# Chemical Engineering 412

*Introductory Nuclear Engineering*

Lecture 18

Nuclear Reactor Theory IV

Reactivity Insertions

# Spiritual Thought

**Mosiah 2:33**

**33** For behold, there is a wo pronounced upon him who listeth to obey that spirit; for if he listeth to obey him, and remaineth and dieth in his sins, the same drinketh damnation to his own soul; for he receiveth for his wages an everlasting punishment, having transgressed the law of God contrary to his own knowledge.

# Reactivity and Δk

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = \frac{\delta k}{k_{eff}}$$

$$k(\$) = \frac{\rho}{\beta}$$

$$\rho = reactivity$$

$$\delta k = delta - k$$

$$\beta = delayed\ neutron\ fraction$$

$$k(\$) = worth$$

$$k_o = \left[ \frac{l}{\beta} + \sum_{i=1}^{G} \frac{a_i}{\lambda_i + \omega} \right]$$

Inhour equation

# Reactivity Equation Solutions

$$\phi_T = \underbrace{A_1 \exp(\omega_1 t)}_{\substack{dominant\ term \\ as\ t \to \infty}} + \underbrace{A_2 \exp(\omega_2 t)}_{approaches\ 0\ rapdily}$$

General solution for single group of delayed neutrons

$$T = \frac{1}{\omega_1}$$

Definition of reactor or stable period

$$\phi_T = \exp\left(\frac{t}{T}\right)$$

General solution for single group of delayed neutrons

$$\rho = \frac{\omega l_p}{1 + \omega l_p} + \frac{\omega}{1 + \omega l_p} \sum_{i=1}^{6} \frac{\beta_i}{\omega + \lambda_i}$$

Reactivity equation for six group model – graphical solution on next page

# Reactivity Equation Solutions

$$T = \frac{\bar{l}}{k_{eff} - 1} = \frac{\bar{l}}{\delta k} = \frac{\beta\tau}{\delta k} =$$

**Reactor period** - The time required for a neutron population to change by a factor of e

$$k_{eff} = 1 + \delta k = 1 + \frac{\beta\tau}{T}$$

$\tau =$ Lifetime of delayed neutrons ~12.8s (U235)

$$T = \frac{\beta\tau}{\delta k} = \frac{\beta\tau}{k_{eff} - 1} = \frac{\beta\tau}{k_{eff}\rho} = \frac{\tau}{k_{eff}\rho(\$)} \approx \frac{\tau}{\rho(\$)}$$

$$\phi(t) = exp\left(\frac{t}{T}\right)$$

Remember, Flux is proportional to power….

$$C \cdot P(t) = exp\left(\frac{t}{T}\right) \qquad T = exp\left(\frac{P(t) \cdot C}{P(0) \cdot C}\right) \qquad T = exp\left(\frac{P(t)}{P(0)}\right)$$

