Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 11

Radiation Interactions with Matter

Spiritual Thought

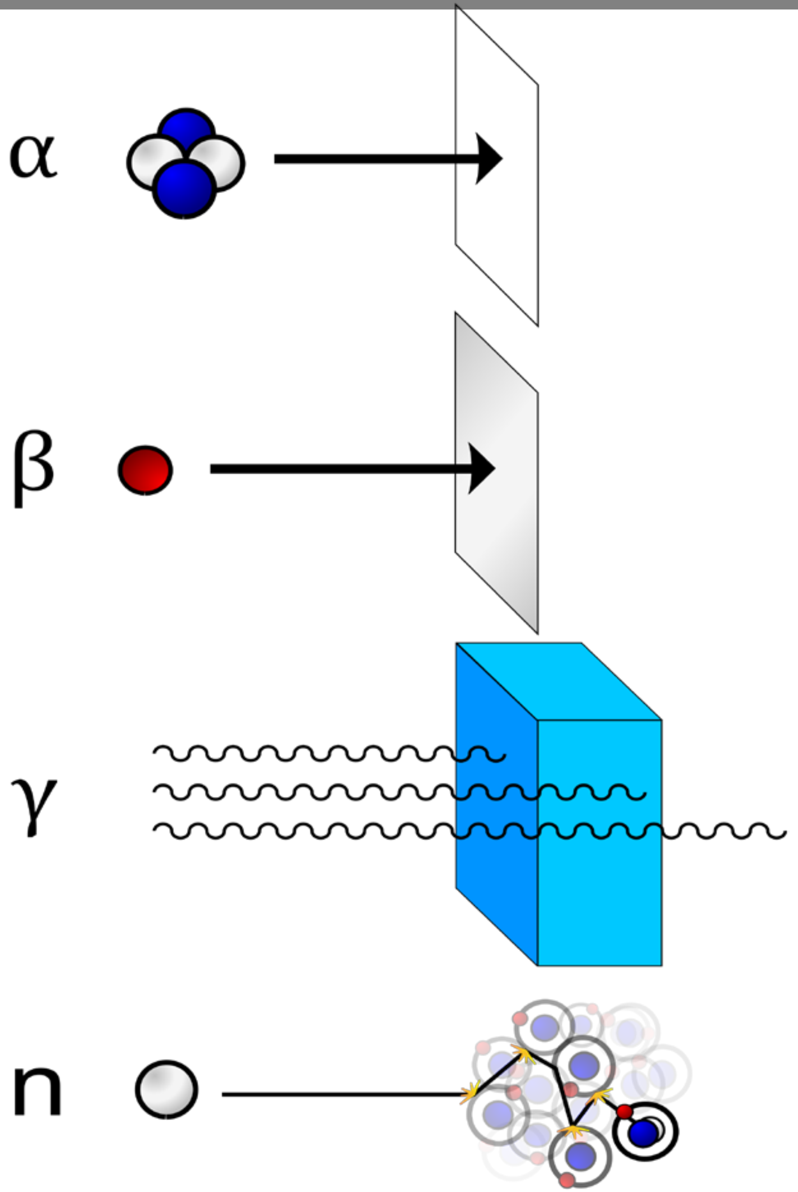
Alma 27:17-18

17 Now the joy of Ammon was so great even that he was full; yea, he was swallowed up in the joy of his God, even to the exhausting of his strength; and he fell again to the earth.

18 Now was not this exceeding joy? Behold, this is joy which none receiveth save it be the truly penitent and humble seeker of happiness.

Radiation Interaction with Matter

- World is awash with radiation
- First step to understanding impact is knowing how it interacts
- Different particles have different effects
- Derive general terms to quantify interactions



Linear Interaction Coefficient

- As a particle passes through a homogeneous material
  - Probability of interaction is constant per differential unit distance traveled
  - Empirically derived

$$\mu_i \equiv \lim_{\Delta x \to 0} \frac{P_i(\Delta x)}{\Delta x}$$

- μ_i is called the macroscopic interaction coefficient
- indicated by Σ_i (except for photons).
- Depends on
  - Particle energy
  - Reaction Type
    - Scattering, absorption, fission, etc.
      - energy-dependent macroscopic linear absorption coefficient
      - linear fission coefficient
      - linear scattering coefficient, etc.
  - Medium type

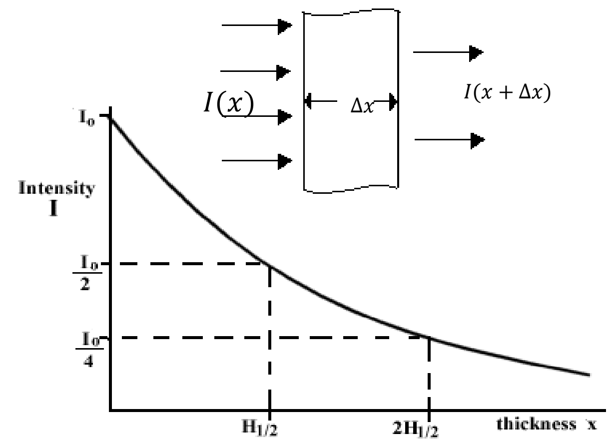
Total Probability of Interaction

- Interaction coefficients are divided into subcategories
  - i.e. total scattering coefficient, Σ_t
    - Linear scattering coefficients, Σ_s
    - Non-linear scattering coefficients
  - Total absorption coefficient, Σ_a
    - Neutron capture
    - Fission
    - Other absorbing interactions
- Total is sum of components
  - Radiation – linear attenuation coefficient
  - Neutrons – Cross Section
  - Photons – linear Interaction Coefficient

$$\mu_t(E) = \sum_i \mu_i(E)$$

Interaction in Material

The fractional amount of a beam that interacts in a differential slice of a material is given by



$$\frac{I(x) - I(x+\Delta x)}{I(x)} = P(x)$$

$$\mu_t \equiv \lim_{\Delta x \to 0} \frac{P_i(\Delta x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{I(x) - I(x+\Delta x)}{\Delta x\, I(x)} = -\frac{1}{I(x)} \frac{dI(x)}{dx}$$

$$\frac{dI(x)}{dx} = -\mu_i I(x) \Rightarrow I(x) = I(0)\exp(-\mu_i x)$$

Interaction Metrics

Interaction probability in distance x

$$P(x) = 1 - \frac{I(x)}{I(0)} = 1 - \exp(-\mu_i x)$$

Non-interaction probability in distance x

$$\bar{P}(x) = 1 - P(x) = \frac{I(x)}{I(0)} = \exp(-\mu_i x)$$

Average penetration distance until interaction, or mean-free-path length (assuming $\mu_i \neq \mu_i(x)$)

Prob. part gets to x

$$\bar{x} = \int_0^\infty x\, p(x) dx = \int_0^\infty x \left[\bar{P}(x) P(dx)\right] dx$$

Prob. part interacts in dx

$$= \int_0^\infty x \, \exp(-\mu_i x)\, \mu_i dx = \frac{\mu_i}{\mu_i^2} = \frac{1}{\mu_i}$$

Conceptual Interpretations

- The linear attenuation coefficient can be thought of in three ways:
  - Probability that a particle interacts in a differential length of material (does not assume constant $\boldsymbol{\mu_i}$)
  - Inverse of the mean free path of a particle (assumes constant $\boldsymbol{\mu_i}$).
  - Related to distance at which half of particles have interacted ($\boldsymbol{x_{1/2,\,i} = \frac{\ln 2}{\mu_i}}$) (assumes constant $\boldsymbol{\mu_i}$)
- Analogous to decay constants
  - Decay probabilities
  - Average lifetimes
  - Half lives.

