Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 16

Nuclear Reactor Theory III

Neutron Transport

Spiritual Thought

With any major decision there are cautions and considerations to make, but once there has been illumination, beware the temptation to retreat from a good thing. If it was right when you prayed about it and trusted it and lived for it, it is right now. Don't give up when the pressure mounts. Certainly don't give in to that being who is bent on the destruction of your happiness.

Jeffrey R. Holland

Homework Notes

- Homework 15 is the OEP and a web problem, posted on the website
- Homework 16 is 2 book problems, 1 web problem.

- Both assignments are due Friday

Topics

- Group 1: Effects of Bio. On Rad Trans.
- Group 2: Frost Damage
- Group 3: Deep Time Analysis
- Group 4: Infiltration
- Group 5: Erosion of Cover
- Group 6: Clay Liner
- Group 7: GoldSim Quality Assurance
- Group 8: Evapotranspiration Cover

- Monday March 5th – problem outline due

One-group Reactor Equation

Mono-energetic neutrons (Neutron Balance)

$$D\nabla^2\phi - \Sigma_a\phi + s = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

v is neutron speed

For reactor, $s = \nu\Sigma_f\phi$

ν is neutrons/fission

In eigenfunction form and at steady state

$$D\nabla^2\phi - \Sigma_a\phi + \frac{\nu}{k}\Sigma_f\phi = 0$$

$$\Rightarrow \nabla^2\phi - \frac{\Sigma_a - \frac{\nu}{k}\Sigma_\mathrm{f}}{D}\phi = \nabla^2\phi + B^2\phi = 0$$

Material Buckling

$$\nabla^2\phi = -B^2\phi$$ eigenfunction form of reactor equation

$$\nabla^2\phi + B^2\phi = 0$$ one-group, steady-state, reactor equation

$$B^2 = \frac{\frac{\nu}{k}\Sigma_f - \Sigma_a}{D}$$ material buckling = (neutron generation – absorption)/diffusion

$$k = \frac{\nu\Sigma_f\phi}{DB^2\phi + \Sigma_a\phi} = \frac{\nu\Sigma_f}{DB^2 + \Sigma_a}$$ reactor multiplication factor

multiplication factor = neutron generation rate/(leakage + absorption)

Fuel utilization and k_∞

$$s = \eta \Sigma_{aF} \phi = \eta \frac{\Sigma_{aF}}{\Sigma_a} \Sigma_a \phi = \eta f \Sigma_a \phi$$

$$f = \frac{\Sigma_{aF}}{\Sigma_a}$$

f = fuel utilization factor – neutrons absorbed by fuel / (those absorbed by fuel + by other means – coolant, moderator, etc.)

$$k_\infty = \frac{\eta f \Sigma_a \phi}{\Sigma_a \phi} = \eta f$$

k_∞ = k-value for infinite (no overall leakage) reactor – a material property

η = fission neutrons generated per absorbed neutron

Operating Critical Reactor Equation

$k = 1$ Reactor operating at steady state

$$-DB^2\phi - \Sigma_a\phi + \frac{k_\infty}{k}\Sigma_a\phi = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

$$-DB^2\phi + (k_\infty - 1)\Sigma_a\phi = 0$$

$$-B^2\phi + (k_\infty - 1)\frac{\Sigma_a}{D}\phi = -B^2\phi + \frac{(k_\infty - 1)}{L^2}\phi = 0$$

$$L^2 = \frac{D}{\Sigma_a}$$ One-group diffusion area

$$B^2 = \frac{k_\infty - 1}{L^2}$$ One-group buckling

Perspective

- Previous equations show
  - how to solve for neutron flux profile $\boldsymbol{\phi}$ as a function of space
  - How to determine critical reactor dimensions
    - $\boldsymbol{B^2} = \frac{\boldsymbol{k_\infty - 1}}{\boldsymbol{L^2}} = \frac{\boldsymbol{(k_\infty - 1)\Sigma_a}}{\boldsymbol{D}}$, for a bare reactor ($B^2_g = B^2_{mat}$)
- First find solutions to the reactor equations
  - 1D, 2D, or 3D
  - Then find dimensions for a critical reactor
- Assumptions:
  - Bare, homogeneous reactors
  - Constant (special and temporal) properties
  - None are valid but, but help to develop insight into reactor operations
- Because source terms are proportional to the flux, the generally inhomogeneous differential equations are now homogeneous equations.