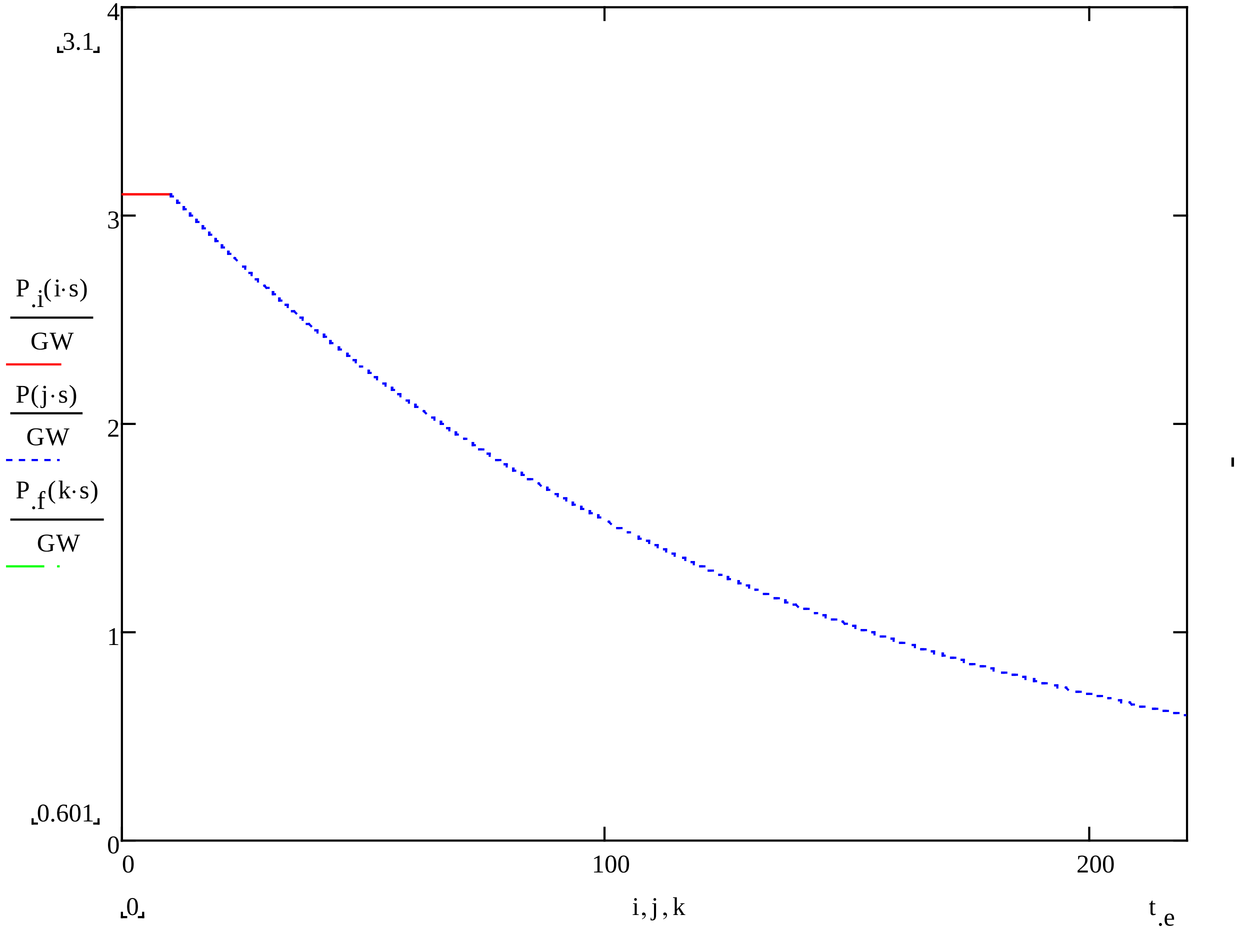
# 1-level Model Parameters

| | $\beta$ | $T_{\frac{1}{2},d}(s)$ | $\tau_d(s)$ |
|---|---|---|---|
| $^{232}Th$ | 0.0203 | 6.98 | 10.07 |
| $^{233}U$ | 0.0026 | 12.4 | 17.89 |
| $^{235}U$ | 0.0064 | 8.82 | 12.72 |
| $^{238}U$ | 0.0148 | 5.32 | 7.68 |
| $^{239}Pu$ | 0.002 | 7.81 | 11.27 |
| $^{241}Pu$ | 0.0054 | 104.1 | 150.18 |
| $^{241}Am$ | 0.0013 | 10 | 14.43 |
| $^{243}Am$ | 0.0024 | 10 | 14.43 |
| $^{242}Cm$ | 0.0004 | 10 | 14.43 |

Source: Laboratoire de Physique Subatomique et de Cosmologie

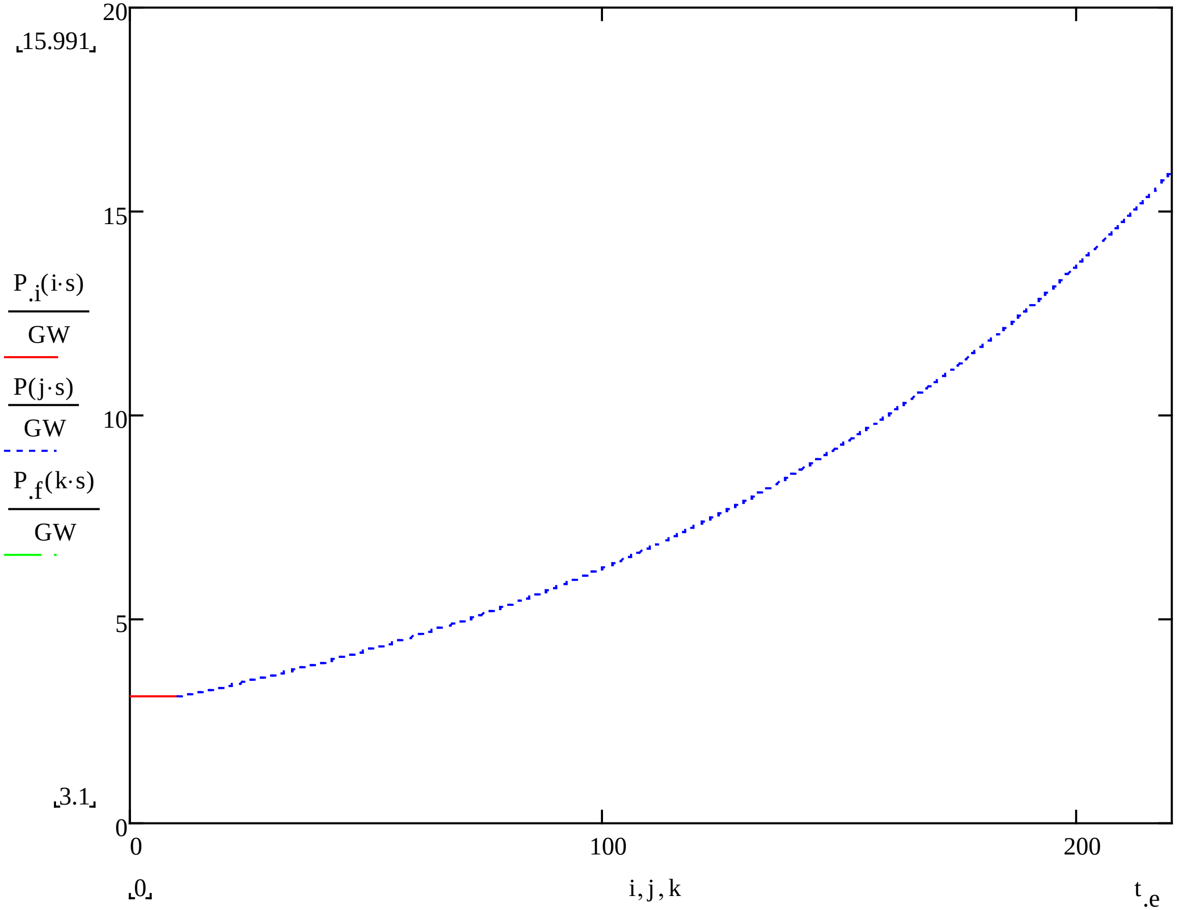
# Exploration 1

- What if we add -$0.1 to AP1000 core?



