Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 12

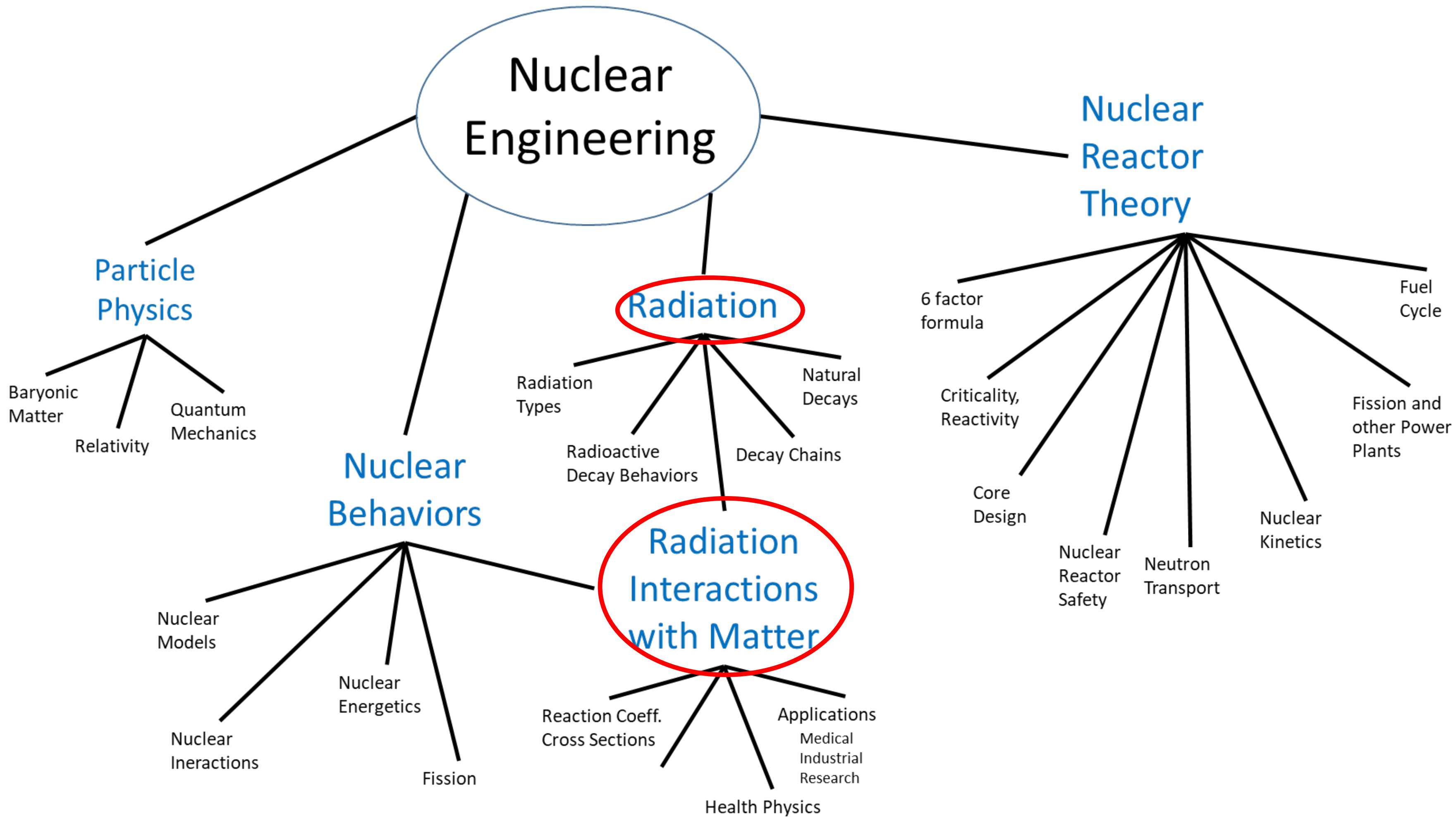
Radiation/Matter Interactions II

Spiritual Thought

“Sadness, disappointment, and severe challenge are *events* in life, not life itself. I do not minimize how hard some of these events are. They can extend over a long period of time, but they should not be allowed to become the confining center of everything you do. The Lord inspired Lehi to declare the fundamental truth, ‘Men are, that they might have joy.’ That is a conditional statement: ‘they *might* have joy.’ It is not conditional for the Lord. His intent is that each of us finds joy. It will not be conditional for you as you obey the commandments, have faith in the Master, and do the things that are necessary to have joy here on earth.”

Elder Richard G. Scott

The BIG Picture



Summary

- Neutron Flux
- ***Reaction Rate***
- Flux Attenuation
- *Photon Interactions*
  - Photoelectric
  - Compton
  - Pair Production
- Range/Stopping Power

Neutron Flux

Number of collisions per unit volume is

$$F(E) = \Sigma_t I = \Sigma_t N_p v$$

Neutron flux density is centrally important

$$\boxed{\phi(\vec{r}) = N_p(\vec{r})v}$$

- ϕ has units of flux (neutrons/area/time)
- Called a flux
  - Scalar, not a vector!
  - Varies significantly with neutron energy.

