump, whose lifetime will be many years. There are a variety of sophisticated ways of doing this, treated in books on plant design [15]. Here we consider the *simplest possible* kind of economic analysis:

$$\text{Purchase price} = \text{PP} \cdot \text{pipe diameter} \cdot \text{pipe length} \qquad (6.39)$$

where the purchase price is what we would have to pay a contractor for both supplies and labor to build the complete pipeline and pump for us and PP is a constant with dimensions \$/[inch (of diameter) · ft (of length)].

$$\text{Annual capital charge} = \text{CC} \cdot \text{purchase price} \qquad (6.40)$$

where capital charge (CC) is a constant, with dimension (1/year) and

$$\text{Annual pumping cost} = \text{PC} \cdot \text{pump power} \qquad (6.41)$$

where pumping cost (PC) is a constant with dimensions \$/[hp · year]. As shown in Fig. 6.20, the maintenance cost is practically independent of the pipe diameter, so we do not include it in the analysis. We then wish to find the minimum of

$$\text{Total annual cost} = \text{PC} \cdot \text{Po} + \text{CC} \cdot \text{PP} \cdot \text{diameter} \cdot \text{length} \qquad (6.42)$$

Assuming that the pipe is horizontal, we may apply Bernoulli's equation from the pump inlet, point 1, to the pipe outlet, point 2, and see that there is no

change in elevation or velocity. We assume that the pressure at the pump inlet is the same as the pressure at the pipe outlet; i.e., the pump has to overcome only the effects of friction. Then from Eq. 6.16 we have

$$\frac{-dW_{a.o.}}{dm} = \mathcal{F} = 2f \frac{\Delta x}{D} V^2 \tag{6.43}$$

$$Po = \frac{-dW_{a.o.}}{dm} \dot{m} = 2f \frac{\Delta x}{D} V^2 \dot{m} \tag{6.44}$$

but we have

$$V = \frac{\dot{m}}{\rho(\pi/4)D^2} \tag{6.45}$$

and therefore

$$Po = \frac{\dot{m}^3 2f \Delta x (4/\pi)^2}{\rho^2 D^5} \tag{6.46}$$

Substituting Eq. 6.43 and the cost of the pipe in Eq. 6.39, we find

$$\text{Total annual cost} = PC \cdot \dot{m}^3 2f \Delta x \left(\frac{4}{\pi}\right)^2 \frac{1}{\rho^2} \cdot \frac{1}{D^5} + CC \cdot \Delta x \cdot PP \cdot D \tag{6.47}$$

We now differentiate the total annual cost with respect to diameter D and set the derivative equal to zero:

$$0 = \frac{d(\text{cost})}{dD} = PC \cdot \dot{m}^3 2f \Delta x \left(\frac{4}{\pi}\right)^2 \frac{1}{\rho^2} \cdot \frac{-5}{D^6} + CC \cdot \Delta x \cdot PP \tag{6.48}$$

Solving for D_{econ}, we find

$$D_{econ} = \left[\frac{10 \cdot PC \cdot \dot{m}^3 f (4/\pi)^2 (1/\rho^2)}{CC \cdot PP}\right]^{1/6} \tag{6.49}$$

This equation shows that the economic pipe diameter is independent of how long the pipe is. This should be no surprise: Both the pumping and capital costs are proportional to the pipe length. The equation also shows that the economic diameter is proportional to the friction factor to the one-sixth power; hence, we can use a rough estimate of the friction factor and make very little error.

Example 6.17. We wish to transport 200 gal/min of water 5000 ft in a horizontal, schedule 40, carbon-steel pipe. We will install a pump to overcome the friction loss. Given the economic data shown below, what is the economic pipe diameter?

$$PC = \frac{\$270}{hp \cdot yr} \qquad PP = \frac{\$2}{\text{in of diameter} \cdot \text{ft of length}} \qquad CC = \frac{0.40}{yr}$$

First we guess that the pipe will have an inside diameter of 3 in. Then from Table 6.2 we have $\varepsilon/D = 0.0018/3 = 0.0006$. The friction factor will probably be about 0.0042. The mass flow rate is 200 gal/min · 8.33 lbm/gal = 1666 lbm/min. Substituting these and the values of PC, CC, and PP in Eq.

