Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 14

Nuclear Reactor Theory I

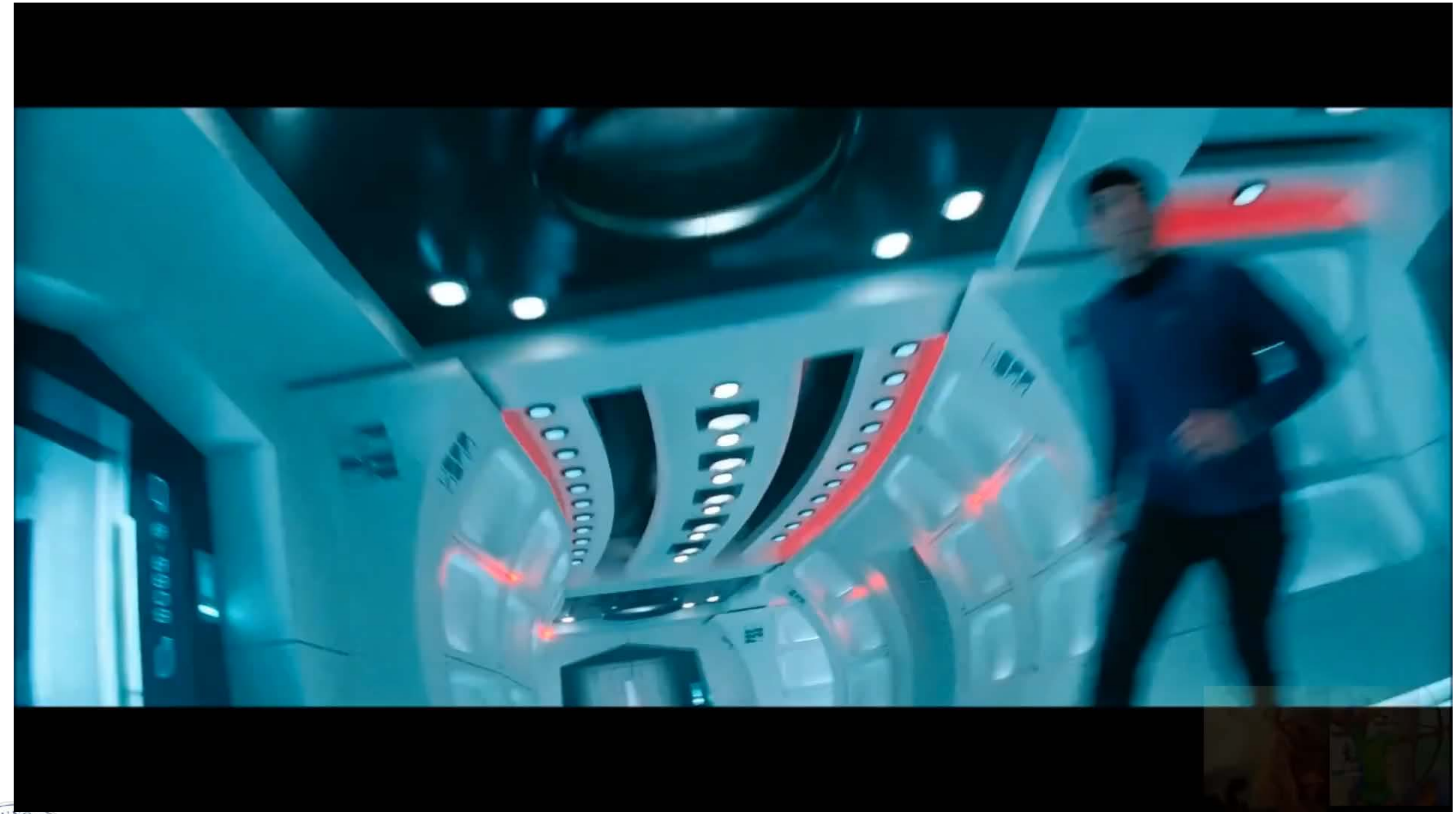
Six Factor Formula

Spiritual Thought

"'Things will work out' may well be President Hinckley's most repeated assurance to family, friends, and associates. 'Keep trying' he will say. 'Be believing. Be happy. Don't get discouraged. Things will work out'"