Non-absorbing Particles

- In many cases (scattering, photons, etc.), interactions do not eliminate the particles
- The total amount of particles
  - Highly complex, calculated with large computations
  - Derive a buildup factor, B(x), that correlates complex behavior with simple expression

$$I(x) = B(x)I(0)\exp(-\mu_i x)$$

- This is especially common in calculating dose (as opposed to total particles).

Microscopic Cross Section

- Probability of interaction is proportional to the concentration of interaction sites/atoms

$$\mu_i = \sum_i = N\sigma_i = \sigma_i \frac{\rho N_a}{A}$$

- σ_i = microscopic cross section, has units of L^2
- $\boldsymbol{N}$ = Number/atom density
- $\boldsymbol{\rho} =$ Mass density
- $\boldsymbol{N_a}$ = Avagadro's number
- $\boldsymbol{A}$ = Atomic mass of the medium

Example

- What is the power generation in a 1cm3 section of U^{235} fuel, assuming a thermal neutron flux of $1x10^{22}$ neutrons/cm^2-s?

Microscopic cross section

- The microscopic cross section
  - Independent of atomic density
  - Based strongly and complexly on particle kinetic energy
  - Play vital roles in nuclear engineering
- Behaviors are empirical!
  - (can be conceptually explained but not always quantitatively predicted by theoretical means)
- Typical unit is barns (1 barn = 1×10^{-24} cm^2)
- 1 barn is approximate physical cross section of a uranium nucleus.

Mass Interaction Coefficient

- Photons – mass interaction coefficient
  - Interaction coefficient (macroscopic) divided by density
  - which depends only weakly on the properties of the medium (for photons)

$$\frac{\mu_i}{\rho} = \frac{\sigma_i N}{\rho} = \frac{N_a}{A}\sigma_i$$

- Homogeneous mixture properties can be determined from

$$\mu_i = \sum_j \mu_{i,j} = \sum_j N_j \sigma_{i,j} \qquad \frac{\mu_i}{\rho} = \sum_j w_j \left(\frac{\mu_i}{\rho}\right)_j$$

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Cross sections for each interaction

$$\sigma_t = \sigma_e + \sigma_i + \sigma_\gamma + \sigma_f + \ldots$$

total cross section

$$\sigma_a = \sigma_\gamma + \sigma_f + \sigma_\alpha + \sigma_p + \ldots$$

absorption cross section

$$\sigma_s = \sigma_e + \sigma_i$$

scattering cross section

$$\sigma_t = \sigma_s + \sigma_a$$

total cross section

t = total
e = elastic scattering
i = inelastic scattering
γ = radiative capture
f = fission
α= alpha (charged) particle
p = proton (charged) particle

Linear Coefficient

$$\mu_i = \sigma_i N = \frac{\sigma_i \rho N_a}{A} = \Sigma_i$$

$$I^0(x) = I^0(0)\exp(-\mu_i x)$$

$$P(x) = 1 - \exp(-\mu_i x)$$

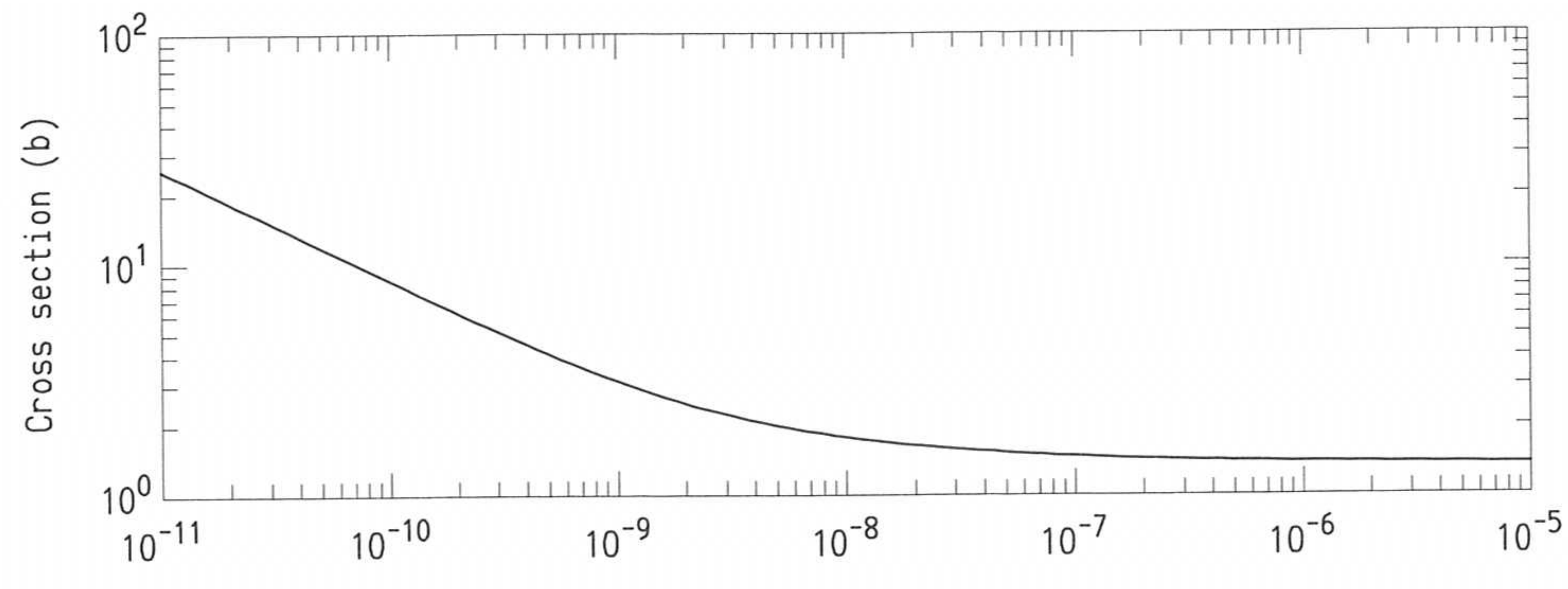
$$\bar{x} = \mu_t \int_0^\infty x \exp(-\mu_i x)\, dx = \frac{1}{\mu_t}, x_{\frac{1}{2}} = \frac{\ln(2)}{\Sigma_t}$$

- μ_i = generic linear interaction
- Σ_i, = macroscopic cross section – neutron interactions
- σ_i = microscopic cross section - isotopic property
    - tabulated as a function of neutron energy.