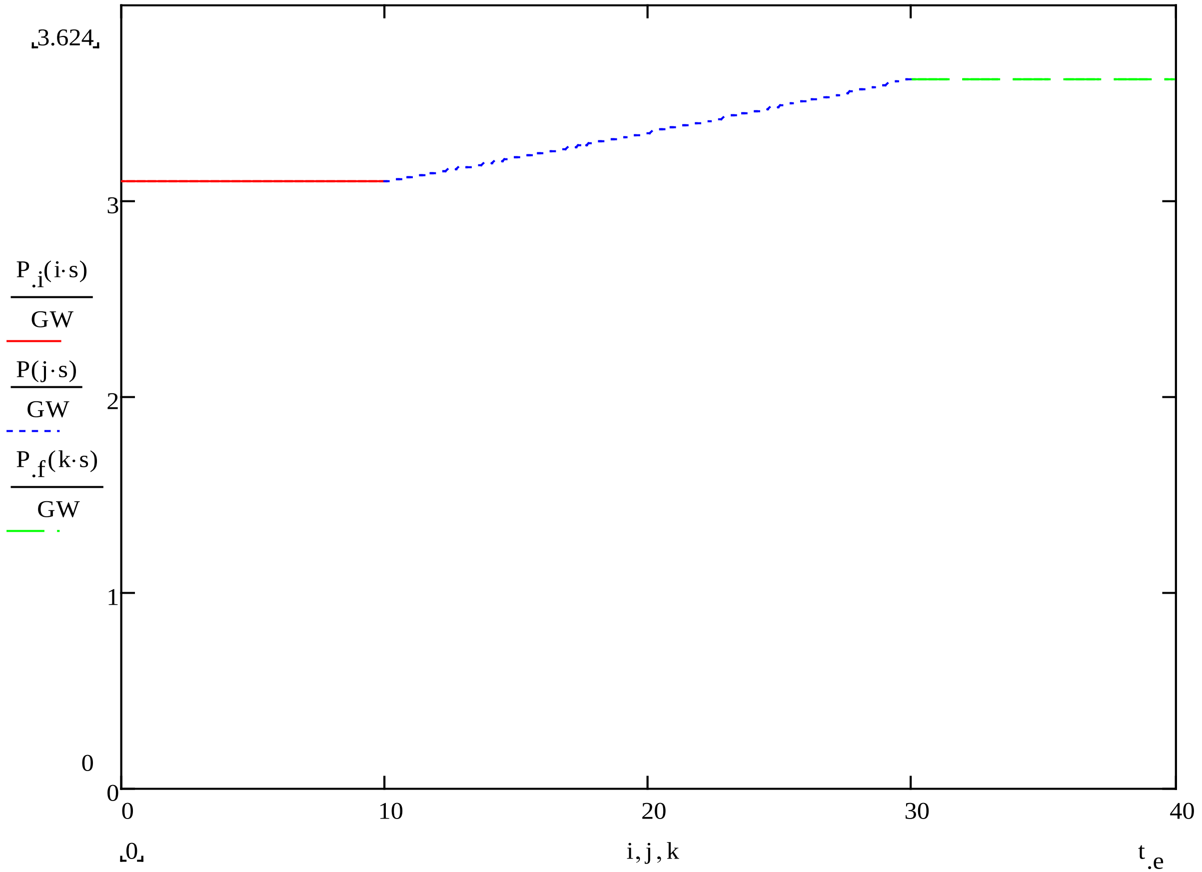
# Exploration 2

- What if we add $0.1 to AP1000 core?

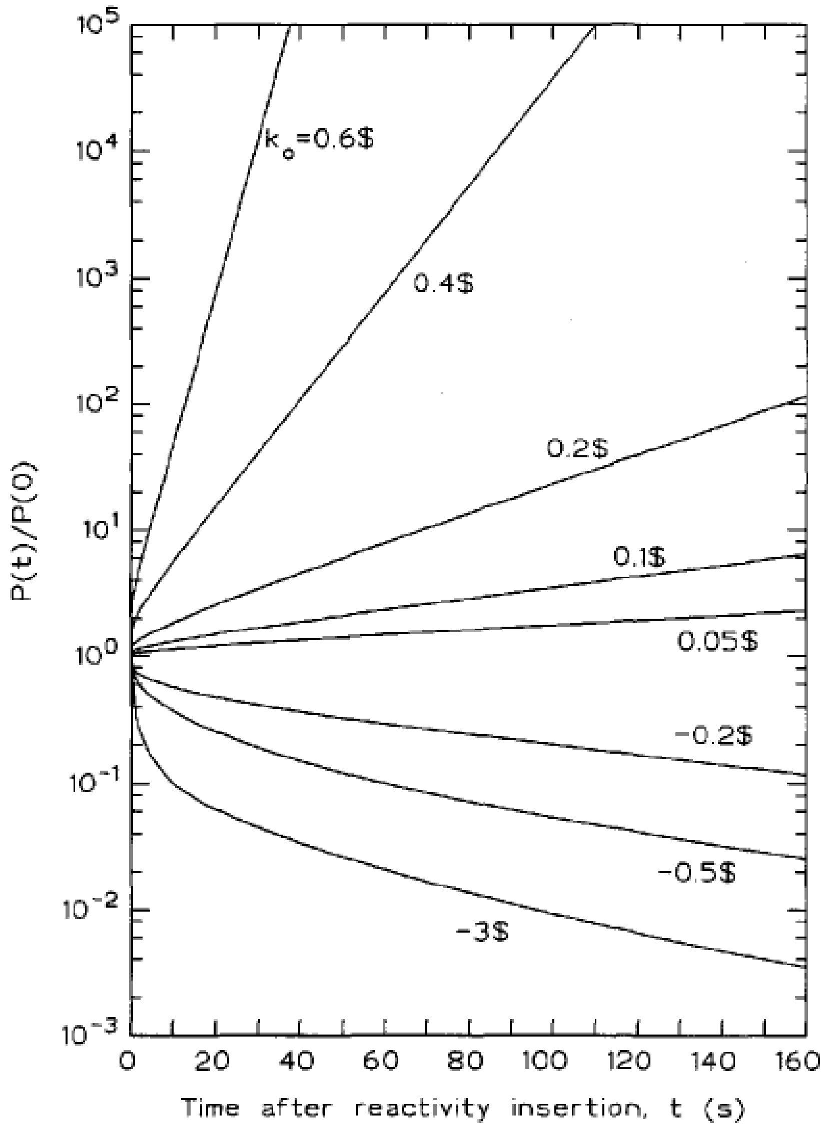


# Exploration 3

- What if we add $0.1 to AP1000 core, then after 10 seconds we add -$0.1?