Elder Jeffrey R. Holland

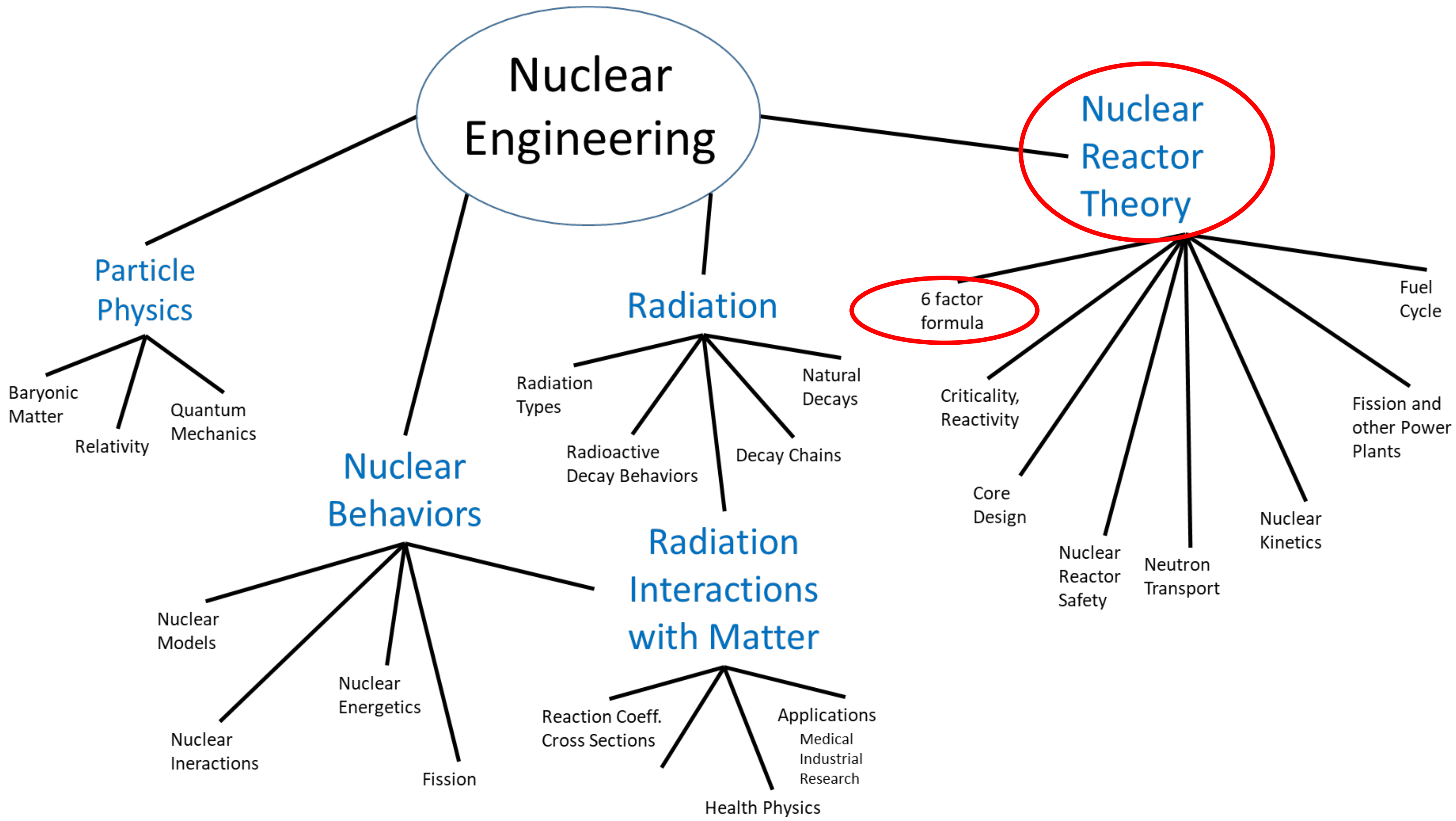
The Death of Kirk



OEP #3 (due 2/21/18)

Curse you Kahn! In the epic battle against the USS Vengeance, The Enterprise is badly damaged, and begins careening towards San Francisco on earth (I know, I know… but on this planet earth, San Francisco is the headquarters for Starfleet). Kirk heroically enters the fusion core chamber to realign the broken fission supports. In so doing, he is exposed to a lethal neutron dose, which permeates the chamber he is in. In this scene, Spock is coming to grips with the fact that his friend is dying, and a “touching” moment ensues in which they touch the glass opposite of each other’s hand with their own hand. In order for Spock to not also die from neutron radiation poisoning, the flux (uncollided) must be reduced by 1000 times. Is this possible?

