Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 15

Nuclear Reactor Theory II

Reactor Design

Multiplication Factor

$$k_{eff} \equiv \frac{neutrons\ at\ point\ in\ cycle}{neutrons\ at\ same\ point\ in\ previous\ generation}$$

$$k_{eff} = \frac{n'}{n}$$

$$k_{eff} = \epsilon p \eta f P_{NL}^{f} P_{NL}^{th}$$

$$k_{\infty} = \epsilon p \eta f$$

Reactor Considerations

- Increase Power?

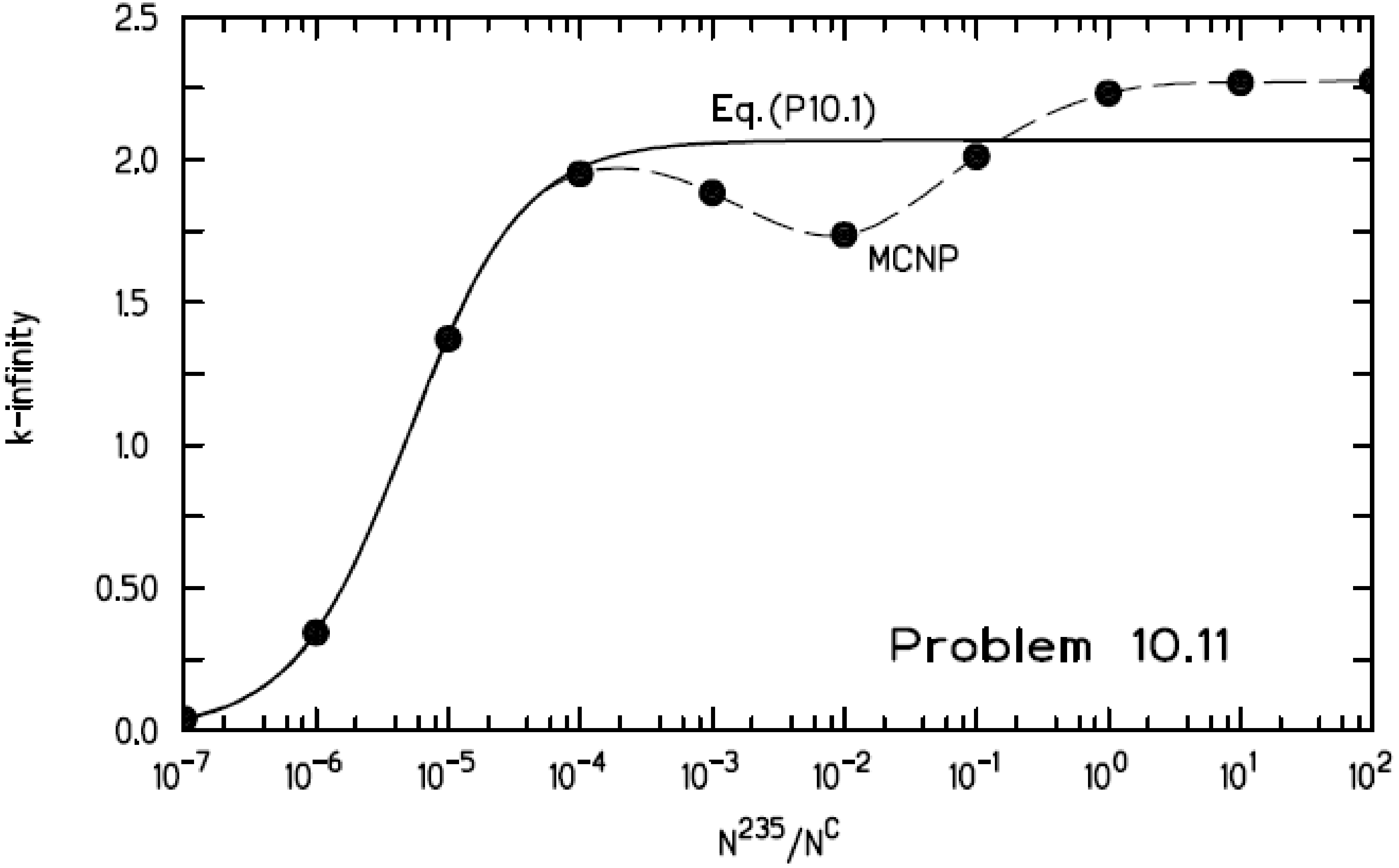
$$k_{eff} > 1$$

- Decrease Power?