6.49 produces

$$D_{econ} = \left[\frac{\dfrac{\$270}{hp \cdot yr} \cdot \left(\dfrac{1666\,lbm}{min}\right)^3 \cdot 10 \cdot 0.0042 \cdot \left(\dfrac{4}{\pi}\right)^2 \cdot \left(\dfrac{ft^3}{62.3\,lbm}\right)^2}{0.4/yr \cdot \$2/(in \cdot ft)}\right]^{1/6}$$

$$\cdot \left(\frac{hp \cdot min}{3.3 \times 10^4\,ft \cdot lbf} \cdot \frac{lbf \cdot s^2}{32.2\,lbm \cdot ft} \cdot \frac{min^2}{3600\,s^2} \cdot \frac{ft}{12\,in}\right)^{1/6}$$

$$= (5.95 \times 10^{-4}\,ft^6)^{1/6} = 0.290\,ft = 3.48\,in = 0.088\,m$$

Because of the approximate nature of the economic data used, a 4-in pipe would probably be selected. It would be appropriate to check the assumed friction factor (Prob. 6.62). ∎

Because calculations such as these are long and tedious, companies that install many pipelines have solved the problem for a large number of cases and have summarized the results in convenient form. The most popular method is to calculate the economic velocity:

$$\text{Economic velocity} = \frac{\text{volumetric flow rate}}{(\pi/4)(\text{economic diameter})^2} \tag{6.50}$$

Substituting for the economic diameter from Eq. 6.49, we find

$$V_{econ} = \frac{\dot{m}/\rho}{\dot{m}(1/\rho^{2/3})f^{1/3} \cdot \text{constant}} = \text{constant} \cdot \frac{1}{f^{1/3}\rho^{1/3}} \tag{6.51}$$

This equation says that for a given set of cost data the economic velocity is independent of the mass flow handled and dependent on only the fluid density and the friction factor. More thorough analyses and far more complicated cost equations lead to substantially the same conclusion. For example, for schedule 40 carbon-steel pipe, Boucher and Alves [16] give the data shown in Table 6.4.

The table refers to turbulent flow only. For laminar flow, the value of f goes up quite rapidly as the viscosity increases, making the economic velocity go down. Oil companies spend more money pumping viscous liquids (crude oils, asphalt, heating oils, etc.) than do any other companies; therefore they have made up the most convenient economic-velocity plots for laminar flow.

TABLE 6.4
Economic velocity for schedule 40, carbon-steel pipe

Fluid density, lbm/ft	Economic velocity, ft/s
100	5.1
50	6.2
10	10.1
1	19.5
0.1	39.0
0.01	78.0

Figure 6.21 shows such a plot. It can be used to rapidly select the economic pipe diameter for laminar flow, subject to the restriction that the economic data on the line to be installed must be the same as those shown on the plot. Figure 6.21 has nomenclature similar to that of Fig. 6.13, and the comments on the latter are applicable here. Figure 6.21 also shows the economic diameter for turbulent flow.

Why does App. A.4 show the velocity in feet per second for all the water flows given? From Table 6.4 and Fig. 6.21 we can see that for water (which is almost always in turbulent flow in industrial equipment) an economic velocity is almost always about 6 ft/s. Thus, working engineers often simply select pipe sizes for water or similar fluids by looking at App. A.4 for the pipe size which gives a velocity of about 6 ft/s (2 m/s).

Table 6.4 and Fig. 6.21 are for one set of costs; for other costs the results are different. However, because of the $\frac{1}{6}$ factor in Eq. 6.49, the different costs change the economic diameter very little (see Prob. 6.66).

6.14 FLOW AROUND SUBMERGED OBJECTS

The flow around a submerged object is generally more complicated than the flow in a straight pipe or channel, because it is two- or three-dimensional. To understand the *details* of the flow around any submerged object, we must first take up the subjects of potential flow and the boundary layer, which we do in Chaps. 10 and 11.

Frequently we are not interested in the details of the flow but only in the practical problem of predicting the force on a body due to the flow of fluid around it. For example, the airplane designer wants to know the "air resistance" of the plane to select the right engine, the submarine designer wants to know the "water resistance" to determine how fast the submarine can go, and the designer of a chimney wants to know the maximum wind force on it to decide how much bracing is needed. These forces are now all called *drag forces*, following aeronautical engineering terminology. By using experimental data on such flows we can treat the problems as if they were one-dimensional.