# Reality



$k_o$=0.6$
0.4$
0.2$
0.1$
0.05$
−0.2$
−0.5$
−3$
P(t)/P(0)
Time after reactivity insertion, t (s)

# Kinetics

- This is how reactor power is controlled
  - Control rods add/subtract worth
- The circumstances we've seen so far are not a real, however.  Why?

- Often a balancing influence is experienced…

- Feedback Mechanisms!

# Isotopic Feedbacks (slow)

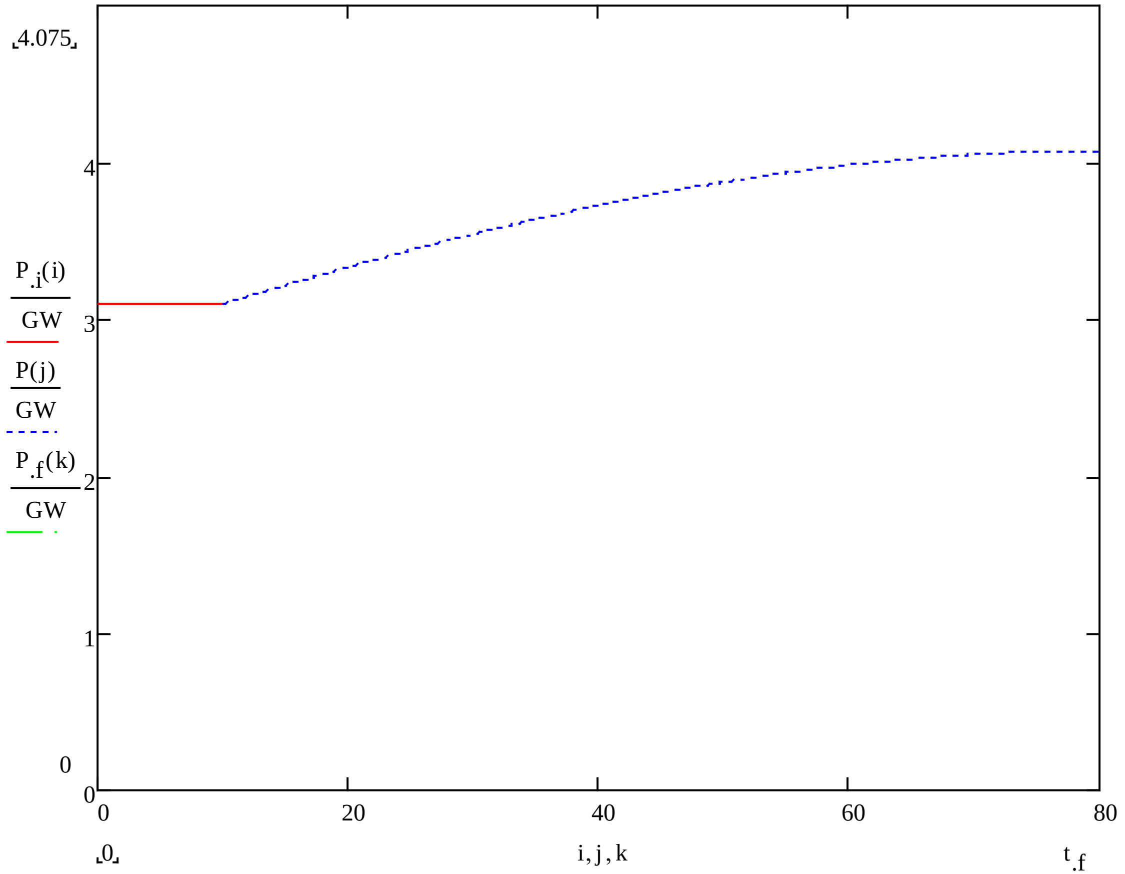
- Fuel Burnup (slow)
  - Decrease in reactivity
- Fuel breeding (slow)
  - Increase in reactivity
- Fission product poisons (moderate–hours)
  - $^{135}Xe$ and $^{149}Sm$
  - Decrease reactivity until decay away
- Burnable Poisons (slow)
  - Decrease reactivity until transmuted away