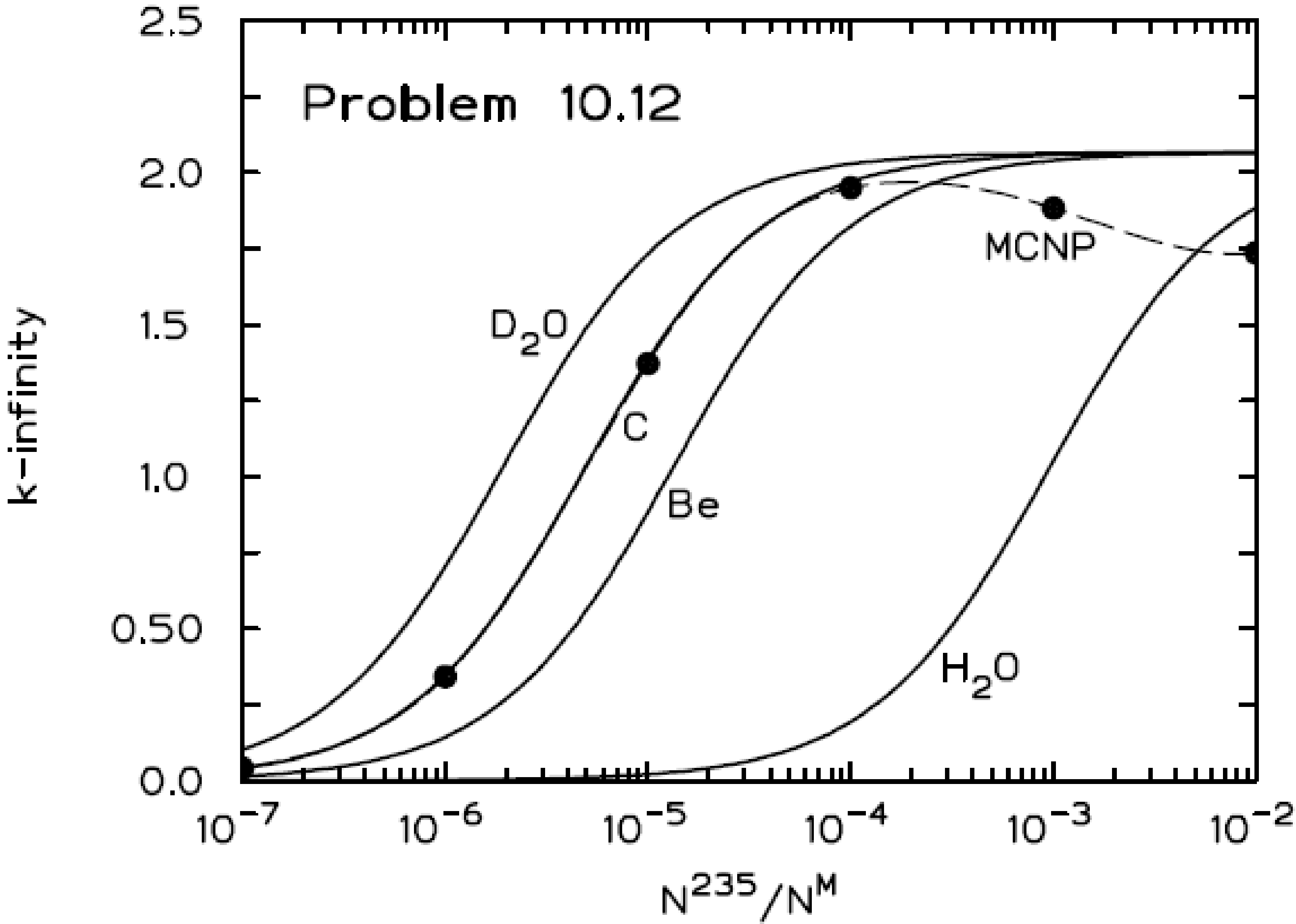
$$k_{eff} < 1$$

- Most Reactors have K_{eff}>1, but cancel excess out with absorptive “poisons”, which are removed with time.
- Reactors designed to not reach prompt supercriticality
- If k_{eff} increases, “feedback” effects resist increase
- What if we want to change amount of fuel or moderator?
  - Impacts various “six factor” parameters
  - Changes k_{eff}

k_∞ variation with fuel:modifier ratio (HM/H)



k_∞ variation with (HM/H)



Neutron Cycle Parameters

Natural uranium and moderator in homogeneous reactor:

Moderator	$(N^M/N^U)_{opt}$	ϵ	η	f	p	k_∞
H_2O	1.64	1.057	1.322	0.873	0.723	0.882
D_2O	272	1.000	1.322	0.954	0.914	1.153
Be	181	1.000	1.322	0.818	0.702	0.759
C	453	1.000	1.322	0.830	0.718	0.787

Heavy water moderation allows homogeneous reactor operation with natural uranium. Candu reactors take advantage of this in principle – though no reactor is a homogeneous reactor.