Bare Slab Reactor Solution

$$\frac{d^2\phi}{dx^2} = -B^2\phi$$ reactor equation

$$\phi\left(\frac{\tilde{a}}{2}\right) = \phi\left(-\frac{\tilde{a}}{2}\right) = \phi'(0) = 0$$ boundary conditions

$$\phi(x) = A\cos(Bx) + C\sin(Bx)$$ general solution

$$C = 0$$ from symmetry or by substitution

$$\phi(x) = A\cos(Bx)$$

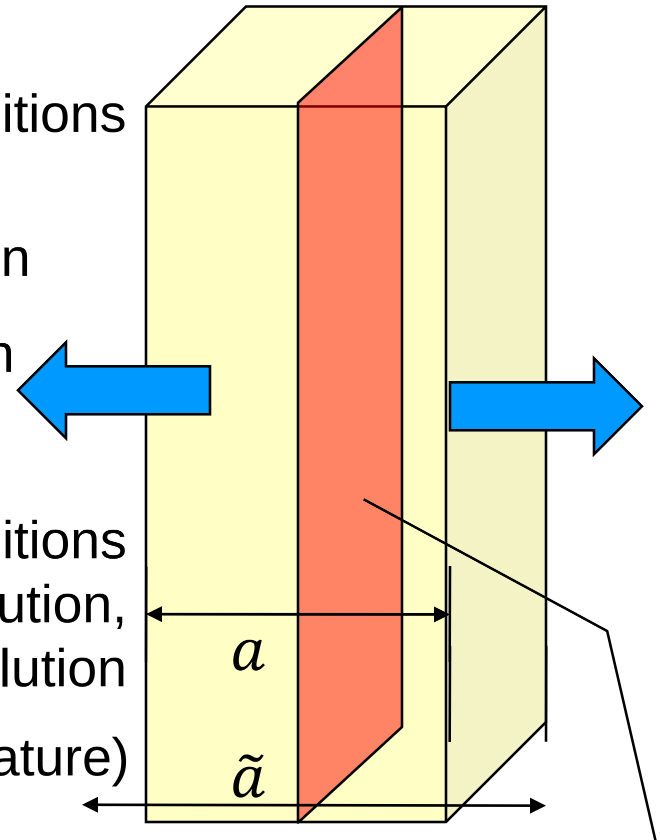
$$\phi(\tilde{a}/2) = 0$$

$$\Rightarrow B_n = \frac{n\pi}{\tilde{a}}$$

Eigenvalues from boundary conditions – all n important in transient solution, only n=1 important for steady solution

$$B_1^2 = -\frac{1}{\phi}\frac{d^2\phi}{dx^2}$$

B_1^2 is buckling (prop. to flux curvature)



Infinite plane indicates no net flux from sides

The constant A is as yet undetermined and relates to the power. There are different solutions to this problem for every power level.