The BIG Picture

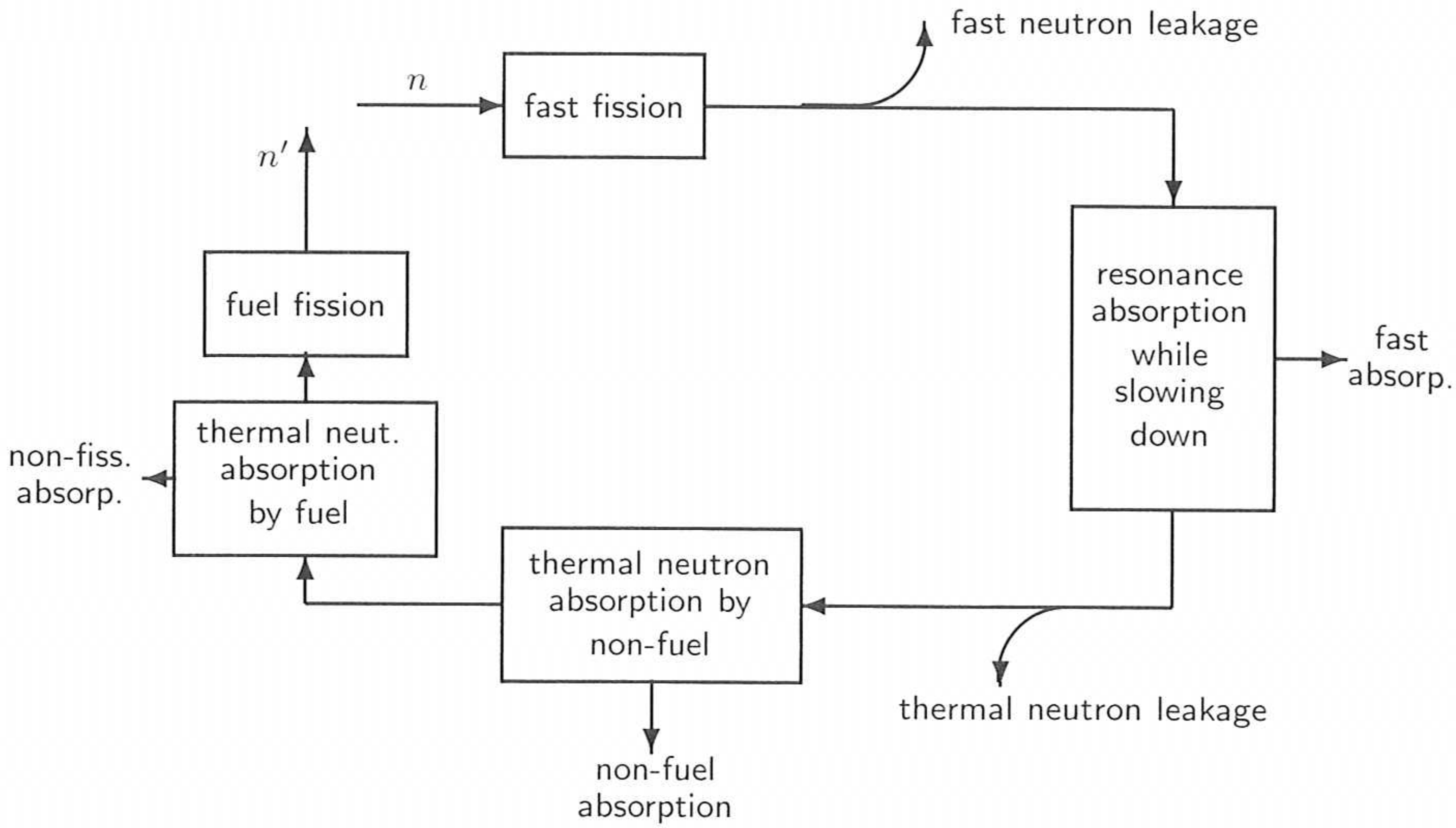


Core Nomenclature

- Fuel – fissile material in core
- Heavy atoms – generally fissile, fertile, and fissionable material
- Moderator – collision material that slows neutrons
- Cladding – fuel containment
- Reflector – core wrapping that minimizes neutron leakage
- Blanket – material used to produce useful isotopes from core
- Shielding – neutron (and other radiation) absorbing material to protect personnel and instrumentation

Fast Neutron Life Cycle

- What happens to fast neutrons?