# Temperature Feedbacks (fast)

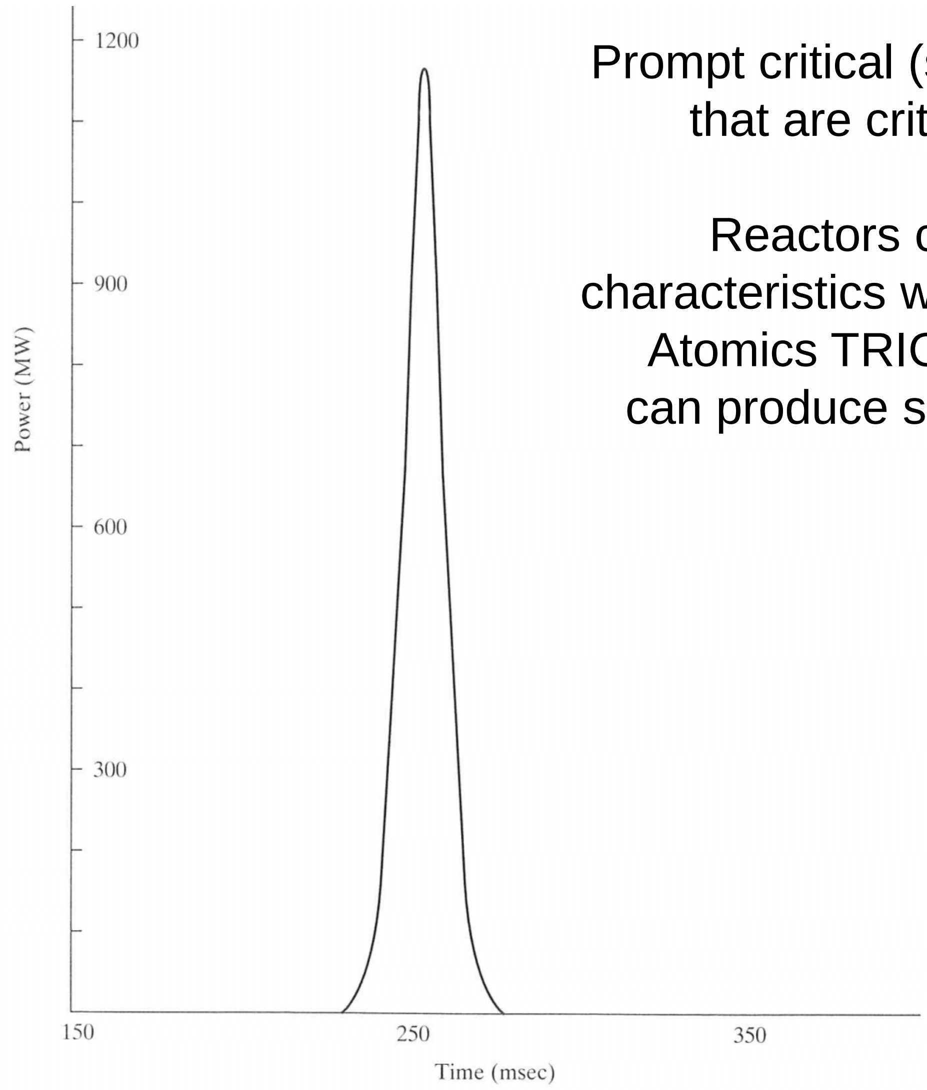
- Atomic concentration changes
  - Moderator coolant density
  - Void coefficient fuel expansion
- Neutron energy distribution changes
  - “harden” spectrum with increased T
  - TRIGA reactor is extreme example
- Resonance interaction changes
  - Doppler – dominant feedback
- Burnable Poisons
  - Geometry changes

# Feedback Effects

- What if we add $0.1 to AP1000 core with void feedbacks included?



# Exotic Reactors



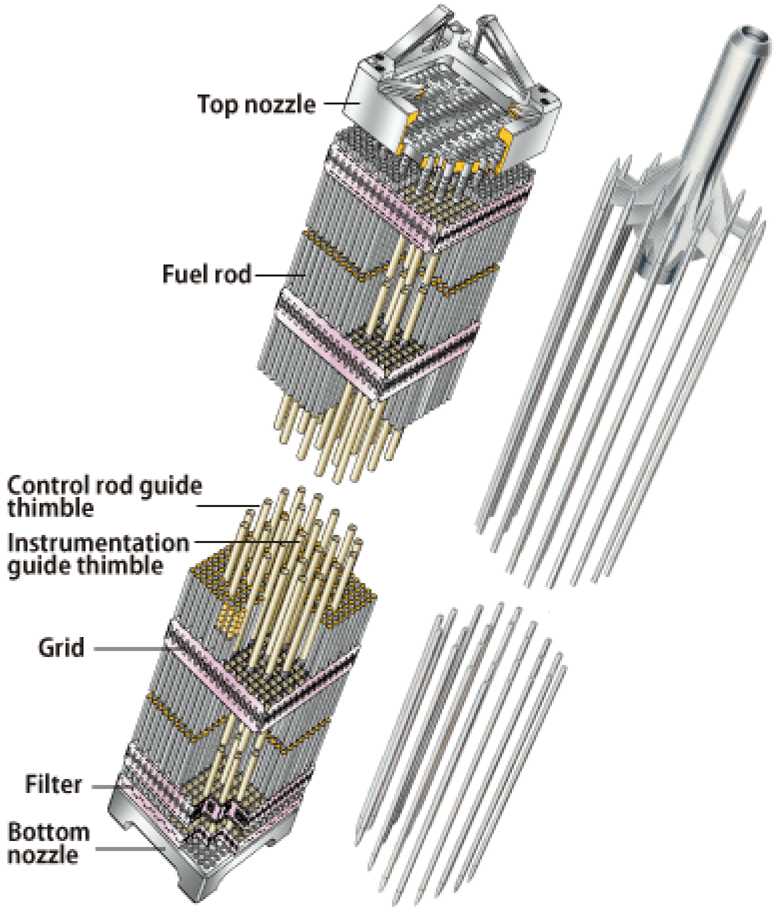

Prompt critical (supercritical) behavior refers to reactors that are critical based on prompt neutrons only and hence have very short periods.

Reactors can be designed with inherent shutdown characteristics when they become supercritical. General Atomics TRIGA reactor is an example. Such reactors can produce short but intense pulses of neutrons (see chart at left).

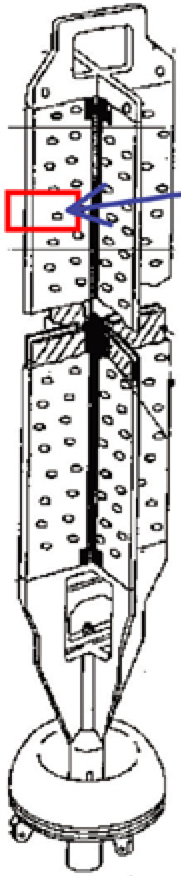
# Ramifications

- For positive reactivity (increases in power), which necessarily must be small, prompt neutron jump is negligible, (flux essentially unchanged in the short term)
- For negative reactivity (decreases in power)
  - can be arbitrarily large
  - prompt neutron change can be very large
  - Up to 96% in the case of  a scram over about 80 seconds.
- Fission product decay accounts for up to 6% of total power (for an equilibrium reactor)
  - not affected by the reactivity change
  - cannot reduce by more than about 93% the power output