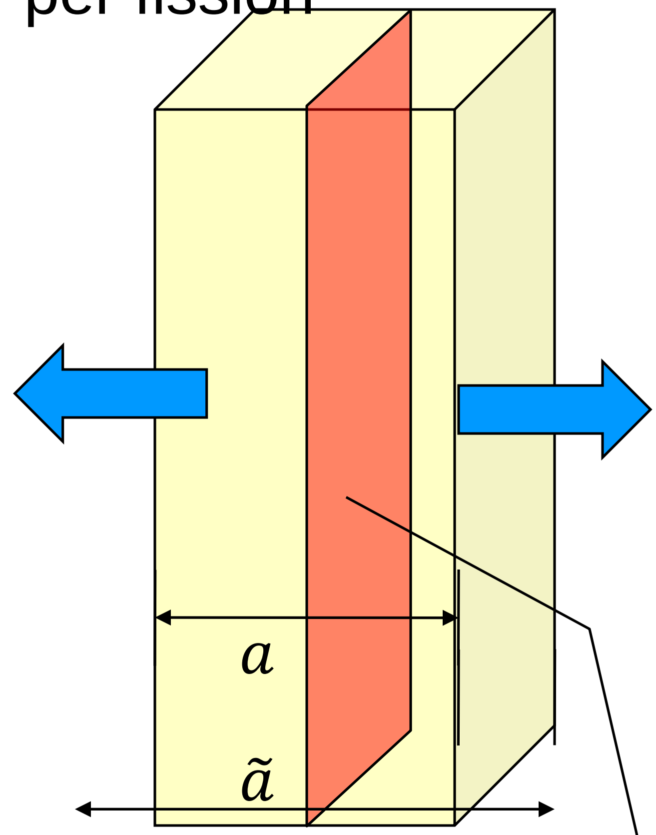
Bare Slab Reactor Power

$$P = E_R \Sigma_f \int_{-a/2}^{a/2} \phi(x)dx$$

E_R is the recoverable energy per fission

$$P = \frac{2A\tilde{a}E_R\Sigma_f}{\pi} \sin\left(\frac{\pi a}{2\tilde{a}}\right)$$

$$\phi(x) = \frac{\pi P}{2\tilde{a}E_R\Sigma_f \sin\left(\frac{\pi a}{2\tilde{a}}\right)} \cos\left(\frac{\pi x}{\tilde{a}}\right)$$



Infinite plane indicates no net flux from sides

Power Scales with flux !