Cross Section Trends

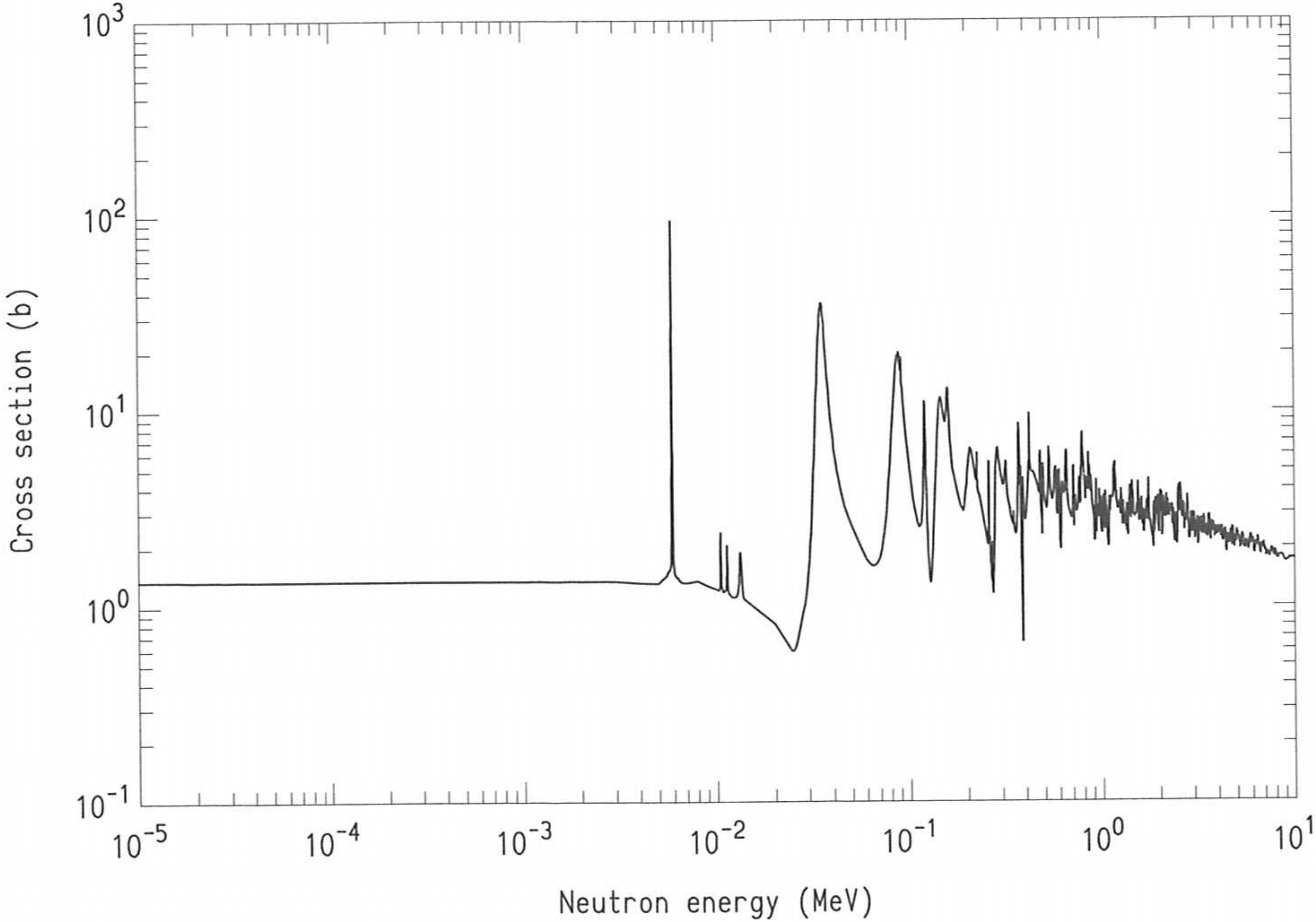
- Most Isotopes
  - Cross sections rise as neutron energy decreases.
  - Resonance regions with narrow and rapidly varying interactions that eventually are not resolvable
- Light isotopes (A < 25)
- Heavy isotopes (A > 150)
- Intermediate

Al Total Neutron Cross Section

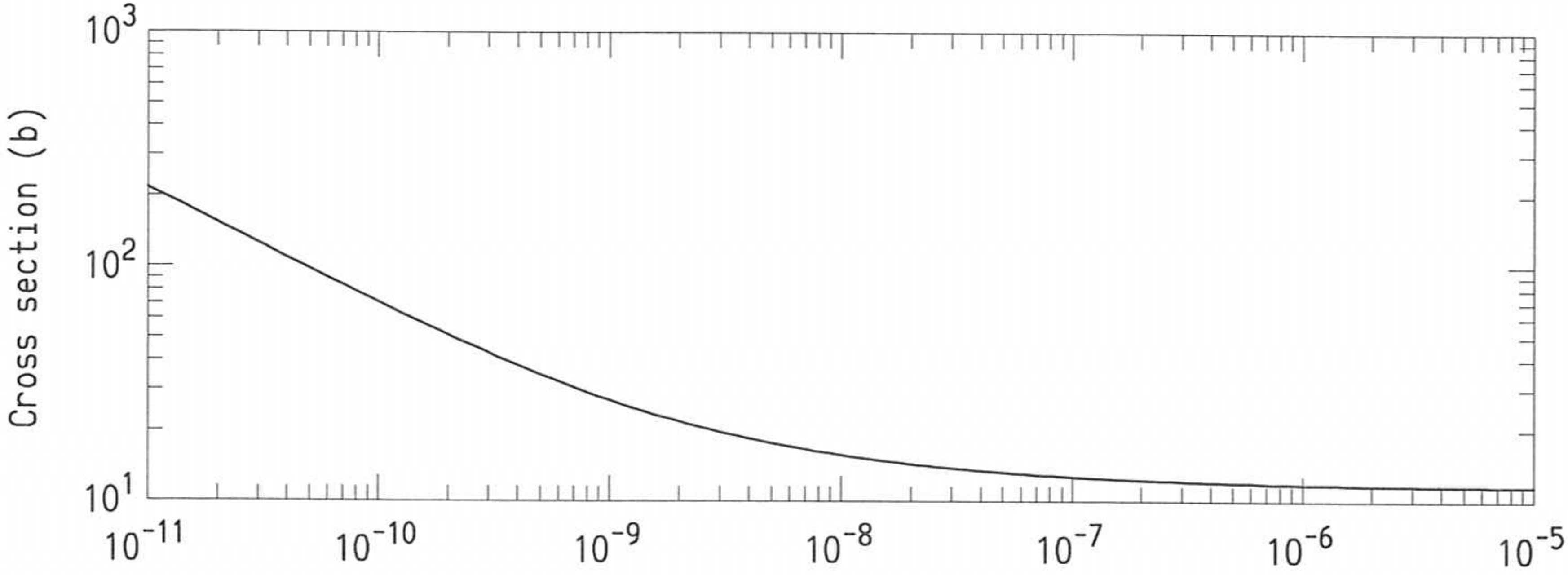


Neutron Energy (MeV)

