Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 17

Nuclear Reactor Theory IV

Nuclear Kinetics

Spiritual Thought

Isaiah 1:18

Come now, and let us reason together, saith the LORD: though your sins be as scarlet, they shall be as white as snow; though they be red like crimson, they shall be as wool.

Homework 17 (due 2/23)

Homework 17 Text

Homework #17

Back to the Future

GROUP WORK OKAY, Due 2/28/18 at beginning of class

(Don't be afraid to "Google" good assumptions!)

Back to the Future

The flux capacitor is the single greatest invention of our time, but the energy requirements are incredible! 1.21 GW? Seriously? Luckily Doc brown had a few jars of plutonium (water shielded of course). Assuming that the water/plutonium ratio in the jar is the same ratio as in the reactor, determine whether there is sufficient plutonium (assume it's pure ^{239}Pu) for the reactor to be critical. Also, determine the power of such a critical reactor, assuming a max thermal flux of 2^{16} neut/cm^2/s.

General Transient Problem

Mono-energetic neutrons

$$D\nabla^2\phi - \Sigma_a\phi + S = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

For a reactor, $S = \nu\Sigma_f\phi$

$$D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

$$\nabla^2\phi - B^2\phi = -\frac{1}{Dv}\frac{\partial\phi}{\partial t}$$

Power Plant Operation

- Try to achieve “steady state” operation at all times
- Set up core and fuel to provide minimal changes
- Despite this, still have three scales of change
  1. Short Term (Grid load transients)
  2. Intermediate Term (Fuel composition changes)
  3. Long Term (Burnup & Depletion)

Three Time Scales (short)

- Short Time Constant (load change)
  - An abrupt change in steam demand/load.
  - A load change is seen as:
    - Reactor pressure change in the BWR
    - Reactor temperature change in a PWR.
  - Higher loads lead to higher pressures/temperatures.
- Assumptions
  - Shape of the flux profile is assumed constant
    - Power changes
    - Magnitude of the neutron flux scales everywhere in the reactor.
  - Assumes a uniform multiplicative change everywhere
    - Spatial variations in time in the reactor are not considered
  - This method is called <u>point kinetics</u>.

Three Time Scales (intermediate)

- Intermediate Time Constant (core composition change)
  - Changing fission product concentrations
    - Generation rates and destruction/decay rates.
  - Many fission products have measurable thermal neutron cross sections
    - ***Change*** the value of k (and k_∞).

$$f = \frac{\Sigma_a^F}{\Sigma_a^F + \Sigma_a^{NF}(V^{NF}/V^F)(\phi^{NF}/\phi^F)}$$

  - Point kinetics can be used if spatial variations in concentrations are negligible.
  - Otherwise, detailed spatial and temporal equations must be used!

Three Time Scales (long)

- Long Time Constant (Fuel Depletion)
  - Treated as a series of steady-state problems

$$D\nabla^2\phi - \Sigma_a\phi = -\lambda\nu\Sigma_f\phi$$

  - Two things are adjusted to maintain $\lambda = 1$ (i.e. $\lambda = 1/k$)
    - Material buckling
    - Reactor dimensions
    - If $\lambda \neq 1$, the equation is not valid
      - Why?
  - In operating reactor, cannot change dimensions (much)
    - *k* is adjusted slowly in time by changing chemical shim conc.
      - Chemical shim is an isotope that absorbs neutrons in the reactor
    - k is also adjusted via the control rods.
  - Managing fuel consumption is a classical example of this type of transient

Prompt Neutrons

- Lifetime, l_p, is time between emission and absorption.
- Neutrons in thermal reactors:
  - Spend more time (most of l_p) in the thermal regime
  - Travel further as fast neutrons
- Average lifetime of a thermal neutron in a an infinite reactor is the mean diffusion time, t_d, and is approximately the same as l_p in an infinite reactor. $l_p \approx t_d$

Assuming $1/v$ behavior (cross section)

$$t(E) = \frac{\lambda_a(E)}{v(E)} = \frac{1}{\Sigma_A(E)v(E)} = \frac{1}{\Sigma_A(E_0)v_0}$$

$$t_d = \overline{t(E)} = \frac{\sqrt{\pi}}{2\bar{\Sigma}_a v_T}$$

Prompt Neutrons

- For mixtures of fuel and moderator in thermal reactors

$$t_d = \overline{t(E)} = \frac{\sqrt{\pi}}{2(\bar{\bar{\Sigma}}_{aF} + \bar{\bar{\Sigma}}_{aM})v_T} = \frac{\sqrt{\pi}}{2\bar{\bar{\Sigma}}_{aM}v_T} \frac{\bar{\bar{\Sigma}}_{aM}}{(\bar{\bar{\Sigma}}_{aF} + \bar{\bar{\Sigma}}_{aM})}$$

TABLE 7.1 APPROXIMATE DIFFUSION TIMES FOR SEVERAL MODERATORS

Moderator	t_d, sec
H_2O	2.1×10^{-4}
D_2O*	4.3×10^{-2}
Be	3.9×10^{-3}
Graphite	0.017

*With 0.25% H_2O impurity.