Spherical Reactor

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = -B^2\phi$$

reactor transport equation

$$\phi(\tilde{R}) = \phi'(0) = 0$$

boundary conditions

$$\phi(r) = A\frac{\sin(Br)}{r} + \mathrm{C}\frac{\cos(Br)}{r}$$

general solution

$$C = 0$$

from symmetry or by substitution

$$\phi(r) = A\frac{\sin(Br)}{r}$$

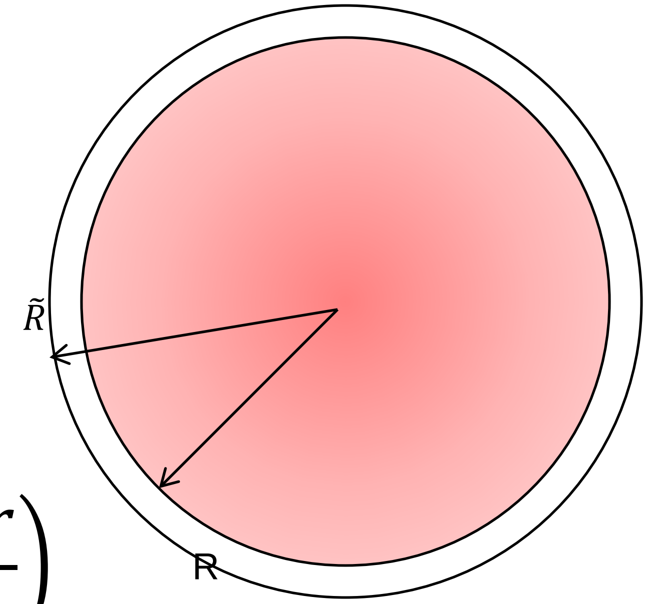
specific solution

$$B_n = \frac{n\pi}{\tilde{R}}$$

Eigen values

$$B_1^2 = \left(\frac{\pi}{\tilde{R}}\right)^2$$

buckling



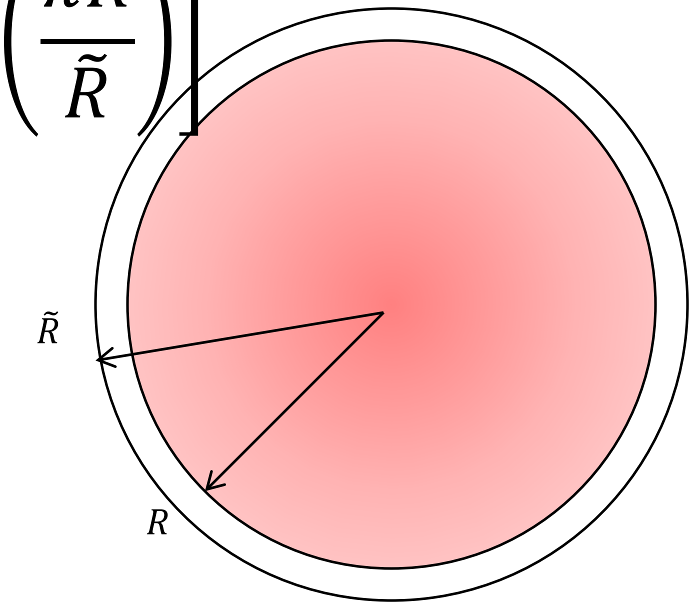
$$\phi(r) = A\frac{\sin\left(\frac{\pi r}{\tilde{R}}\right)}{r}$$

Spherical Reactor Power

Integrate over 2 symmetric dimensions – transform volume integral to radial integral

$$P = E_R \Sigma_f \iiint \phi(r) dV = 4\pi E_R \Sigma_f \int_0^R r^2 \phi(r) dr$$

$$P = 4\pi E_R \Sigma_f A \frac{\tilde{R}}{\pi}\left[\frac{\tilde{R}}{\pi}\sin\left(\frac{\pi R}{\tilde{R}}\right) - R\cos\left(\frac{\pi R}{\tilde{R}}\right)\right]$$



again, power is proportional to flux and highest at center

$$\phi(r) = \frac{P \sin\left(\frac{\pi r}{\tilde{R}}\right)}{4\, E_R \Sigma_f R^2 r}$$

Infinite Cylindrical Reactor

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right) = -B^2\phi = \frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr}$$

reactor transport equation

$$\phi(\tilde{R}) = \phi'(0) = 0; |\phi(r)| < \infty$$

boundary conditions

$$\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} + \left(B^2 - \frac{m^2}{r^2}\right)\phi = 0$$

zero-order (m=0) Bessel equation

$$\phi(r) = AJ_0(Br) + CY_0(Br)$$

general solution involves Bessel functions of first and second kind

$$\phi(r) = AJ_0(Br)$$

flux is finite

$$B_n = \frac{x_n}{\tilde{R}}$$

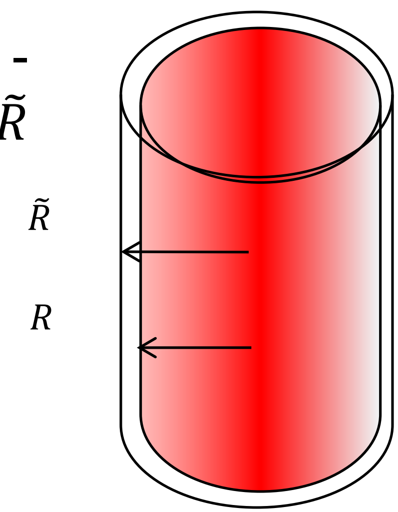
roots of Bessel functions - ϕ is zero at boundary $\tilde{R}$

$$B_1^2 = \left(\frac{x_1}{\tilde{R}}\right)^2 = \left(\frac{2.405}{\tilde{R}}\right)^2$$