Neutron Flux

The collisions of neutrons of all energies is given by

$$F = \int_0^\infty \Sigma_i\,(E)\phi(E)dE$$

All volumetric reactions (fissions, scattering, absorption, etc.) are proportional to ϕ

$$\boxed{\hat{R}_i(\vec{r}) = \Sigma_i \phi(\vec{r})}$$

For reactions other than with neutrons, the same equation applies, with μ_i replacing Σ_i

Showing All Dependencies

Flux generally depends on energy, time, and position

$$\phi(\vec{r},E,t) = N_p(\vec{r},E,t)v(E)$$

Volumetric reaction has same dependencies

$$\hat{R}(\vec{r},E,t) = \mu_i(\vec{r},E,t)\phi(\vec{r},E,t) = \Sigma_i(\vec{r},E,t)\phi(\vec{r},E,t)$$

Commonly, neutrons are divided into energy groups by energy, so that within each group:

$$\boxed{\hat{R}(\vec{r},E,t)dE = \Sigma_i(\vec{r},E,t)\phi(\vec{r},E,t)dE}$$

Example

- What is the power generation in a 1cm^3 section of U^{235} fuel, assuming a thermal neutron flux of $1x10^{22}$ neutrons/cm^2-s?

Fissile Nuclide Thermal Data

$$\alpha = \frac{\sigma_\gamma}{\sigma_f}$$

capture-to-fission ratio

$$\nu$$

neutrons per fission

$$\eta = \nu \frac{\sigma_f}{\sigma_a} = \nu \frac{\sigma_f}{\sigma_\gamma + \sigma_f} = \frac{\nu}{1+\alpha}$$

neutrons per absorption

TABLE 3.4 THERMAL (0.0253 e V) DATA FOR THE FISSILE NUCLIDES*

	σ_a†	σ_f	α	η	ν
^{233}U	578.8	531.1	0.0899	2.287	2.492
^{235}U	680.8	582.2	0.169	2.068	2.418
^{239}Pu	1011.3	742.5	0.362	2.108	2.871
^{241}Pu	1377	1009	0.365	2.145	2.917

*From *Neutron Cross-Sections*, Brookhaven National Laboratory report BNL-325, 3rd ed., 1973.

†$\sigma_a = \sigma_\gamma + \sigma_f$.

Integral values

Fissions over a defined time interval

$$\mathcal{F}(\Delta t) = \int_{t_1}^{t_2} \iiint_V \int_0^{\infty} \Sigma_i(\vec{r}, E, t)\phi(\vec{r}, E, t)\, dE dV dt$$

For time-independent properties (common apprx.)

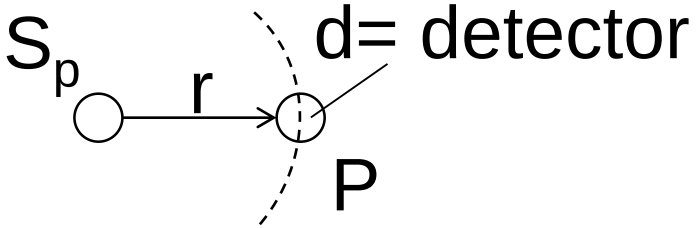
$$\mathcal{F}(\Delta t) = \iiint_V \int_0^{\infty} \Sigma_i(\vec{r}, E)\Phi(\vec{r}, E)\, dE dV$$

Where the fluence, $\Phi(\vec{r}, E, t)$

$$\Phi(\vec{r}, E) = \int_{t_1}^{t_2} \phi(\vec{r}, E, t) dt \approx \Delta t \phi(\vec{r}, E, t) dt$$

Uncollided Flux

In a vacuum

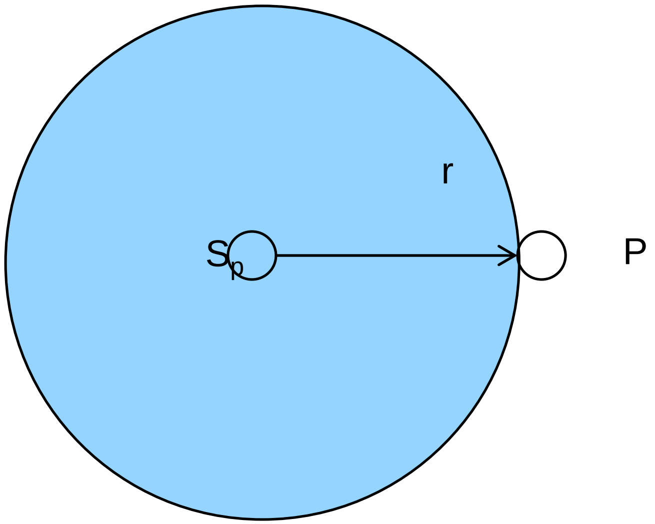


$$\phi^0(r) = \frac{S_p}{4\pi r^2}$$

$$R^0(r) = \Delta V_d \hat{R}_i(\vec{r}) = \Delta V_d \mu_d \phi^0(\vec{r}) = \frac{\Delta V_d \mu_d S_p}{4\pi r^2}$$

Uncollided Flux

Through homogeneous medium



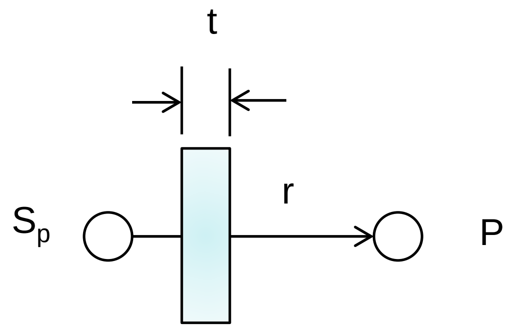
$$\phi^0(r) = \frac{S_p}{4\pi r^2}\exp(-\mu r)$$

$$R^0(r) = \frac{\Delta V_d \mu_d S_p}{4\pi r^2}\exp(-\mu r)$$

Uncollided Flux

Through homogeneous slab

$$\phi^0(r) = \frac{S_p}{4\pi r^2}\exp(-\mu t)$$



$$R^0(r) = \frac{\Delta V_d \mu_d S_p}{4\pi r^2}\exp(-\mu t)$$

$$= \frac{\Delta V_d \mu_d S_p}{4\pi r^2}\exp\left(-\sum_i \mu_i\ t_i\right)$$

Quantitative Treatment

Gamma cross sections

$$\sigma_\gamma = \sigma_{pe} + \sigma_{pp} + \sigma_C = \sigma_{pe} + \sigma_{pp} + Z_e \sigma_C$$