Al Total Neutron Cross Section

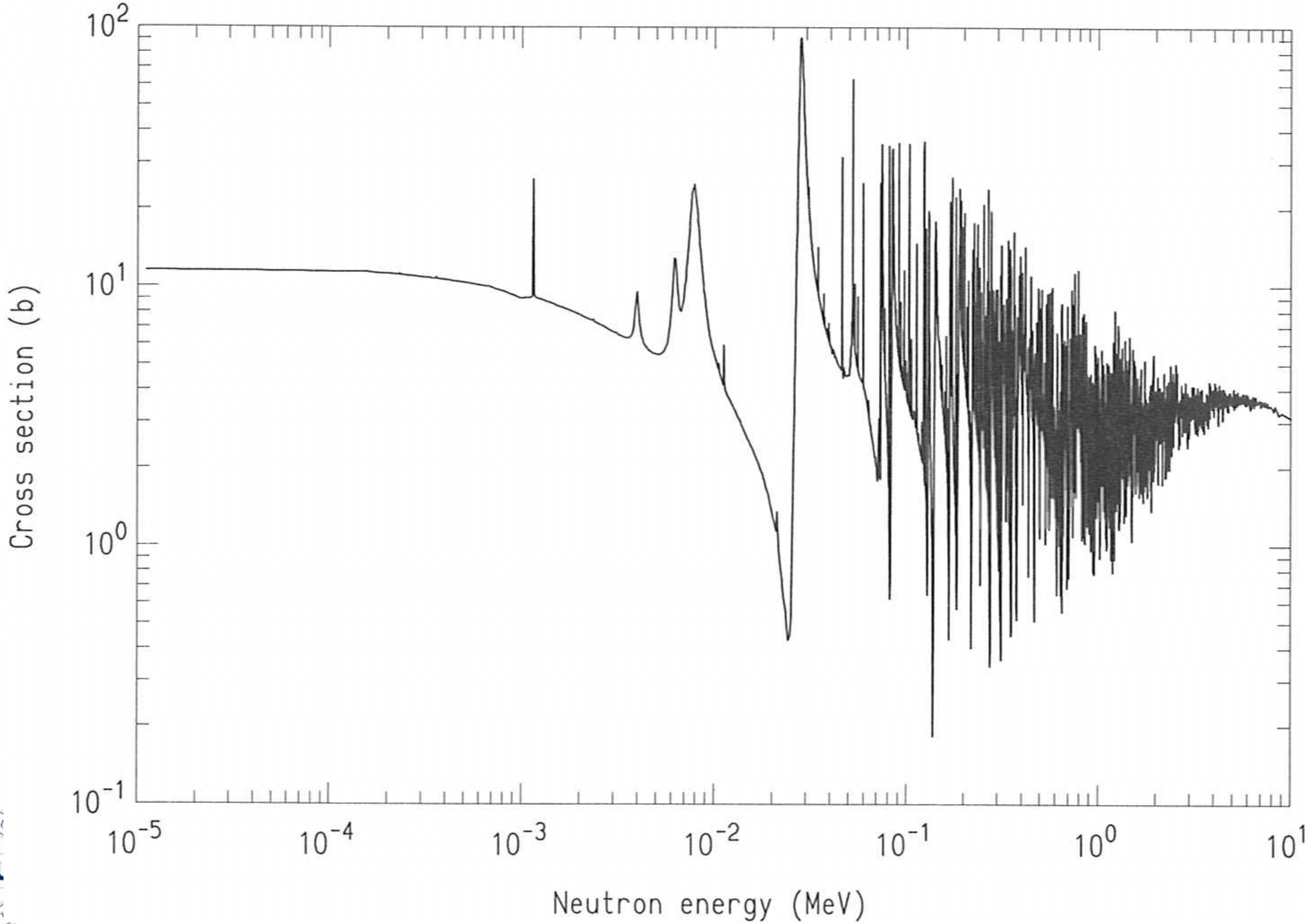


Fe total neutron cross section

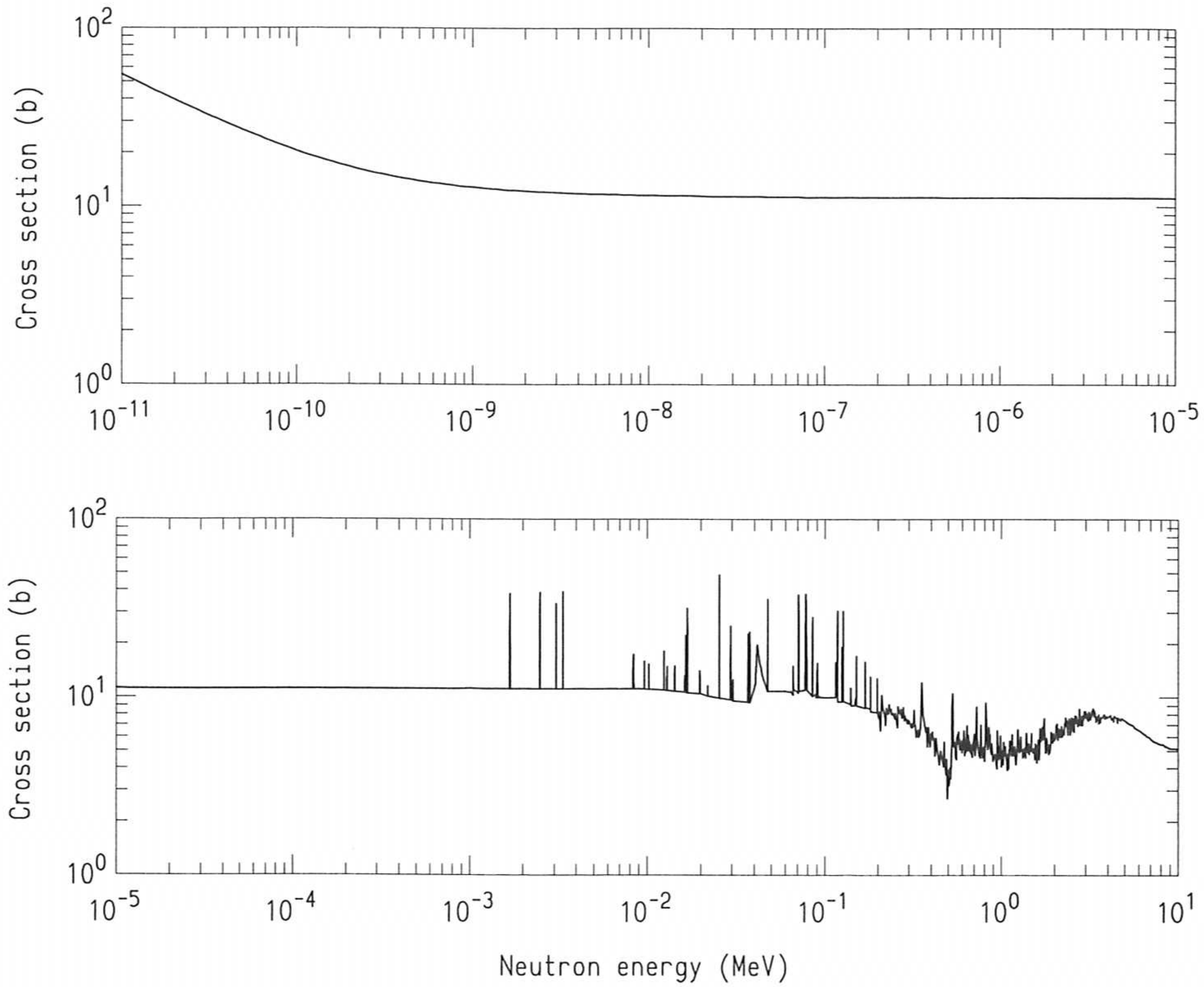


Neutron Energy (MeV)