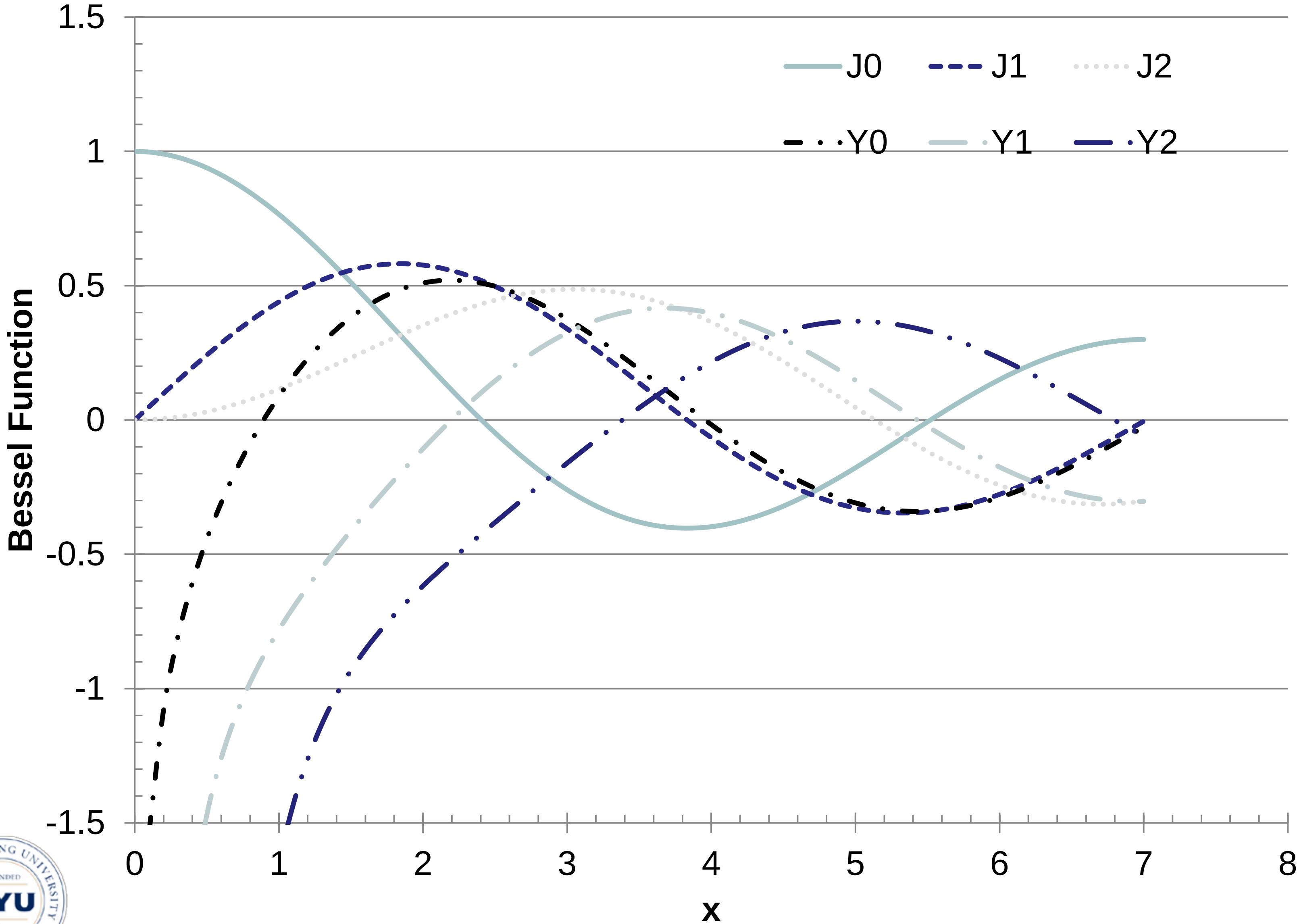
first root

$$\phi(r) = AJ_0\left(\frac{2.405\,r}{\tilde{R}}\right)$$

solution (power production determines A)