A Few Parameters

$$\nu = neutrons / fission$$

Total (prompt and delayed) neutrons produced per fission

$$\alpha = \frac{\sigma_\gamma}{\sigma_f}$$

Capture to fission ratio

$$\eta = \nu \frac{\sigma_f}{\sigma_a} = \nu \frac{\sigma_f}{\sigma_\gamma + \sigma_f} = \frac{\nu}{1+\alpha}$$

Neutrons released per absorption (> 1 converter, > 2 breeder)

$$k = \frac{neutrons\ after\ one\ generation}{original\ neutrons}$$

Multiplication factor

$$C = \frac{fissile\ atoms\ produced}{fissile\ atoms\ consumed}$$

conversion/breeding ratio (>1 breeder reactor)

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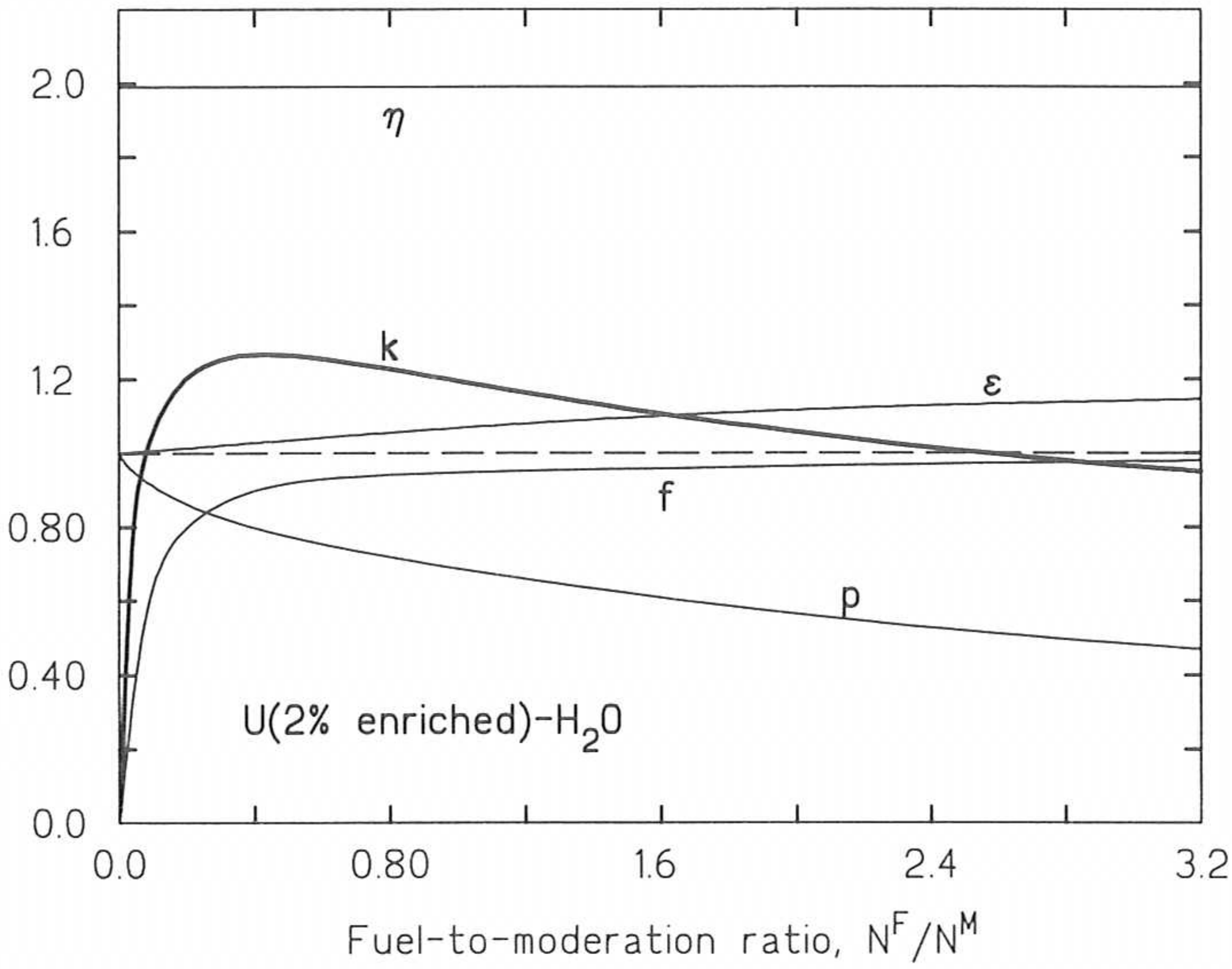
Typical Parameter Variation