Fe total neutron cross section

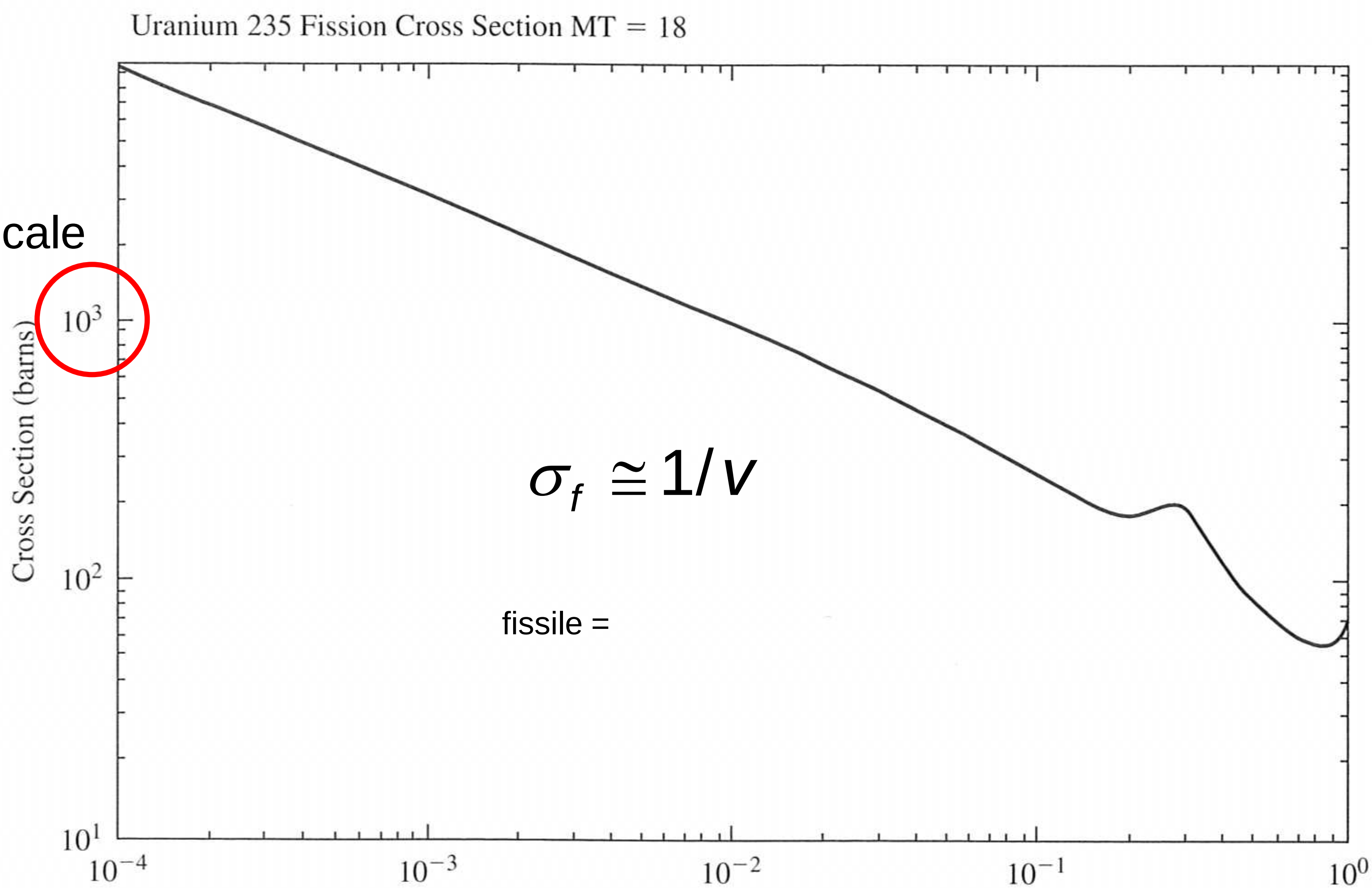


Lead Total Neutron Cross Section



Fission Cross Sections ^{235}U

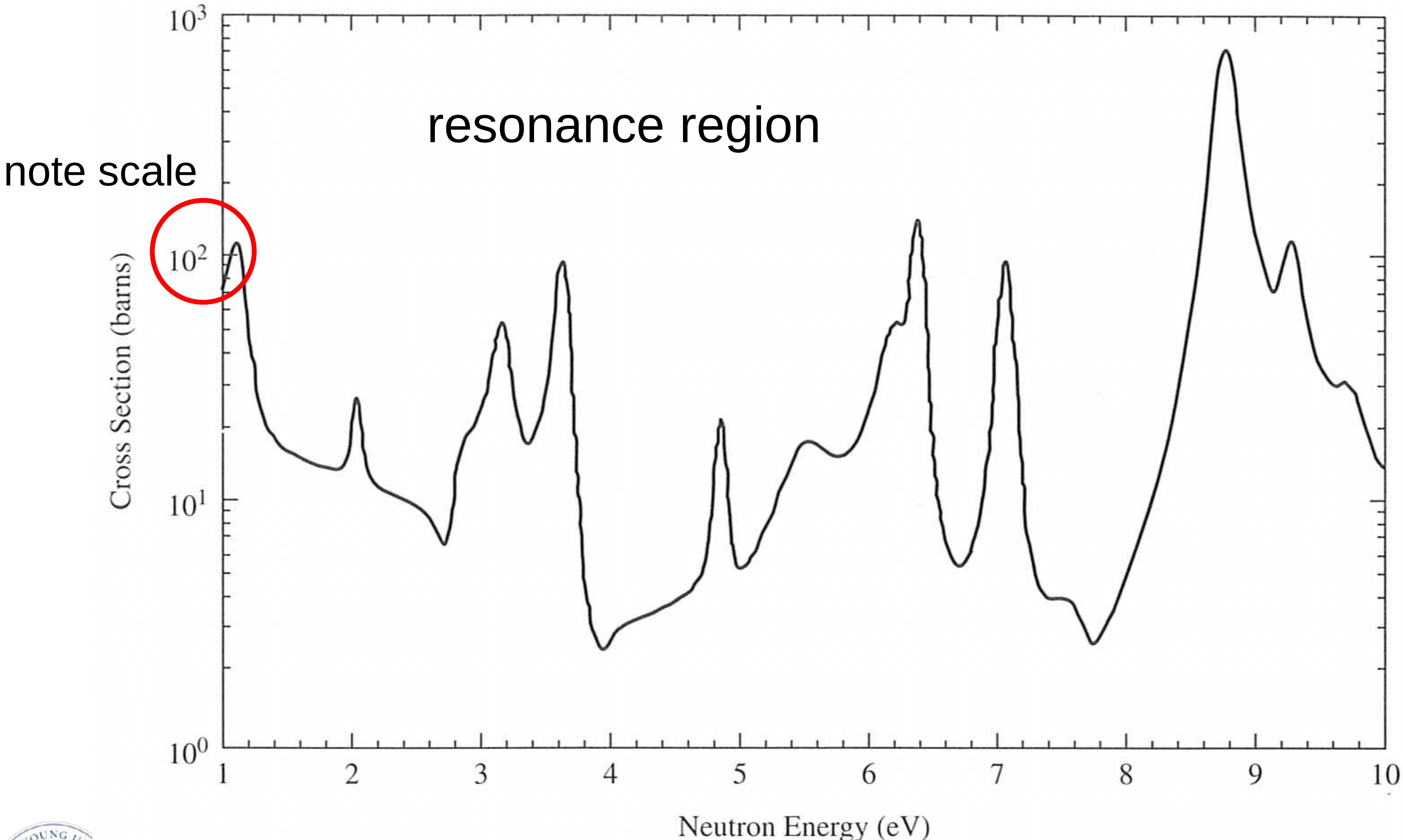