Fast Neutron Factors

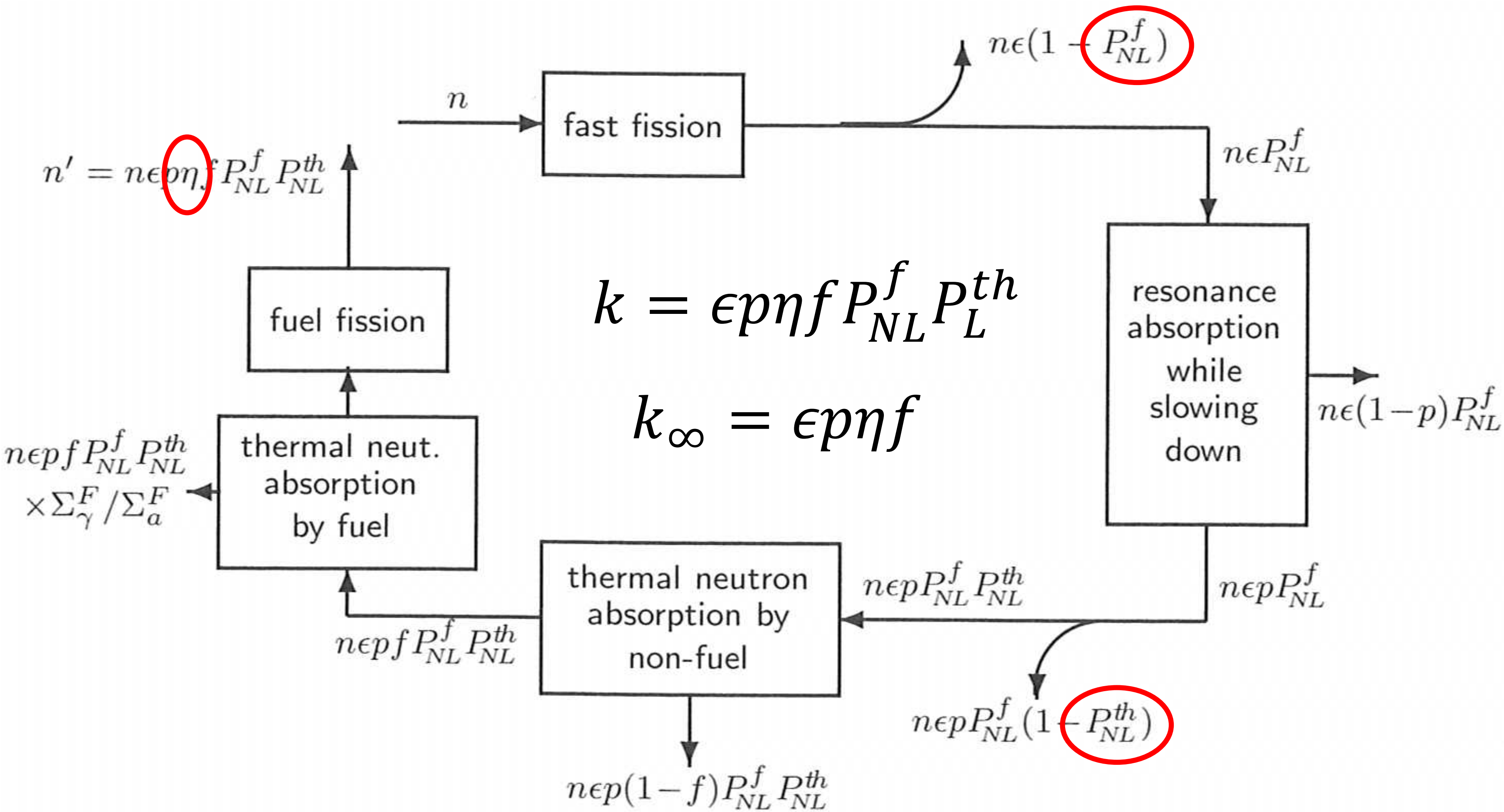
- Fast Fission, ε
- Resonance Escape Probability, p
- thermal utilization, f
- Fission factor, η
- Fast non-leakage probability, P_{NL}^{f}
- Thermal non-leakage probability, P_{NL}^{th}
- Investigate for two reactor types:
  - Homogenous
  - Real (heterogeneous)