Bessel Functions



Infinite Cylindrical Reactor Power

$$P = E_R \Sigma_f \iiint \phi(r) dV = 2\pi E_R \Sigma_f \int_0^R r\phi(r) dr$$

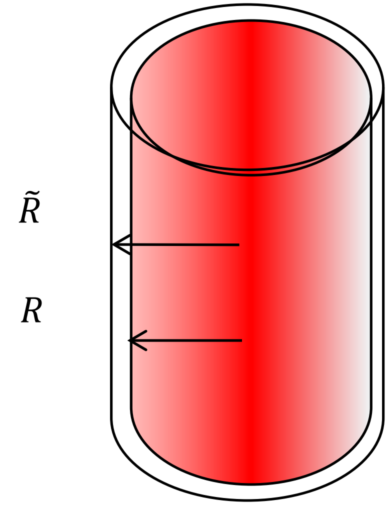
transform volume integral to radial integral – becomes power per unit length

$$P = 2\pi E_R \Sigma_f \int_0^R r J_0\left(\frac{2.405r}{R}\right) dr$$

$$\int_0^R x' J_0(x') dx' = x J_1(x)$$

$$P = \frac{2\pi E_R \Sigma_f R^2 A J_1(2.405)}{2.405}$$

$$\phi(r) = \frac{0.738P}{E_R \Sigma_f R^2} J_0\left(\frac{2.405r}{R}\right)$$



again, power is proportional to power and highest at center

Finite Cylindrical Reactor

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial \phi}{\partial r}\right)+\frac{\partial^2 \phi}{\partial z^2}=-B^2\phi$$

reactor transport equation

boundary conditions

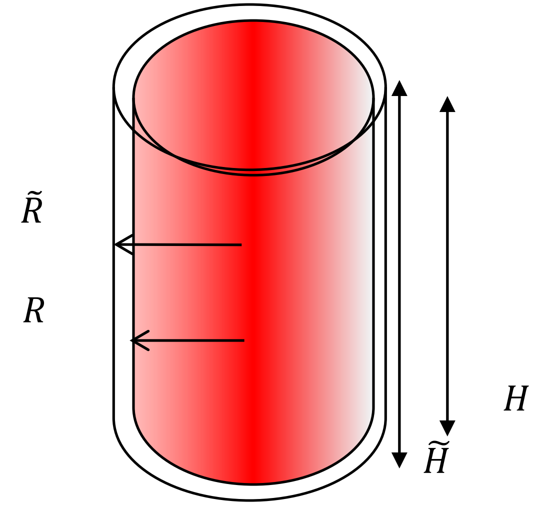
$$\phi\left(\tilde{R},z\right)=\phi'(0,z)=\phi\left(r,\frac{\widetilde{H}}{2}\right)=\phi\left(r,-\frac{\widetilde{H}}{2}\right)=0$$

$$\phi\left(r,z\right)=R(r)Z(z)$$

separation of variables

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial \phi}{\partial r}\right)+\frac{\partial^2 \phi}{\partial z^2}=\frac{Z}{r}\frac{\partial}{\partial r}\left(\frac{\partial R}{\partial r}\right)+R\frac{\partial^2 Z}{\partial z^2}R=-B^2R(r)Z(z)$$

$$\frac{1}{R}\frac{\partial}{\partial r}\left(\frac{\partial R}{\partial r}\right)+\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}R=-B^2$$



since R and Z vary independently, both portions of the equation must equal (generally different) constants, designated as B_R and B_Z, respectively

Finite Cylinder Solution

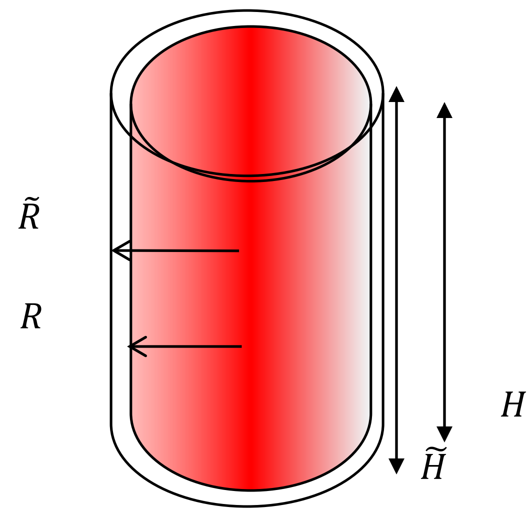
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) = -B_R^2 R$$ a problem we already solved, w/ same bcs

$$\Rightarrow R(r) = A\,J_0\left(\frac{2.405}{\tilde{R}}r\right)$$

$$\frac{\partial^2 Z}{\partial z^2} = -B_Z^2 Z$$ again a problem we already solved, w/ same bcs

$$\Rightarrow Z(z) = A\cos\frac{\pi z}{\tilde{H}}$$

$$\phi(r,z) = A\,J_0\left(\frac{2.405}{\tilde{R}}r\right)\cos\frac{\pi z}{\tilde{H}}$$