note scale



$\sigma_f \cong 1/v$

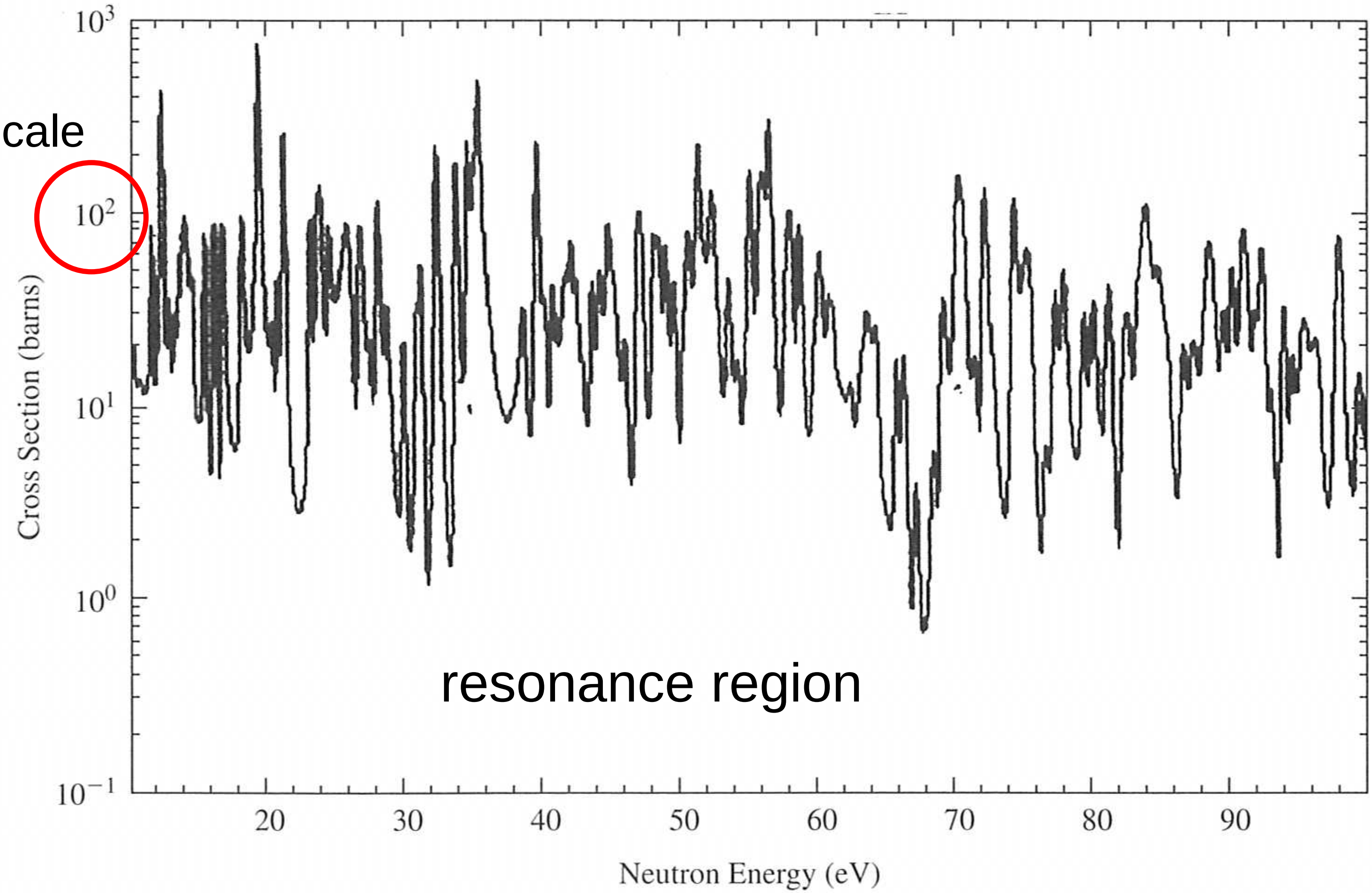
fissile =

Fission Cross Sections ^{235}U



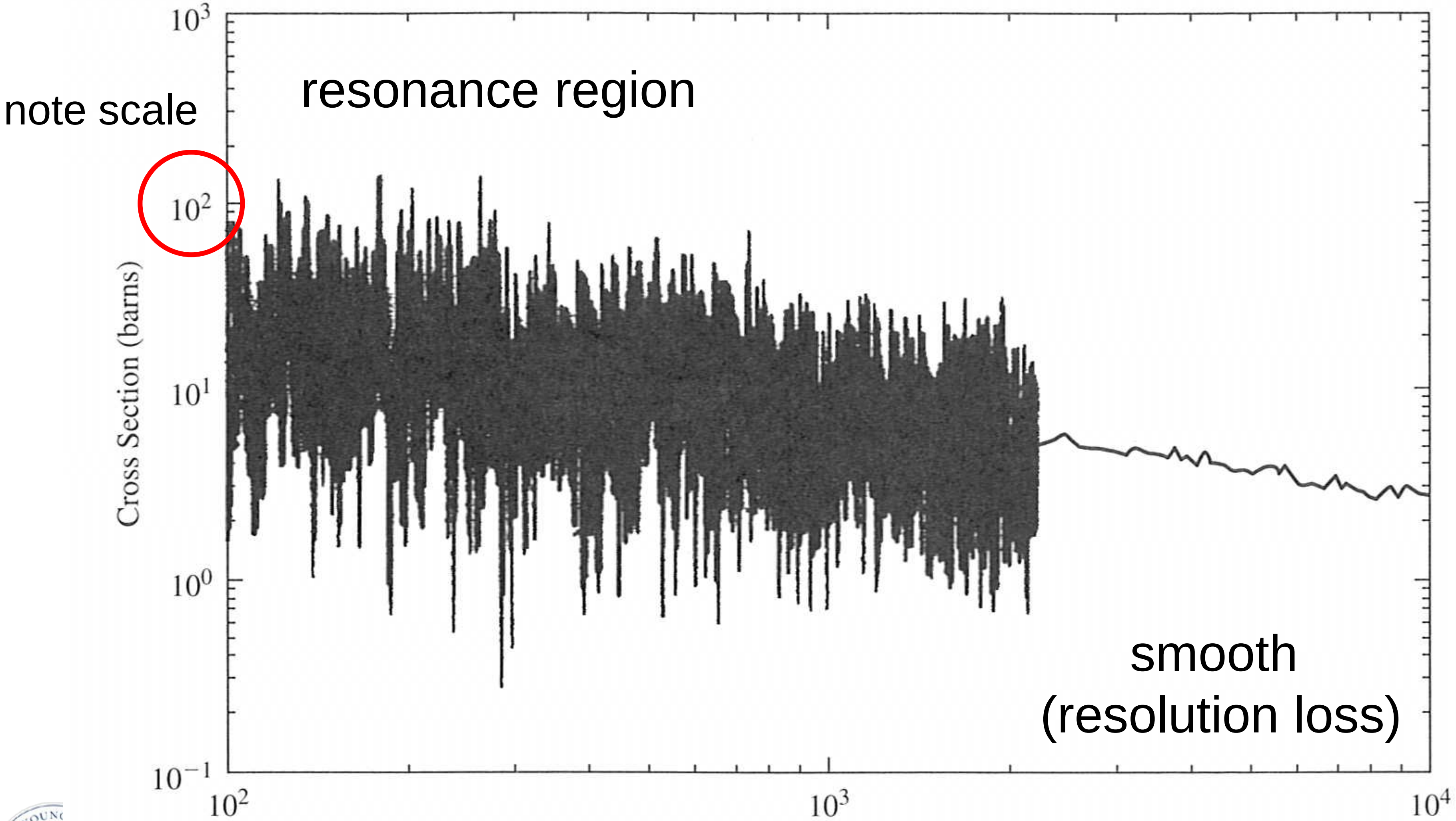
Fission Cross Sections ^{235}U