Attenuation coefficients

$$\mu = N\sigma_\gamma = \mu_{pe} + \mu_{pp} + \mu_C$$

Mass attenuation coefficients

$$\mu^* = \frac{\mu}{\rho} = \frac{\mu_{pe}}{\rho} + \frac{\mu_{pp}}{\rho} + \frac{\mu_C}{\rho}$$

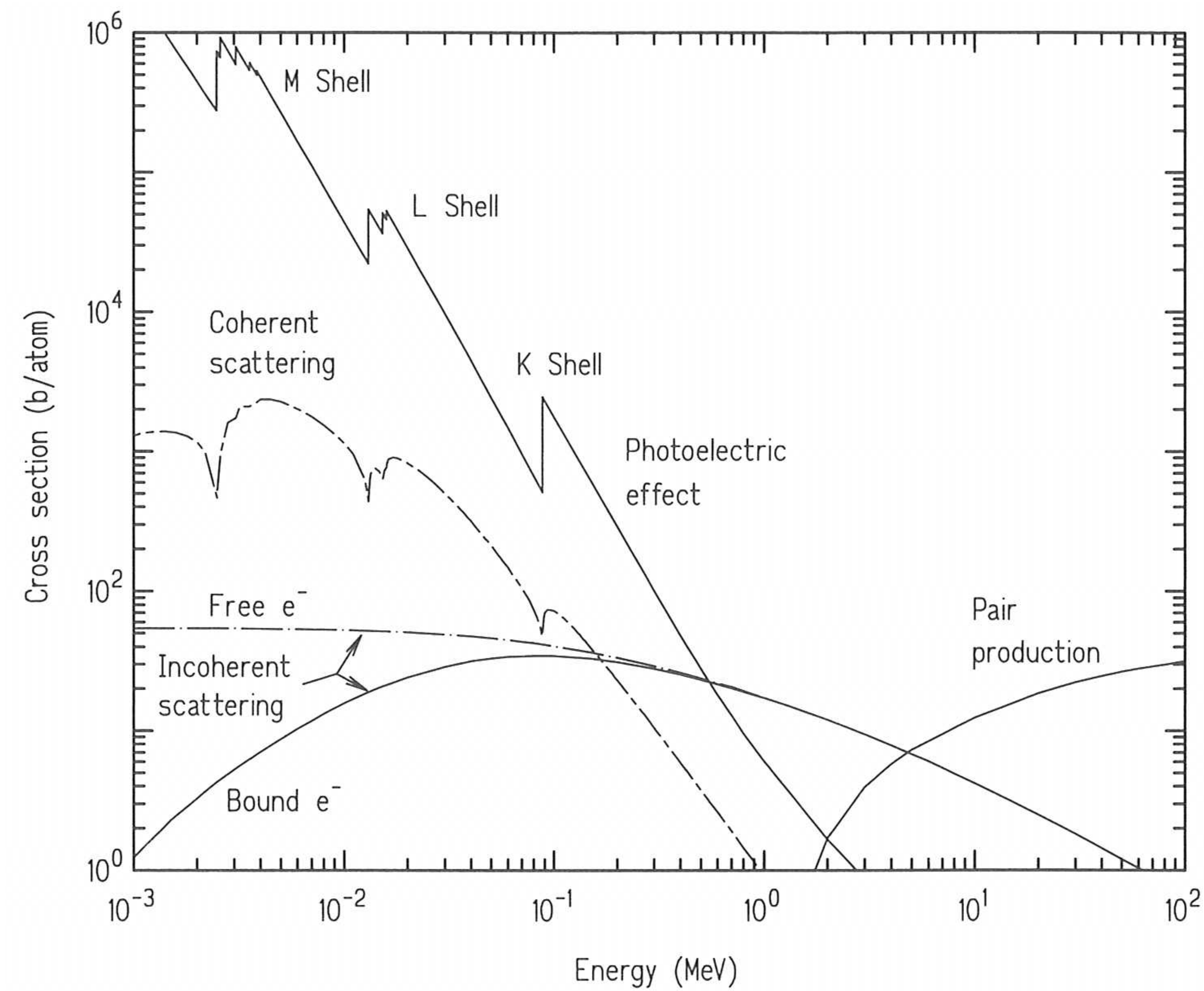
Photon (γ-ray) Interactions

- Photoelectric effect
  - Generally decreasing absorption with increasing energy
  - Indicative of elemental/nuclear structure.
  - γ-ray absorbed
  - Low energy products
- Pair Production
  - Increasing absorption with increasing energy
  - Depends on Z (happens in Coulomb field near nucleus)
  - γ-ray absorbed
  - Low energy products
- Compton Effect
  - Generally decreasing absorption with increasing energy
  - Depends on electron structure and hence Z
  - γ-ray remitted
  - Small energy change

Photoelectric effect

- Photoelectric effect
  - γ-ray is absorbed
  - depends on γ-ray energy and Z
  - electron ejection, generally from the K, L, or M levels
  - X-ray or Auger electron emission follows
  - electron and x-ray/Auger emission generally low energy compared to γ-ray
  - cross section designated as σ_{pe} with interactions/volume given by $IN\sigma_{pe}$ as usual
  - Cross section depends on Z^n where n is about 4, as on following graph (2^{nd} from here)