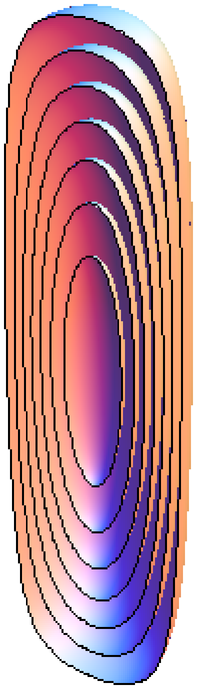
$$B^2 = B_R^2 + B_H^2$$

solution is the product of the infinite cylinder and infinite slab solutions

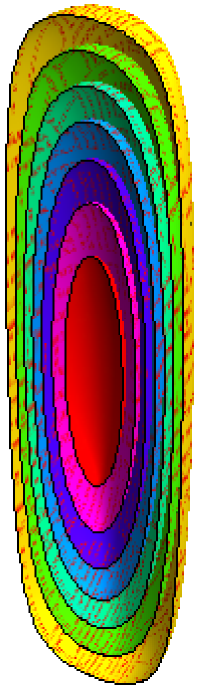
Buckling is higher than for either the infinite plane or the infinite cylinder. Buckling generally increases with increasing leakage, and there are more surfaces to leak here than either of the infinite cases.

Neutron Flux Contours

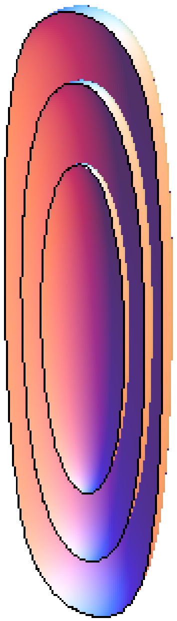
Neutron flux in finite cylindrical reactor