note scale

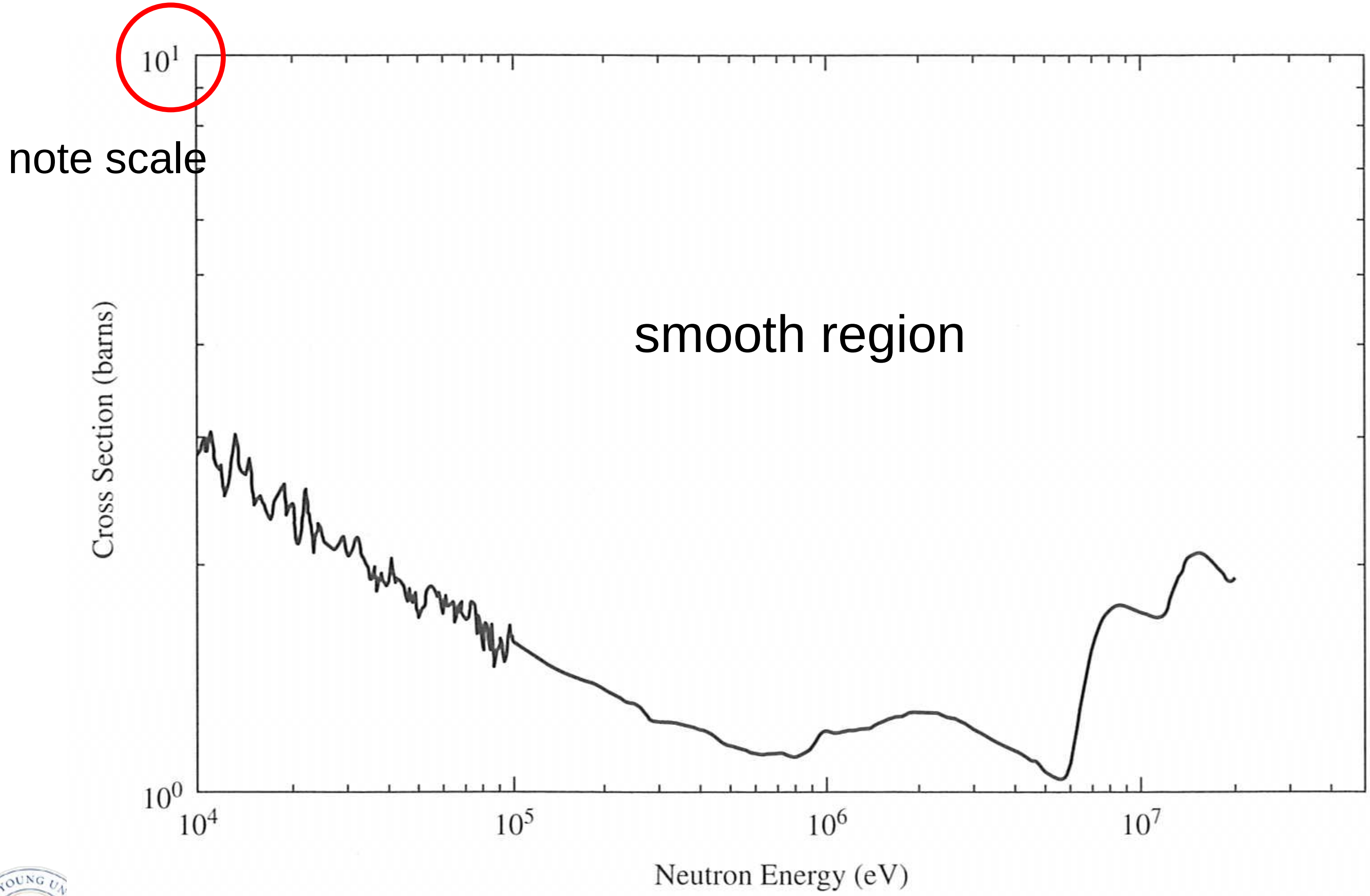


resonance region

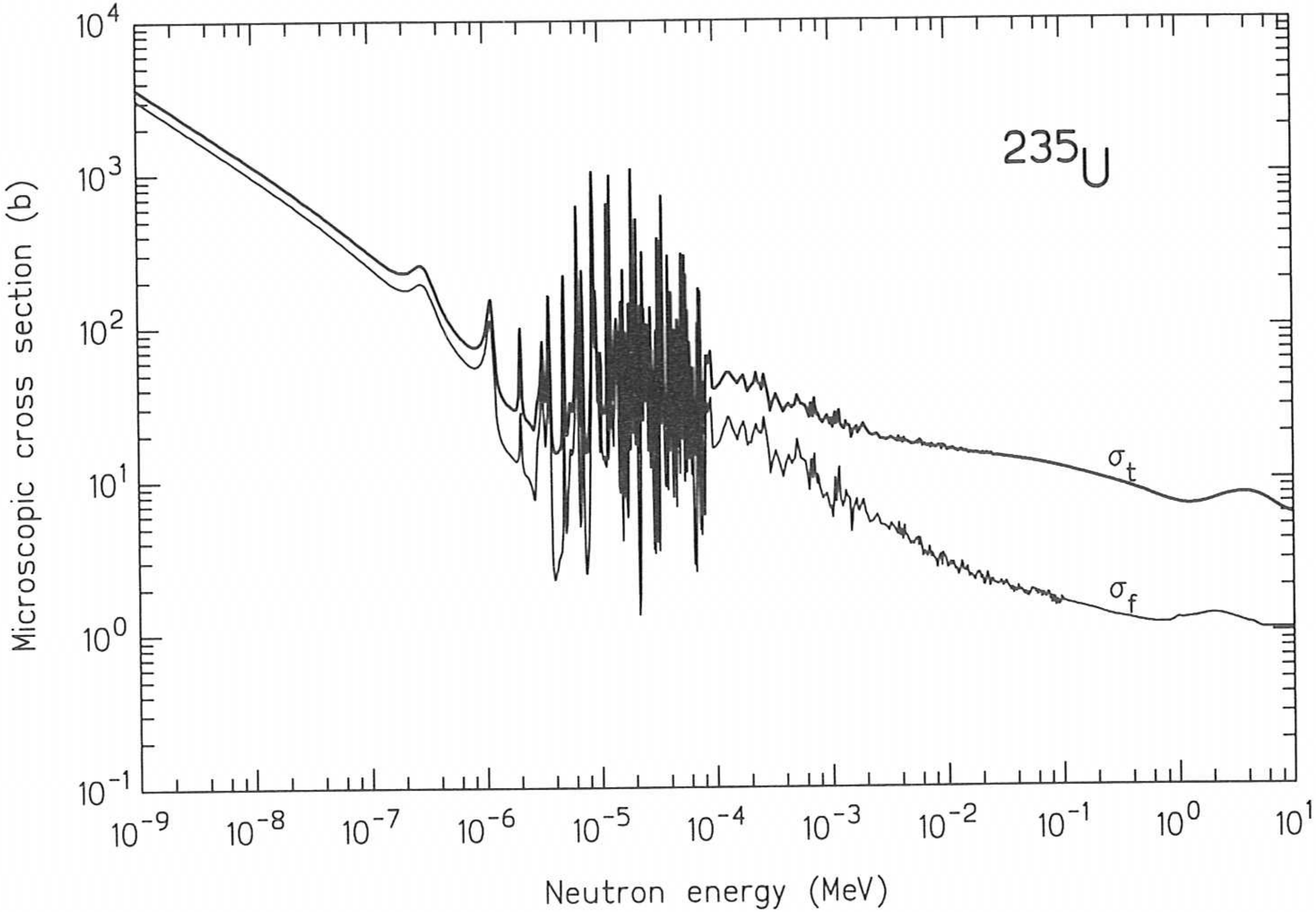
Fission Cross Sections ^{235}U



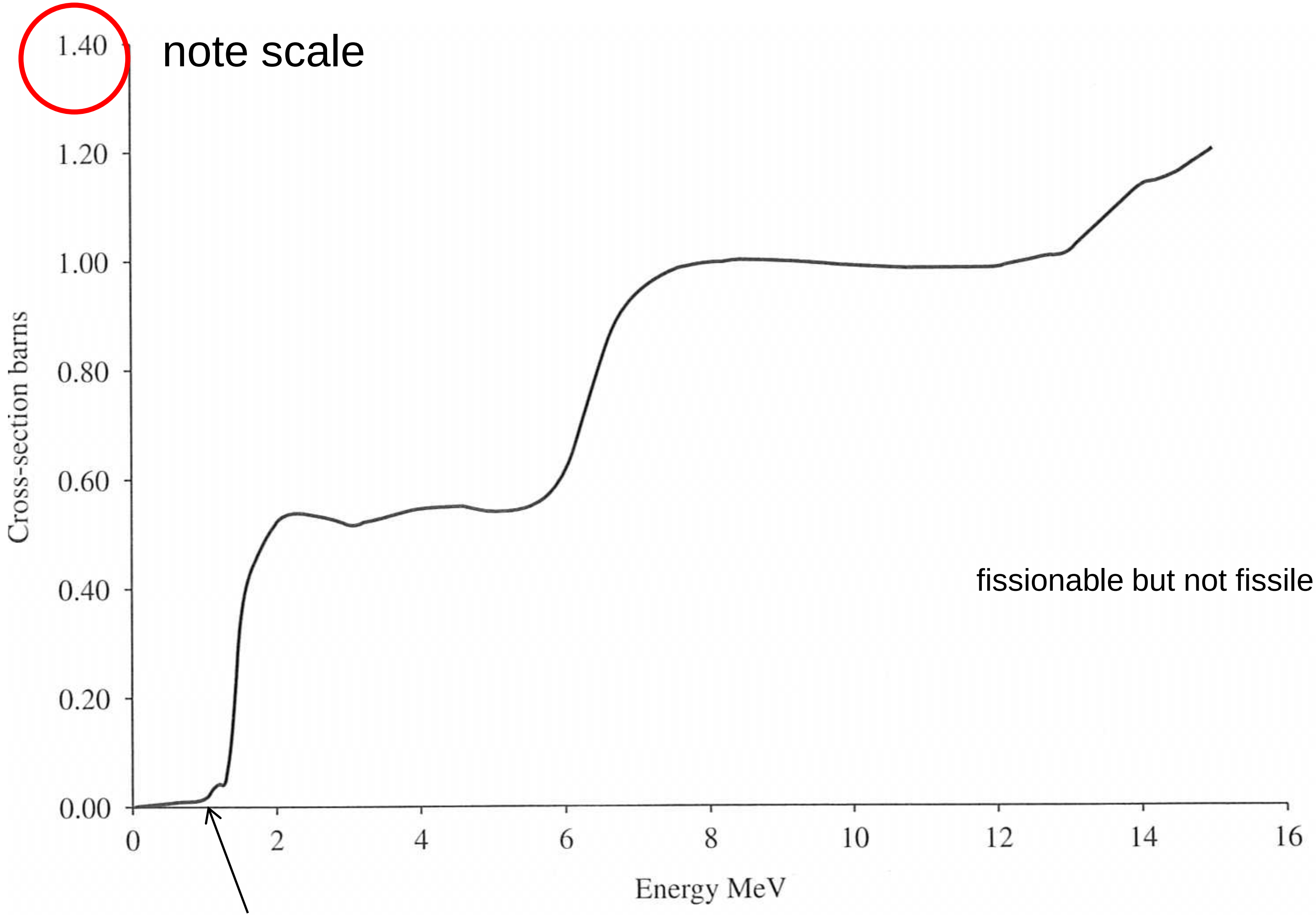
Fission Cross Sections ^{235}U



Cross section over entire range



Fission Cross Section of ^{238}U



threshold energy (> resonance region energy)

Fissionable Cross Sections