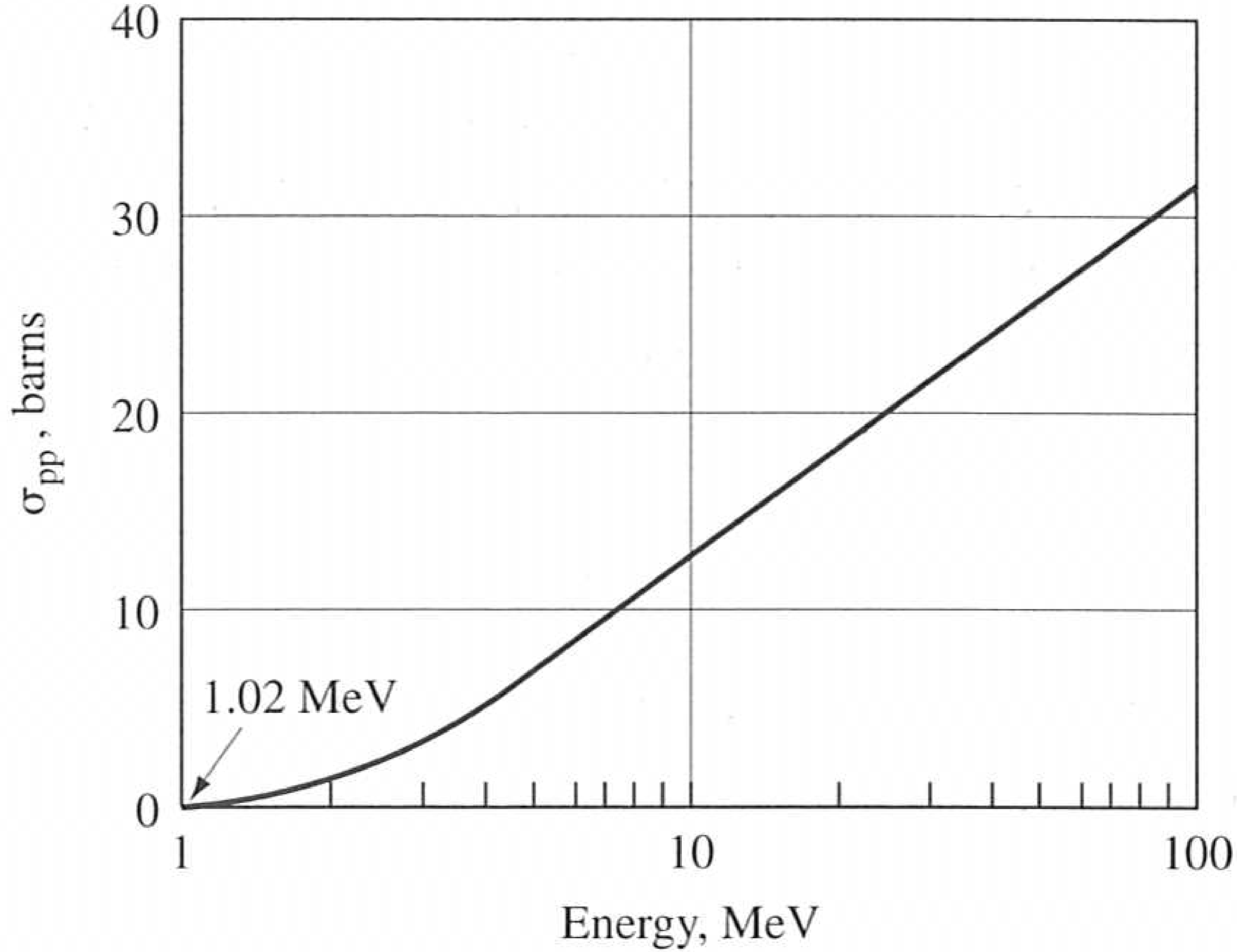
Photon Interactions in Lead



Pair Production

- Creates negatron (electron) and positron from the photon/ γ-ray
- Minimum energy threshold of 1.02 MeV
  $E_{min} = 2m_e c^2$
- Product particles lose kinetic energy (thermally) prior to recombining in annihilation radiation
- Cross section approximately proportional to Z^2

Pair Production



Compton Effect

- Elastic scattering of photon by an electron
- Scattered photon has nearly same energy as initial photon (same except for rebound energy of electron)
- In terms of energy