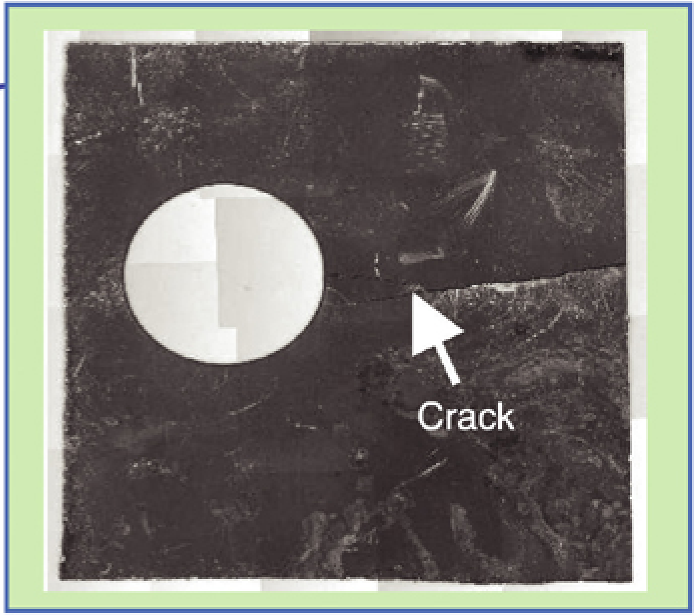
# Cluster Control Rods



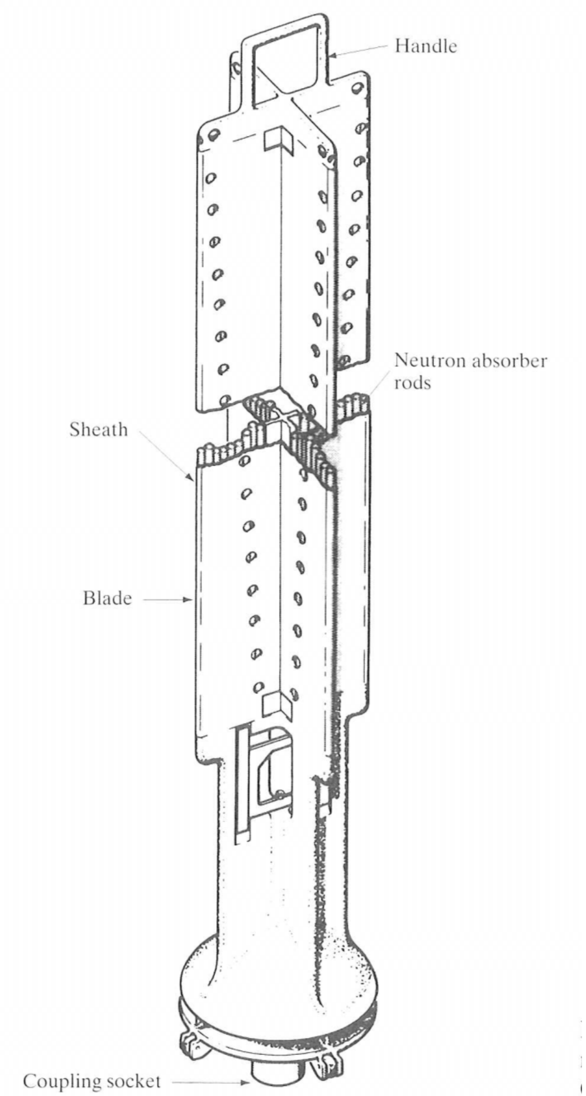

# Cruciform Control Rods



Control rod

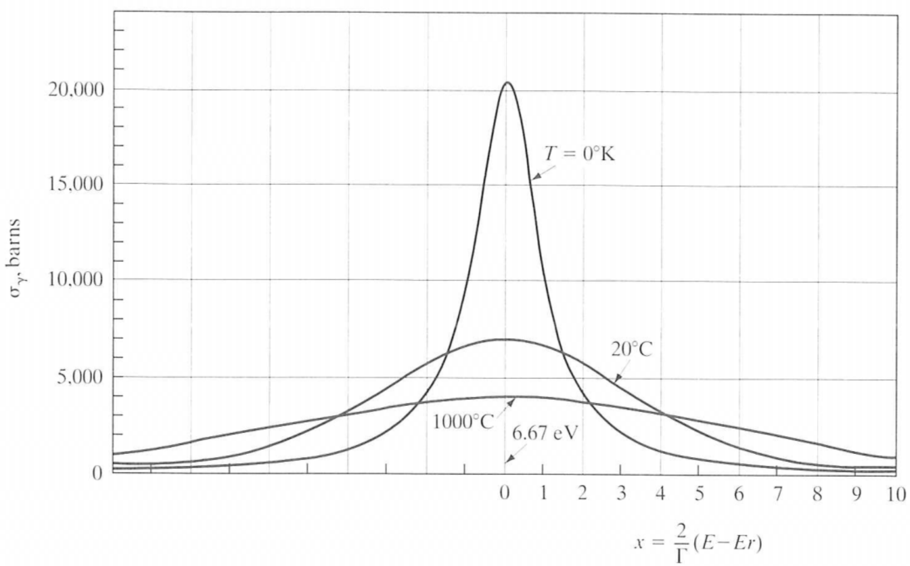



Cracks in the control rod



# Temperature Dependence

$$\alpha_T = \frac{d\rho}{dT} = \frac{d}{dT}\left(\frac{k-1}{k}\right) = \frac{1}{k^2}\frac{dk}{dT} \cong \frac{1}{k}\frac{dk}{dT}$$




Breit-Wigner describes absorption profile at 0 K but Doppler effect broadens peaks, with little change in area, at higher temperatures.