$$\frac{\sqrt{\pi}}{2\bar{\bar{\Sigma}}_{aM}v_T}$$ moderator diffusion time

$$\frac{\bar{\bar{\Sigma}}_{aM}}{\bar{\bar{\Sigma}}_{aF} + \bar{\bar{\Sigma}}_{aM}} = 1 - f$$ f=fuel utilization factor

$$t_d = t_{dM}(1 - f)$$

- In fast reactors, prompt neutron lifetimes are much shorter, on the order of 10^{-7} seconds

Simple Kinetics Model

$$\Delta n(t) \equiv \ell' \frac{dn(t)}{dt} = \left(k_{eff} - 1\right) n(t)$$

$$\frac{dn(t)}{dt} = \frac{k_{eff} - 1}{\ell'} n(t)$$

$$\Rightarrow n(t) = n(0) \exp\left(\frac{k_{eff} - 1}{\ell'} t\right)$$

- For ^{235}U
  - $\ell' = 2.1\text{x}10^{-4}$ s
  - $k_{eff} - 1 = 0.001$
  - and $t = 1s$,
- $n/n^0 = 117$ (22,027 if $\ell' = 10^{-4}$ as in text)
- Far too rapid to control!!!

Delayed Neutrons

TABLE 3.5 DELAYED NEUTRON DATA FOR THERMAL FISSION IN ^{235}U*

Group	Half-Life (sec)	Decay Constant (l_i, sec^{-1})	Energy (ke V)	Yield, Neutrons per Fission	Fraction (β_i)
1	55.72	0.0124	250	0.00052	0.000215
2	22.72	0.0305	560	0.00346	0.001424
3	6.22	0.111	405	0.00310	0.001274
4	2.30	0.301	450	0.00624	0.002568
5	0.610	1.14	—	0.00182	0.000748
6	0.230	3.01	—	0.00066	0.000273
				Total yield:	0.0158
				Total delayed fraction (β):	0.0065

*Based in part on G. R. Keepin, *Physics of Nuclear Kinetics*, Reading, Mass.: Addison-Wesley, 1965.

For 1-group model, $T_{\frac{1}{2}}$ for ^{235}U is about 8.87 s and τ is about 12.8 s.

Delayed Neutron Fractions

Group	^{235}U		^{233}U		^{239}Pu	
	Half-life (s)	fraction β_i	Half-life (s)	fraction β_i	Half-life (s)	fraction β_i
1	55.7	0.00021	55.0	0.00022	54.3	0.00007
2	22.7	0.00142	20.6	0.00078	23.0	0.00063
3	6.22	0.00127	5.00	0.00066	5.60	0.00044
4	2.30	0.00257	2.13	0.00072	2.13	0.00068
5	0.610	0.00075	0.615	0.00013	0.618	0.00018
6	0.230	0.00027	0.277	0.00009	0.257	0.00009
total	-	0.0065	-	0.0026	-	0.0021

Reactors with delayed neutrons

$$\bar{\ell}_p = (1-\beta)\ell_p + \beta\left(\ell_p + \tau\right) \approx \ell_p + \beta\tau$$

τ is lifetime of delayed neutrons = $\frac{T_{1/2}}{\ln 2} \approx 12.8\ s$

For $\delta k \ll \beta$

$$\frac{n(t)}{n_0} = \exp\left(\frac{k_{eff} - 1}{\bar{\ell}_p}\right) = \exp\left(\frac{t}{T}\right)$$

$$T = \frac{\bar{\ell}_p}{k_{dff} - 1} = \frac{\beta\tau}{\delta k}$$

- For ^{235}U, T = 83 s, k_{eff} -1 = 0.001,
- n/n^0 = 1.012
- This can be controlled!

Reactivity and Worth

$$\rho \equiv \frac{k_{eff} - 1}{k_{eff}} = \frac{\delta k}{k_{eff}}$$

reactivity ρ and δk

$$k(\$) \equiv \frac{\rho}{\beta}$$

β is delayed neutron fraction
worth can be measured in units of $k(\$)$ or k

- cents?
- Percent Mil?

Power Changes

$$T = \frac{\beta\tau}{\delta k} = \frac{\beta\tau}{keff \cdot \rho} = \frac{\tau}{keff \cdot k(\$)} \sim \frac{\tau}{k(\$)}$$

- T = Reactor Period (units of time)
  - Time required to increase reactor power (or neutron flux) by 2.72

$$\frac{t}{T} = ln\left[\frac{P(t)}{P(0)}\right]$$

Example 1

- Following a reactor scram in which all the control rods are inserted into a power reactor, how long is it before the reactor power decreases to 0.0001 of the steady-state power prior to shutdown? (Assume a reactor period of -80 s)

1-level Model Parameters

	β	$T_{\frac{1}{2},d}(s)$	$\tau_d(s)$
^{232}Th	0.0203	6.98	10.07
^{233}U	0.0026	12.4	17.89
^{235}U	0.0064	8.82	12.72
^{238}U	0.0148	5.32	7.68
^{239}Pu	0.002	7.81	11.27
^{241}Pu	0.0054	104.1	150.18
^{241}Am	0.0013	10	14.43
^{243}Am	0.0024	10	14.43
^{242}Cm	0.0004	10	14.43

Source: Laboratoire de Physique Subatomique et de Cosmologie