Six Factor Formula

$$n' \equiv n\epsilon p\eta f P_{NL}^{f} P_{NL}^{th}$$

n'	next generation neutrons
n	neutrons produced per thermal fission
ϵ	ratio of total neutrons to thermal neutrons (1.0-1.08)
p	resonance escape probability (0.8-0.9)
f	thermal utilization = $\frac{\Sigma_a^F \phi^F V^F}{\Sigma_a^F \phi^F V^F + \Sigma_a^{NF} \phi^{NF} V^{NF}}$ (0-1)
η	fission factor $= \nu \frac{\Sigma_f^F}{\Sigma_a^F}$ (2.0-2.2)
P_{NL}^{f}	non-leakage of fast neutrons = $\exp -B_c^2 \tau$ (near 1)
P_{NL}^{th}	non-leakage of thermal neutrons = $\frac{1}{1+L^2 B_c^2}$ (near 1)

Quantitative Neutron Cycle



$$k = \epsilon p \eta f P^f_{NL} P^{th}_{L}$$

$$k_\infty = \epsilon p \eta f$$

Thermal Fission Factor, η

For a homogenous reactor: $\phi_F = \phi_{NF}$

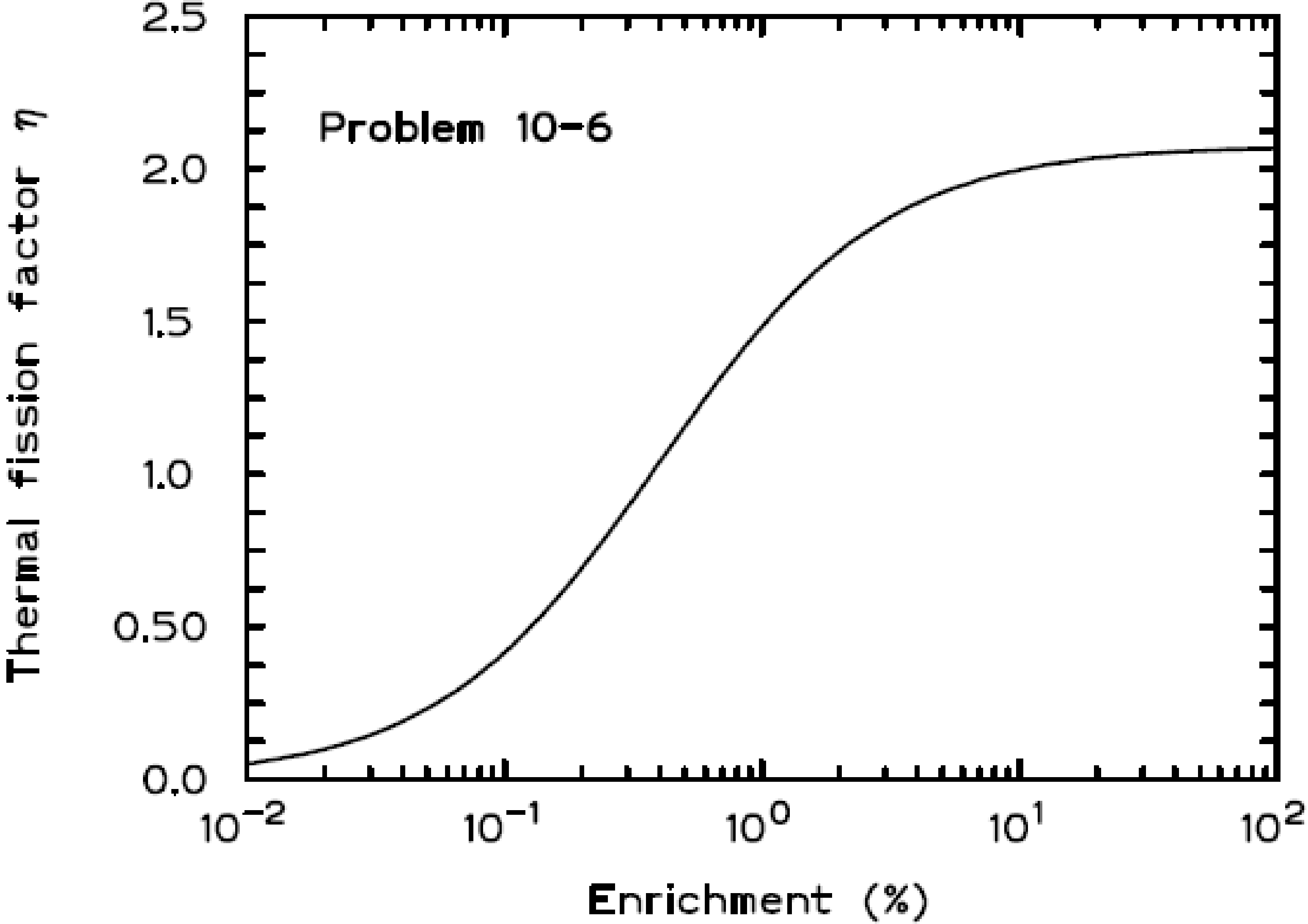
$$\eta \equiv \nu \frac{\Sigma_f}{\Sigma_a} = \nu \frac{\sigma_f^{235} N^{235}}{\sigma_a^{235} N^{235} + \sigma_a^{238} N^{238}} = \frac{\nu^{235} \sigma_f^{235}}{\sigma_a^{235} + \sigma_a^{238} \left(\frac{N^{238}}{N^{235}} \right)}$$