3D contours of neutron flux at high power



3D contours w color scaled to magnitude – intermediate power



3D contours of neutron flux at low power

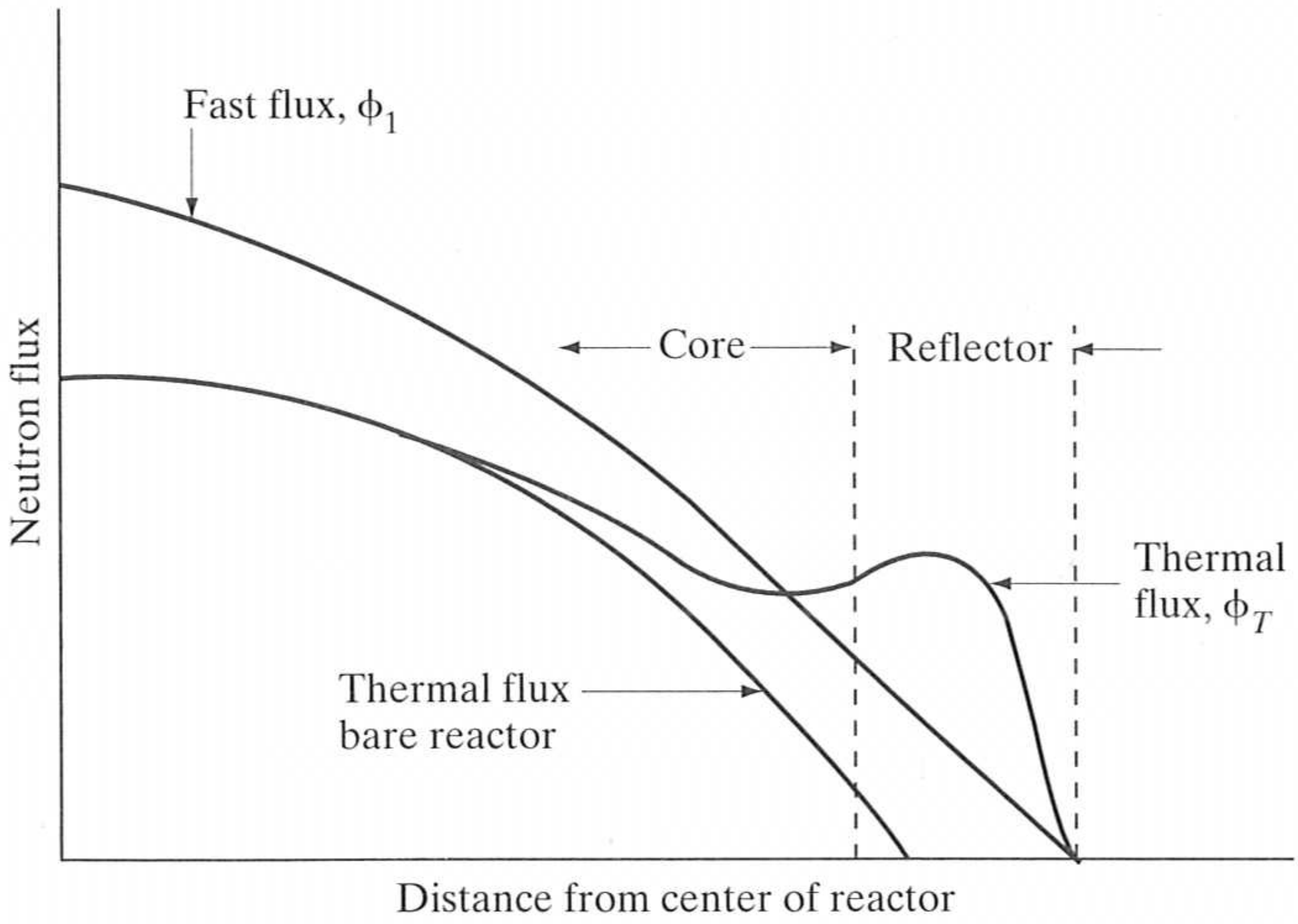
Bare Reactor Summary

geometry	Buckling (B^2)	Flux	A	$\Omega = \dfrac{\phi_{\max}}{\phi_{av}}$
$plate - 1D$	$\left(\dfrac{\pi}{a}\right)^2$	$A\cos\dfrac{\pi x}{a}$	$1.57P/aE_R\Sigma_f$	1.57
$plate - 3D$	$\left(\dfrac{\pi}{a}\right)^2 + \left(\dfrac{\pi}{b}\right)^2 + \left(\dfrac{\pi}{c}\right)^2$	$A\cos\dfrac{\pi x}{a}\cos\dfrac{\pi y}{b}\cos\dfrac{\pi z}{c}$	$3.85P/VE_R\Sigma_f$	3.88
$cylinder - 1D$	$\left(\dfrac{2.405}{R}\right)^2$	$A\,\mathrm{J}_0\left(\dfrac{2.405}{R}\right)$	$0.738P/R^2E_R\Sigma_f$	2.32
$cylinder - 3D$	$\left(\dfrac{2.405}{R}\right)^2 + \left(\dfrac{z}{H}\right)^2$	$A\,\mathrm{J}_0\left(\dfrac{2.405}{R}\right)\cos\dfrac{\pi z}{H}$	$3.63P/VE_R\Sigma_f$	3.64
$sphere$	$\left(\dfrac{\pi}{R}\right)^2$	$\dfrac{A}{r}\cos\dfrac{\pi r}{R}$	$P/4R^2E_R\Sigma_f$	3.29

Some details

- Reflected reactors lend themselves less easily to analytical solution – commonly reactors are considered as sphere equivalents rather than trying to solve the equations.
- Reasonable representation for fast neutrons – not for thermal reactors
- Reflector savings in size is typically about the thickness of the extrapolated distance.

Flux Comparisons



Thermal Flux Variations