$$E' = \frac{EE_e}{E\left(1 - \cos\vartheta\right) + E_e}$$

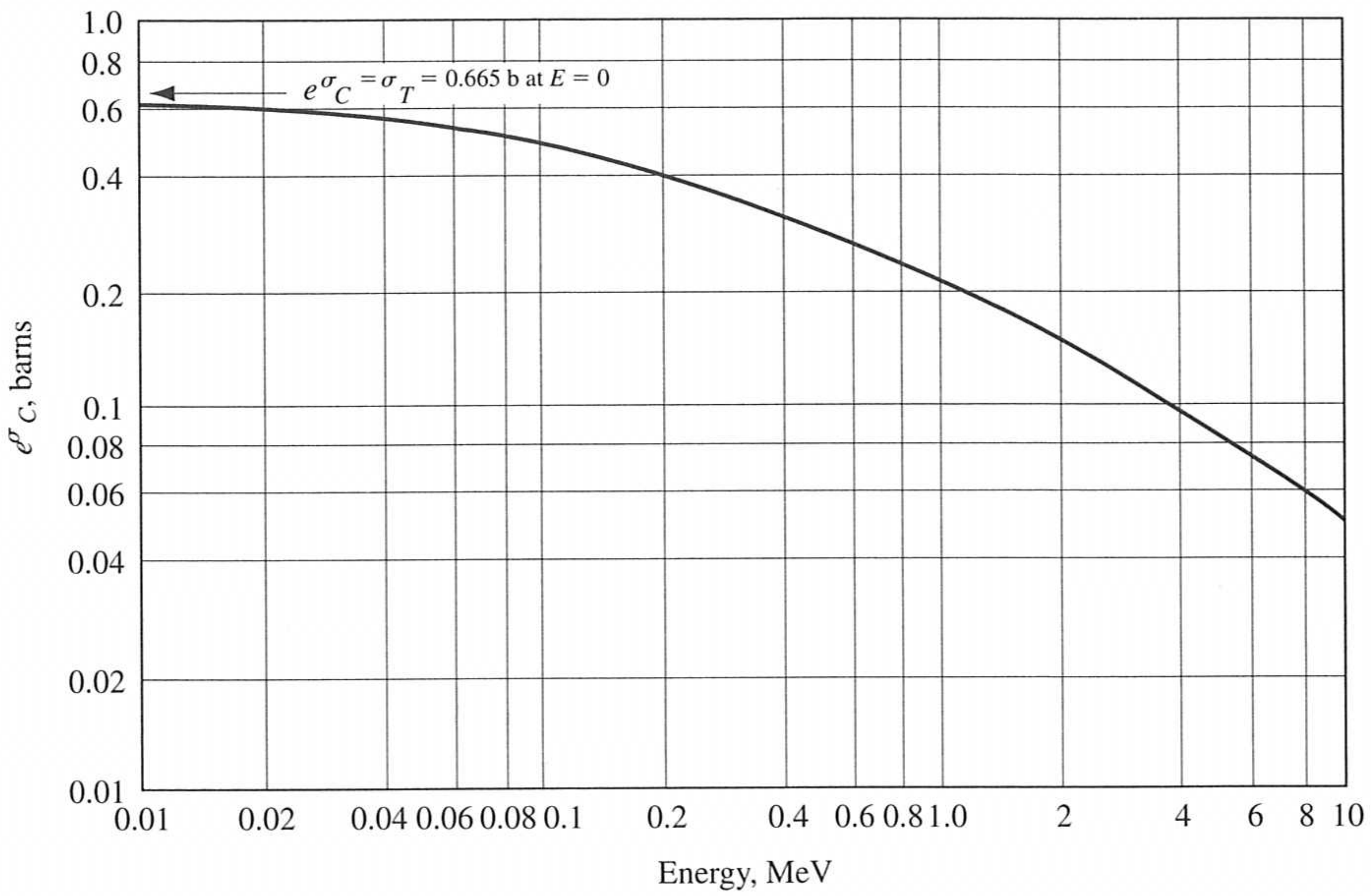
- In wavelength

$$\lambda' - \lambda = \lambda_C\left(1 - \cos\vartheta\right)$$

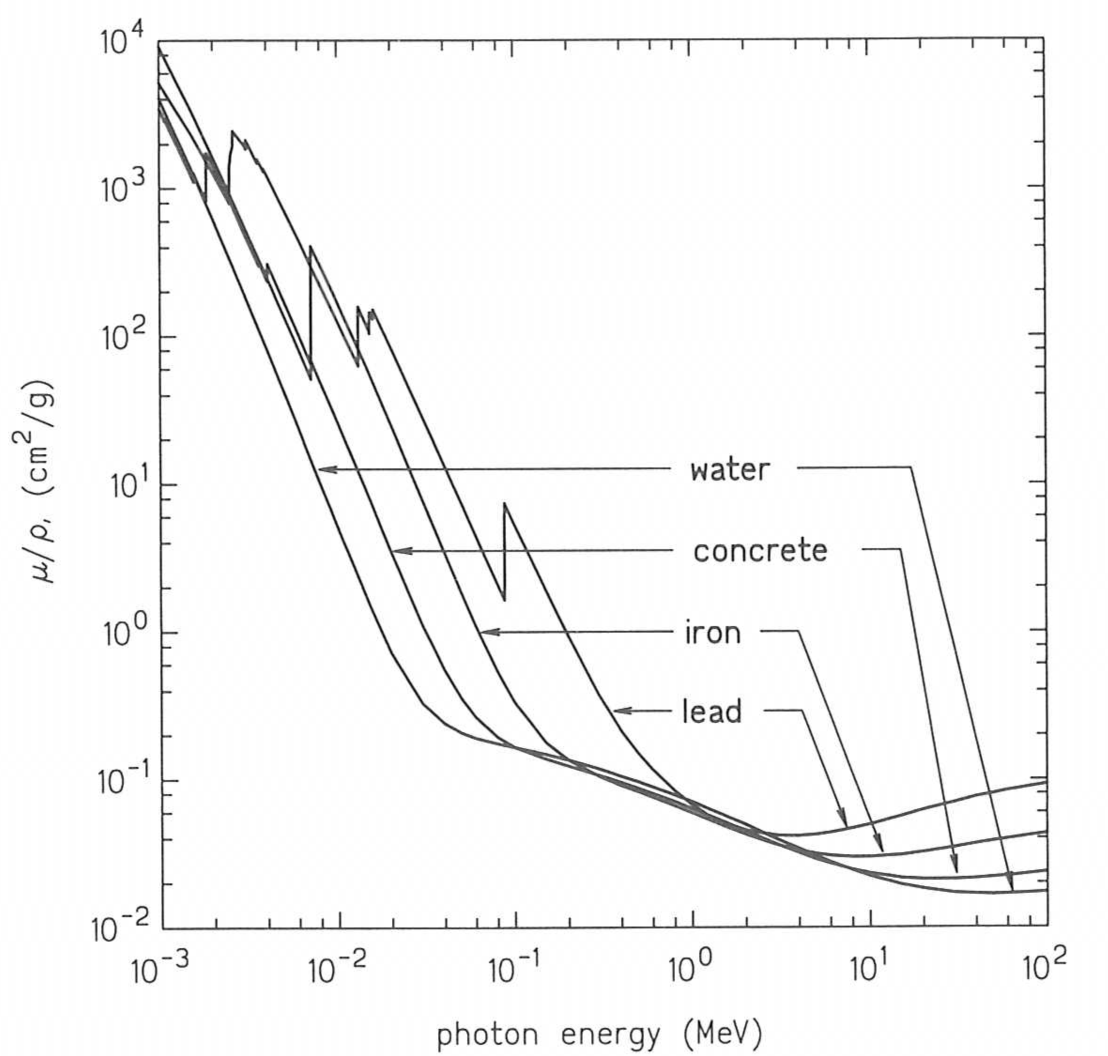
$$\lambda_C = \frac{h}{m_e c} = 2.426x10^{-10}cm$$