$$\sigma_\gamma(E) = \frac{\lambda_r^2 g}{4\pi} \frac{\Gamma_n \Gamma_\gamma}{(E - E_r)^2 + \frac{\Gamma^2}{4}}$$

- *$\alpha_T$ = temperature reactivity feedback coefficient*
- *If $\alpha_T$* > 0,
  - Unstable
  - increases and decreases in temperature run away to meltdown or shutdown without operator response.
- *If $\alpha_T$* < 0,
  - Stable
  - Increases and decreases in temperature self regulate and the reactor stabilizes.
- Different $\alpha$'s for fuel/moderator
  - Different timescales
    - Fuel is most rapid
    - $\alpha_{prompt}$
- NRC requires negative $\alpha_{prompt}$ values for licenses

# Xenon (Iodine, Tellurium)

Xenon-135 has a high absorption cross section (2.65x10[6] b in thermal region) and is the most significant absorbing poison.

$$\underset{\substack{\uparrow \\ Fission}}{^{135}Te} \xrightarrow[11\,sec]{\beta^-} \underset{\substack{\uparrow \\ Fission}}{^{135}I} \xrightarrow[6.7\,hr]{\beta^-} \underset{\substack{\uparrow \\ Fission}}{^{135}Xe} \xrightarrow[9.2\,sec]{\beta^-} {^{135}Cs} \xrightarrow[2.3x10^6\,yr]{\beta^-} {^{135}Ba}\ (stable)$$

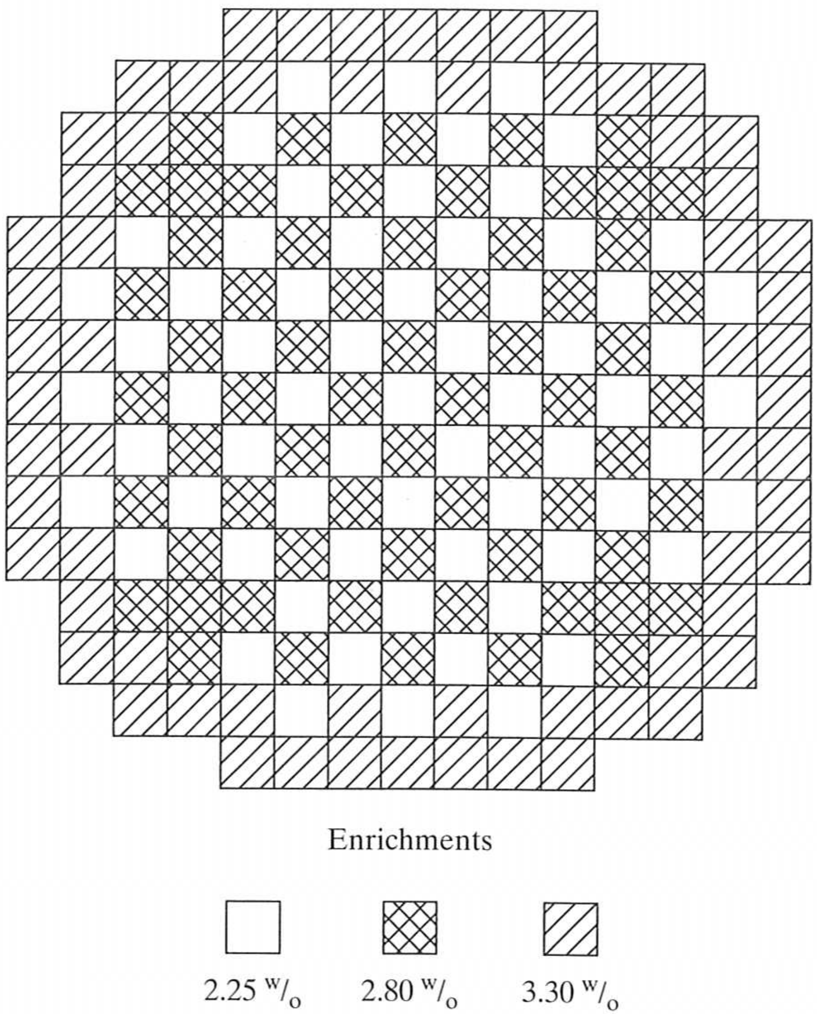
$$\frac{dI}{dt} = \underbrace{\gamma_I \overline{\Sigma}_f \phi_T}_{fission\ yield} - \underbrace{\lambda_I I}_{natural\ decay}$$