$$e = \frac{N^{235}}{N^{235} + N^{238}}$$

$$\Rightarrow \frac{N^{238}}{N^{235}} = \frac{1-e}{e}$$

$$\eta = \frac{\nu^{235} \sigma_f^{235}}{\sigma_a^{235} + \sigma_a^{238} \left(\frac{1-e}{e} \right)}$$

η variation with fuel enrichment



Conceptual Definitions

- L = ½ distance thermal neutron travels from point of thermalization to absorption
- τ = 1/6 mean squared distance from initial (birth) point to thermalization point
- Neutrons
  - Travel further in fast spectrum
    - 30 vs. 6 cm in light water
  - Spend far more time as thermal neutrons
- Probability of non-leakage
  - Approaches 100% as reactor dimensions become infinite
  - Both fast and thermal neutrons.

Non-leakage probabilities

$$P_{NL}^{th} = \frac{1}{1 + L^2 B_c^2}$$

L thermal diff. length

D thermal diff. coefficient

Σ_a absorption cross-section

f fuel utilization factor

$$L^2 \equiv \frac{D}{\Sigma_a} = \frac{D^M}{\Sigma_a^M} \frac{\Sigma_a^M}{\Sigma_a^M + \Sigma_a^F} = L_M^2 \left(1 - \frac{\Sigma_a^F}{\Sigma_a^M + \Sigma_a^F}\right) = L_M^2 (1 - f)$$

- B_c critical buckling – comes from reactor geometry
  - Table 10.6
  - Derived from Neutron Diffusion Equation
  - Balance of size and geometry of reactor

Non-leakage probabilities

$$P_{NL}^{f} = \exp(-B_c^2 \tau)$$

τ Fermi age

$$\tau \approx \tau_M$$

B_c critical buckling – comes from reactor geometry (Table 10.6)

Critical Bare Reactor Summary

geometry	Buckling (B_g^2)	Flux	A	$\Omega = \frac{\phi_{max}}{\phi_{av}}$
$plate - 1D$	$\left(\frac{\pi}{a}\right)^2$	$A\cos\frac{\pi x}{a}$	$1.57P/aE_R\Sigma_f$	1.57
$plate - 3D$	$\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$	$A\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}\cos\frac{\pi z}{c}$	$3.85P/VE_R\Sigma_f$	3.88
$cylinder - 1D$	$\left(\frac{2.405}{R}\right)^2$	$A\,J_0\left(\frac{2.405}{R}\right)$	$0.738P/R^2E_R\Sigma_f$	2.32
$cylinder - 3D$	$\left(\frac{2.405}{R}\right)^2 + \left(\frac{z}{H}\right)^2$	$A\,J_0\left(\frac{2.405}{R}\right)\cos\frac{\pi z}{H}$	$3.63P/VE_R\Sigma_f$	3.64
$sphere$	$\left(\frac{\pi}{R}\right)^2$	$\frac{A}{r}\cos\frac{\pi r}{R}$	$P/4R^2E_R\Sigma_f$	3.29