- Serious shielding problem

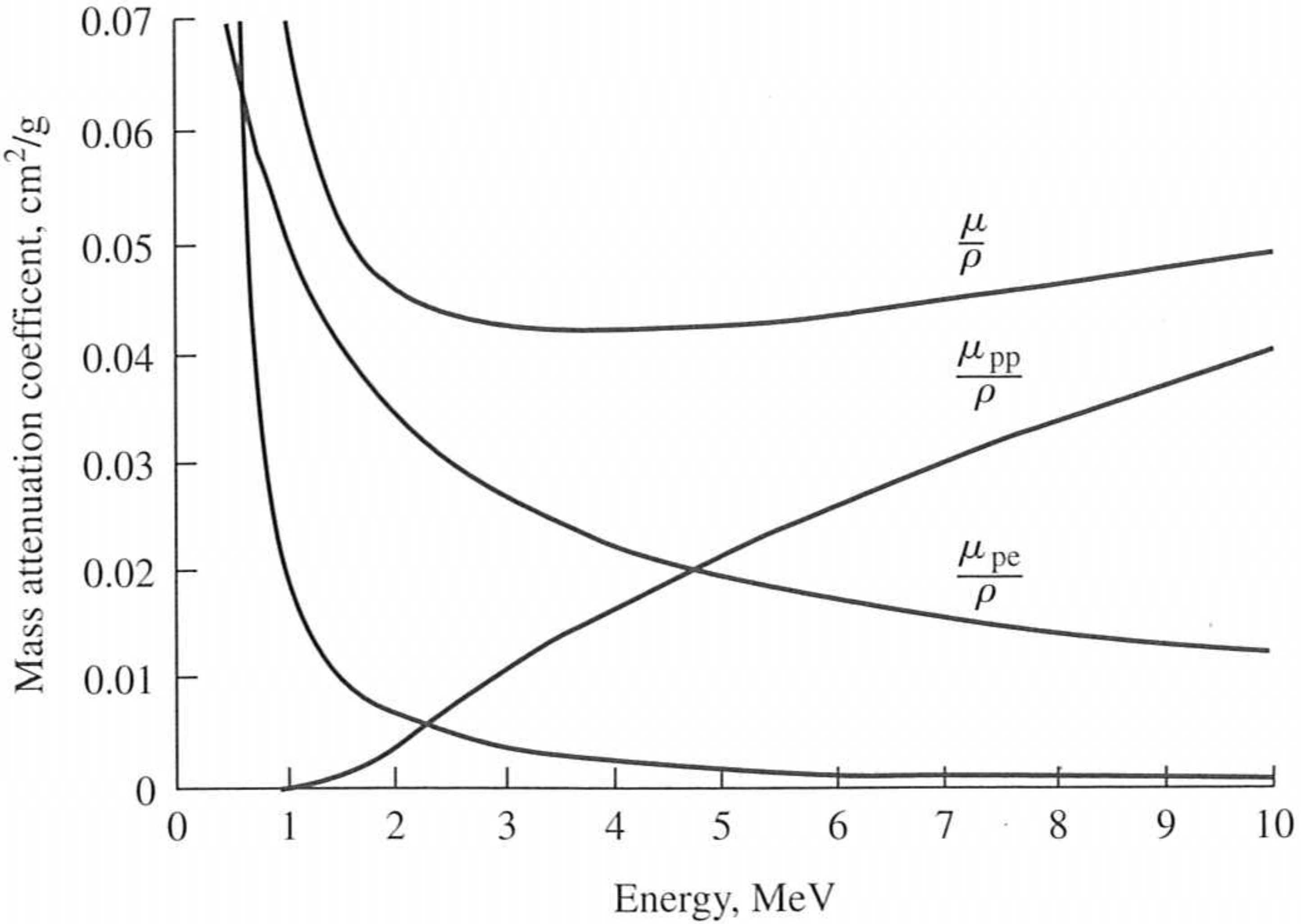
Compton Effect



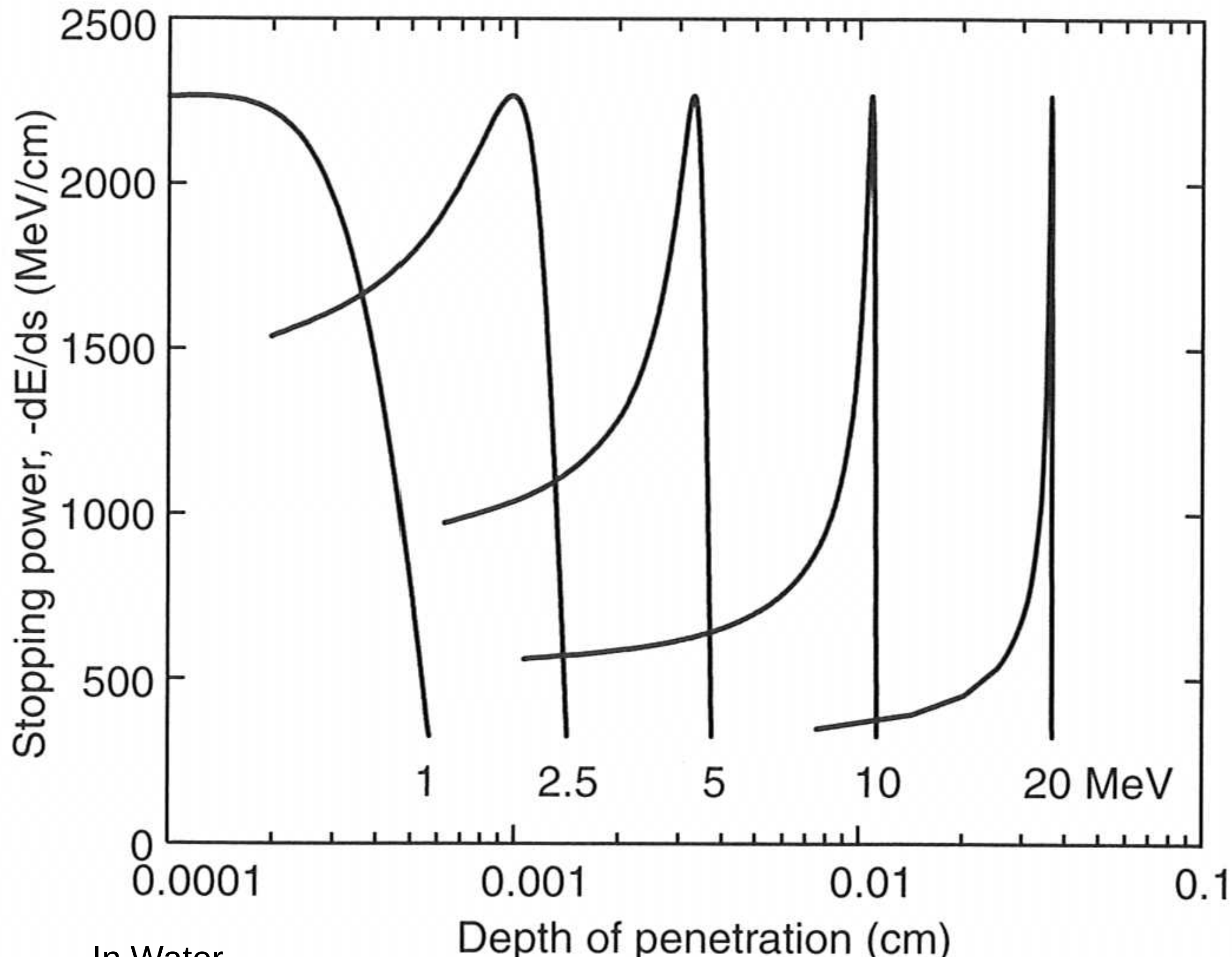
Total Mass Interaction Coefficients



Iron Mass Interaction Coefficients



α particles



In Water

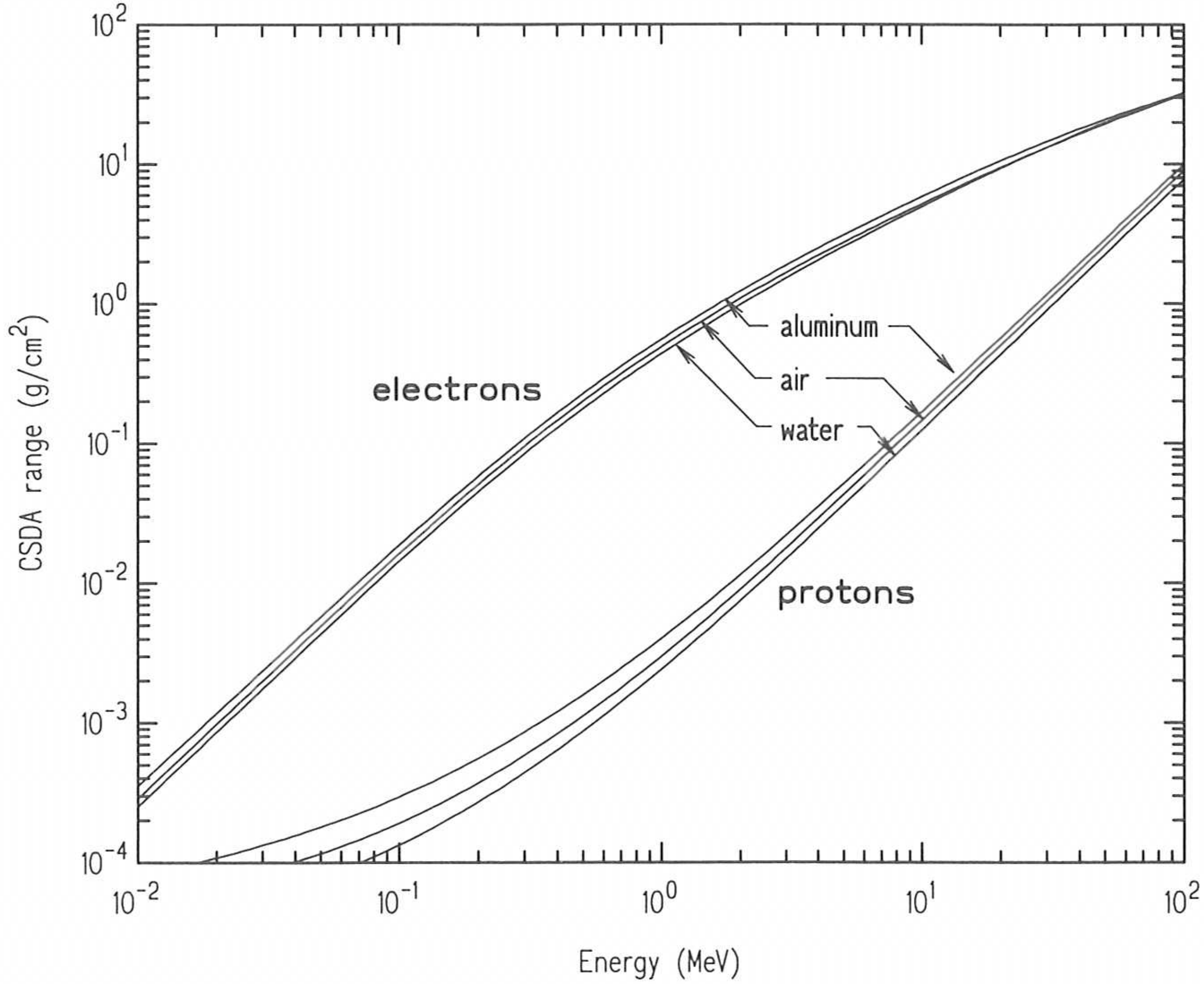
Particle Range & Stopping Power

$$-\left(\frac{dE}{ds}\right)_{col} = \rho\left(\frac{Z}{A}\right)z^2 f(I,\beta)$$