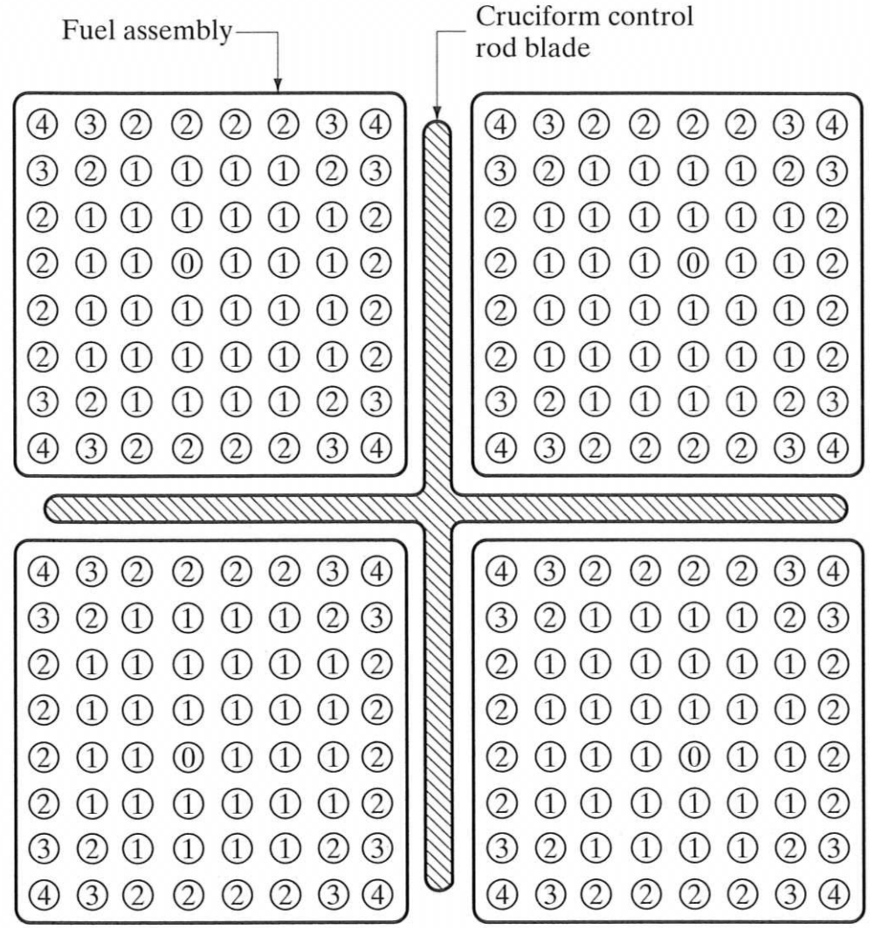
$$\frac{dX}{dt} = \underbrace{\lambda_I I}_{Iodine\ decay} + \underbrace{\gamma_X \overline{\Sigma}_f \phi_T}_{fission\ yield} - \underbrace{\lambda_X X}_{natural\ decay} - \underbrace{\overline{\sigma}_{aX} \phi_T X}_{absorption\ decay}$$

$$= \lambda_I I + \gamma_X \overline{\Sigma}_f(t)\phi_T(t) - \left(\lambda_X + \overline{\sigma}_{aX}\phi_T\right)X$$

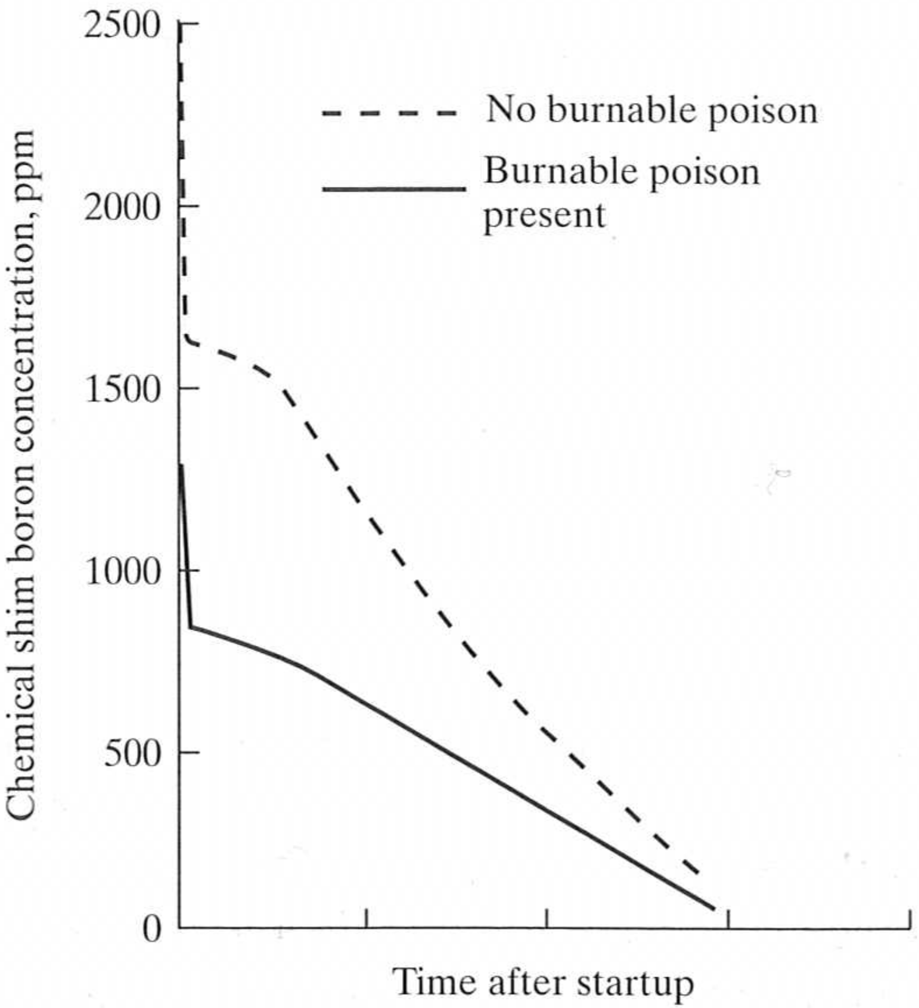
$$\left(T_{1/2}\right)_{X,eff} = \frac{0.693}{\lambda_X + \overline{\sigma}_{aX}\phi_T(t)}$$

# Fuel Loading Patterns



# Burnable (absorbing) poisons




Burnable poison forms products with lower adsorption cross sections, compensating for accumulation of other poisons. Boron and gadolinium oxides (gadolina) are examples.

# Typical Control Worths

**TABLE 7.7** TYPICAL REACTIVITY WORTHS FOR CONTROL ELEMENTS 3,000 MWT LIGHT-WATER REACTOR

| | PWR | BWR |
|---|---|---|
| Excess reactivity at 20°C | \$45 | \$38 |
| No Xe or Sm | $k = 1.41$ | $k = 1.33$ |
| Total control rod worth | \$11 | \$26 |
| | ~60 clusters | 140–185 rods |
| Fixed burnable poisons | \$13 | \$18 |
| Chemical shim worth | \$26 | – |
| Net reactivity | –\$5 | –\$6 |