Homogeneous reactor, 2% enrichment

Core Design Factors

- Design equation $k_{eff} = \epsilon p \eta f P_{NL}^{f} P_{NL}^{th} = k_{\infty} P_{NL}^{f} P_{NL}^{th}$
- $\boldsymbol{\epsilon \approx 1}$ varies to slightly over 1 as enrichment increases and with core heterogeneity
- $\boldsymbol{p < 1}$ increases with increasing enrichment and with core heterogeneity
- Usually $\boldsymbol{\epsilon p}$ will be provided or can be surmised (near unity for fully enriched reactor)
- $\boldsymbol{\eta = \frac{\nu \Sigma_f^F}{\Sigma_a^F}}$ depends only on type of fuel (see table 10.1 in text)
- $\boldsymbol{f = \frac{\Sigma_a^F}{\Sigma_a^F + \Sigma_a^{NF} (V^{NF}/V^F)(\phi^{NF}/\phi^F)}}$ depends on enrichment and heterogeneity.

  For homogeneous reactors, $\boldsymbol{f = \frac{\Sigma_a^F}{\Sigma_a^F + \Sigma_a^{NF}} = \frac{\sigma_a^F}{\sigma_a^F + \sigma_a^{NF}\left(\frac{N^{NF}}{N^F}\right)}}$

Reactor Design, cont'd

- $P_{NL}^{th} = \frac{1}{1+L^2 B_c^2}$
  - depends on reactor buckling/size/geometry and type of moderator
  - see table 10.2 in text for L_M (and τ)
  - L is ½ distance neutron travels from time it becomes thermal until it is absorbed in an infinite core
  - For homogeneous cores, $L^2 = L_M^2(1-f)$
  - B_c is tabulated in tables for various shapes and is computed by solving the diffusion w/point source (Lagrange equation) PDE.
- $P_{NL}^{f} = \exp\left(-B_c^2 \tau\right)$
  - τ is also tabulated for various moderators
  - τ = equals 1/6 the square of the distance from birth (fast neutron) to thermal
  - For dilute fuel-moderator mixtures, τ_M is good approximation for τ.

Example – Homogeneous, ^{239}Pu

- Pure ^{239}Pu fuel and reactor is a water-moderated($\frac{N^{NF}}{N^F}$=1), water-cooled bare infinite cylinder. How large of radius for criticality?
  - $\epsilon p \eta f P_{NL}^{f} P_{L}^{th} = 1 = k_{eff}$
  - $\epsilon p \approx 1$ (pure fuel makes p close to 1 and ϵ slightly greater than 1)
  - $\eta = 2.11$ (Fuel property – Table 10.1)
  - $f = (749+271)/\left[(749+271)+2\left(0.333+\frac{0.00019}{2}\right)\left(\frac{N^{NF}}{N^F}\right)\right]$ = 0.999
  - $P_{NL}^{th} = \frac{1}{1+L^2B_c^2} = \frac{1}{L_M^2(1-f)B_c^2} = \frac{1}{2.85^2(0.001)\left(\frac{2.405}{R}\right)^2}$
  - $P_{NL}^{f} = \exp(-B_c^2\tau) = \exp\left(-\left(\frac{2.405}{R}\right)^2 27\right)$
  - $\epsilon p \eta f P_{NL}^{f} P_{L}^{th} = 1 = 1(2.11) f \frac{\exp\left(-\left(\frac{2.405}{R}\right)^2 27\right)}{2.85^2(1-f)\left(\frac{2.405}{R}\right)^2}$ = 68.8$R^2\frac{N^F}{N^{NF}} exp(-\frac{156.6}{R^2})$