$$R = R_a\left(\frac{\rho_a}{\rho}\right)\sqrt{\frac{M}{M_a}}$$

$$P_s = \frac{R_a}{R} = 3100\,\frac{\rho}{\sqrt{M}}$$

p^+/e^- Stopping Power



Electron Radiative Stopping Power

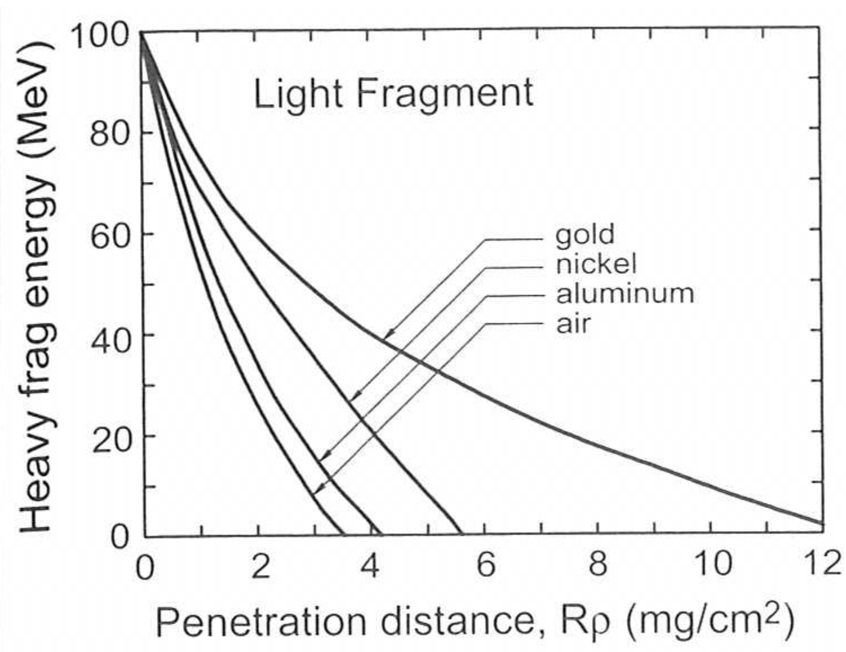
- A charged particle moving through a collection of atoms emits photons as it is deflected in the atomic fields. This is called Bremsstrahlung. The computation of this effect is complex.