Multiplication Factor

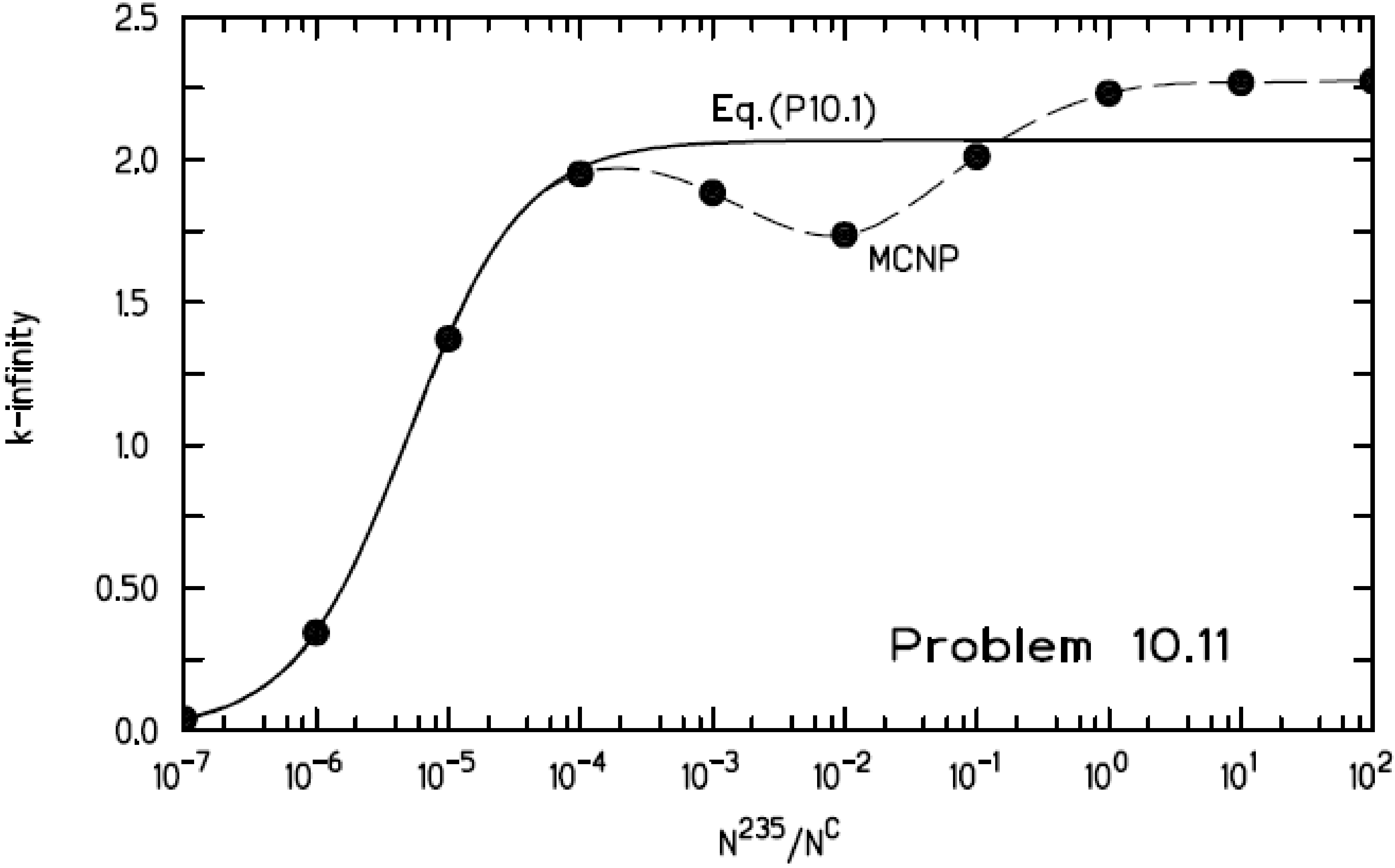
$$k_{eff} \equiv \frac{neutrons\ at\ point\ in\ cycle}{neutrons\ at\ same\ point\ in\ previous\ generation}$$

$$k_{eff} = \frac{n'}{n}$$

$$k_{eff} = \epsilon p \eta f P_{NL}^{f} P_{NL}^{th}$$

$$k_{\infty} = \epsilon p \eta f$$

k_∞ variation with fuel:modifier ratio (HM/H)



k_∞ variation with (HM/H)