Designing Size and Enrichment

$$k = \epsilon p \eta f P_{NL}^{f} P_{L}^{th} = 1$$

For fully enriched reactors, $\epsilon p \approx 1$

$$\eta = \frac{\nu^{235}\sigma_f^{235}}{\sigma_a^{235}} = 2.42\,\frac{587}{687} = 2.068$$

$$f = \frac{\sigma_a^{235}}{\sigma_a^{235} + \sigma_a^{H_2O}\left(\frac{N^{H_2O}}{N^{235}}\right)} = \frac{687}{687 + 0.666(800)} = 0.5632$$

$$\left(\sigma_a^{H_2O} = 2\sigma_\gamma^{H} + \sigma_\gamma^{O}\right)$$

$$k_\infty = \eta f = (2.068\,)0.5632 = 1.165$$

Size & Enrichment, cont'd

$$k_{eff} = P_{NL}^{f} P_{NL}^{th} k_{\infty}$$

$$P_{NL}^{th} = \frac{1}{1 + L^2 B_c^2}$$

$$P_{NL}^{f} = \exp\left(B_c^2 \tau\right)$$

$$k_{eff} = P_{NL}^{f} \frac{k_{\infty} \exp\left(-B_c^2 \tau\right)}{1 + L^2 B_c^2}$$

B_c^2 depends only on geometry (Table 10.6).

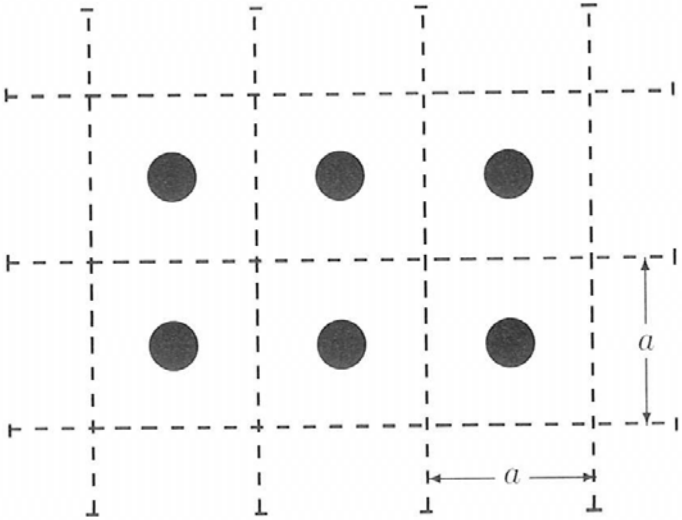
For a cube with side *L*,

$$B_c^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2 = 3\left(\frac{\pi}{L}\right)^2$$

Heterogeneous vs. homogeneous

- Heterogeneous cores change the reactor parameters :
  - $\boldsymbol{p}$ resonance escape probability
    - increases significantly
    - neutrons slow primarily in the moderator
    - no (or controlled amounts of) highly absorbing nuclides.
  - $\boldsymbol{\epsilon}$ fast fission
    - Increases slightly
    - fast neutrons are primarily surrounded by fissionable and fissile nuclides
  - $\boldsymbol{f}$ thermal utilization at fixed fuel loading ($\boldsymbol{N^F/N^{NF}}$)
    - Lower in heterogeneous reactor
    - Thermal neutron flux in fuel rod is less than that in moderator
  - $\boldsymbol{\eta}$ thermal fission factor
    - unchanged
    - depends only on the type of fuel
  - P_{NL}^{f}, P_{NL}^{th} Leakage probabilities
    - Unchanged
    - Depend primarily on reactor shape and size

Dependence on Design



Fuel with carbon moderator

local optimum – heterogeneous cores allow even graphite-moderated reactors to operate on natural uranium

Pitch a (cm)	ϵ	η	f	p	k_∞
12	1.027	1.322	0.972	0.742	0.979
16	1.027	1.322	0.947	0.848	1.090
20	1.027	1.322	0.916	0.900	1.120
21	1.027	1.322	0.907	0.909	1.121
22	1.027	1.322	0.898	0.917	1.119
26	1.027	1.322	0.860	0.940	1.098
30	1.027	1.322	0.818	0.955	1.060

Example 1:

A reactor designer has 15 kg each of a water/fuel (95%/5%) mixture for ^{241}Pu and fully enriched ^{235}U. If he shapes each mass into a sphere can he reach criticality with either one?