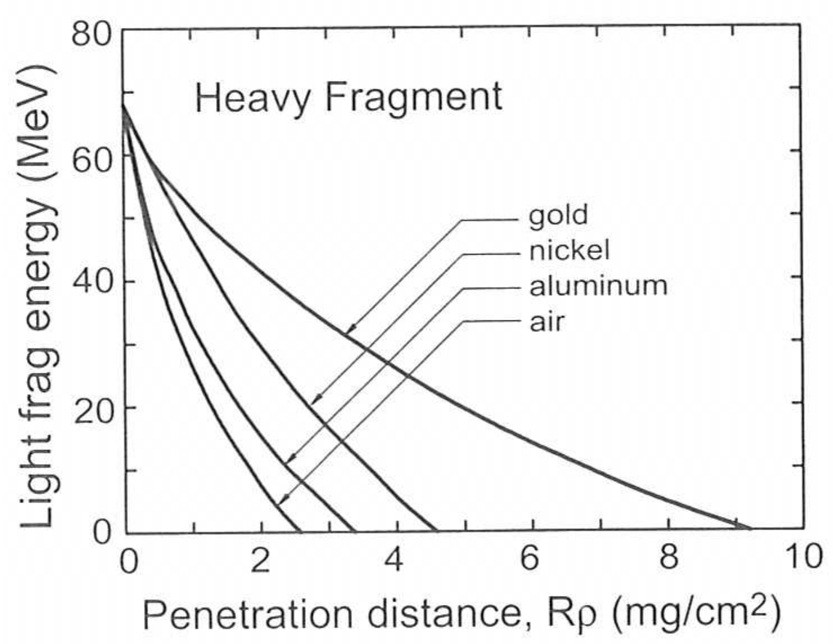
$$\left(-\frac{dE}{ds}\right)_{rad} = \frac{\rho N_a}{A}(E + m_e c^2) Z^2 F(E, Z)$$

- Relativistic, heavy charged particles with rest mas ***M***

$$\frac{\left(-\frac{dE}{ds}\right)_{rad}}{\left(-\frac{dE}{ds}\right)_{coll}} \approx \frac{EZ}{700}\left(\frac{m_e}{M}\right)^2$$

Fission Fragment Penetration





Fission fragments travel locally