Probably the first systematic investigation of drag forces was undertaken by Isaac Newton [17], who dropped hollow spheres from the inside of the dome of St. Paul's Cathedral in London and measured their rate of fall. He calculated that the drag force on a sphere should be given by

$$\text{Drag force} = F = \pi r^2 \rho_{\text{air}} \frac{V^2}{2} \qquad (6.52)$$

Subsequent workers found that this equation had to be modified by introducing a coefficient, which we call the *drag coefficient* C_d. This coefficient is not a constant equal to 1, as Newton believed, but varies with varying conditions, as we will see. Introducing it and dividing both sides of Eq. 6.52 by the cross-sectional area of the sphere, we find

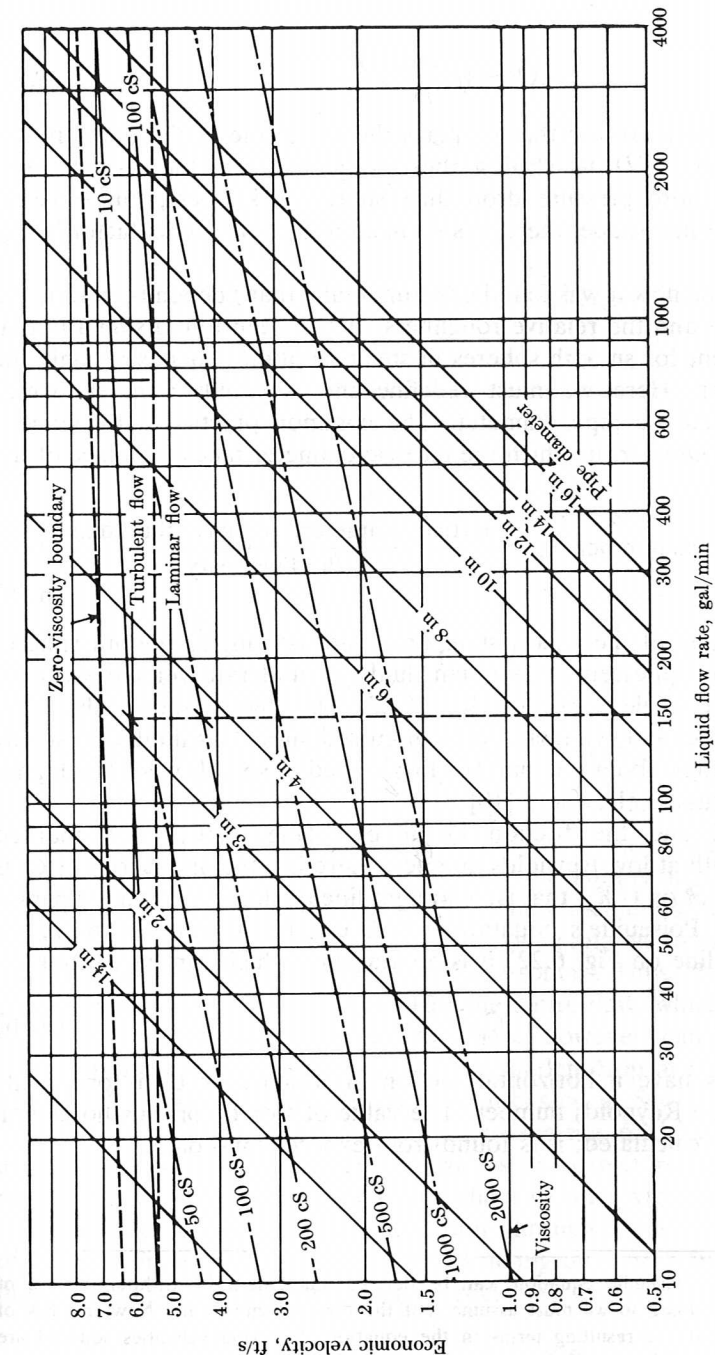$$\frac{F}{A} = C_d \rho \frac{V^2}{2} \qquad (6.53)$$

FIGURE 6.21
Economic pipe size for pumped liquids (carbon-steel pipe). Assumptions: Pumping cost = $135 per horsepower-year; line cost = $1 per inch of diameter per foot; fixed charges per year on line = 0.40 times line cost; liquid specific gravity = 0.80 (not very critical). Examples: for 150 gal/min and 200 cSt, use the 4-in line; for 150 gal/min and 10 cSt, use the 3-in line. (These prices are from the early 1960s. Current prices are higher, but they have risen more or less together, so that the economic pipe sizes have not changed much.) (Courtesy of the Board of Engineers, Standard Oil Company of California.)