Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 2

Quantum Mechanics I

Relativity

Spiritual Thought

Each of you finds yourself in circumstances unique to you alone. Each must practice the acts of your own character. If you would be truly happy, these acts must be acts of selflessness. Selflessness will turn sadness into a cheerful countenance. Selflessness produces kindness and dispels hypocrisy. Selflessness develops love, confidence, and trust. It is the vehicle of generosity. It is the resource of God to answer the prayers of his children.

-Elder William R. Bradford

Soccer Ball off a truck?

Newtonian vs. Maxwellian

- $\sum F = \frac{dm\vec{v}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$
  - One form/solution in non-moving reference
  - Same solution at any constant v reference
    - x'=x-vt, y'=y, z'=z, t'=t
- Maxwell Equations (electricity & magnetism)
  - One form/solution in non-moving reference
  - DIFFERENT form in constant v reference
    - Lorentz transformation:

$x' = \frac{x-vt}{\sqrt{1-{v^2}/{c^2}}}$, y'=y, z'=z, $t' = \frac{t-{vx}/{c^2}}{\sqrt{1-{v^2}/{c^2}}}$

Special Relativity Summary

2 Postulates:

- All physical laws should have the same form in any inertial (constant relative velocity) reference frame.
- The speed of light in free space (a vacuum) is independent of the relative velocity of the light source and the observer.

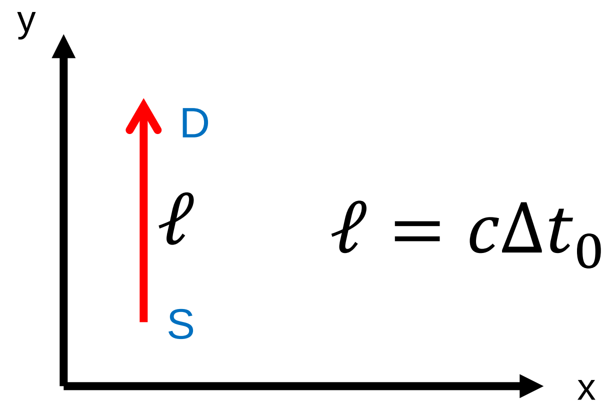
4 Results:

Relative to a fixed observer, moving objects exhibit

- Mass increase: $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$
- Length decrease: $L = L_0\sqrt{1 - v^2/c^2}$
- Time dilation (slower): $\Delta t = \frac{\Delta t_0}{\sqrt{1-v^2/c^2}}$
- Energy and mass are equivalent: $E = mc^2$

Time Dilation Derivation

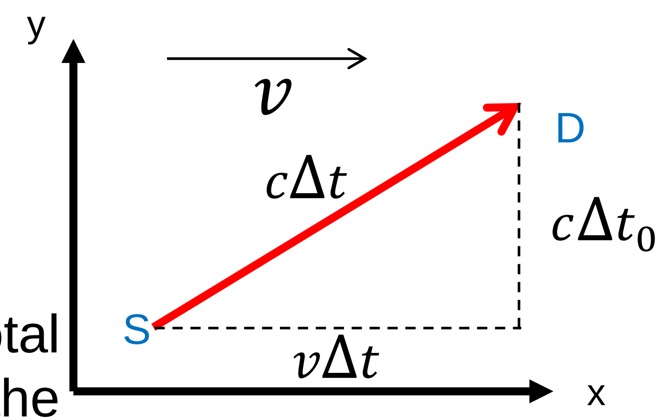
Measurements in stationary reference frames have subscript 0s and are called proper measurements.



stationary system

$$(c\Delta t)^2 = (v\Delta t)^2 + (c\Delta t_0)^2$$

$$\Rightarrow \Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

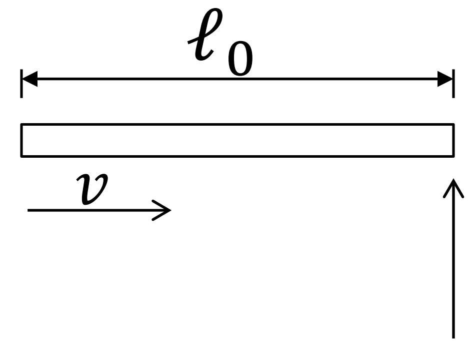


constant velocity system

Conventional behavior indicates that the total velocity in the moving case relative to the stationary observer (the diagonal) is the vector sum of two components. If the velocity components are fixed (premise #2), elapsed time must be different.

Length Decrease

The apparent length of a constant-velocity sled can be measured by a stationary observer as the time it takes to pass a fixed point times the velocity



$$\ell = v\Delta t_0 \Rightarrow v = \frac{\ell}{\Delta t_0}$$

A sled passenger measures the length (ℓ_0 - since there is no relative velocity) as the time it takes to pass a fixed point times the velocity.

$$\ell_0 = v\Delta t = v\frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \frac{\ell}{\Delta t_0}\frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \ell = \ell_0\sqrt{1 - v^2/c^2}$$

Mass Increase

Two people, one static and one traveling at constant relative velocity v, toss identical and elastic balls towards each other, causing a head-on collision of the balls. The stationary observer tosses the ball at velocity $u = h/\Delta t_0$ and perceives the other ball to be approaching at a velocity $w = h/\Delta t$. The relationship between u and w is

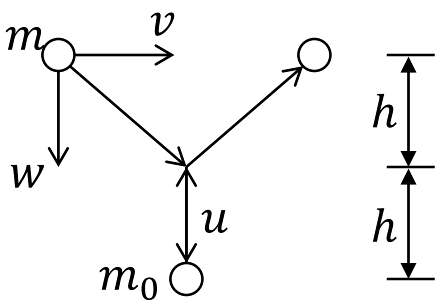
$$w = \frac{h}{\Delta t} = \frac{h\sqrt{1 - v^2/c^2}}{\Delta t_0} = u\sqrt{1 - v^2/c^2}$$

Conservation of momentum requires that the sum of the momentum (in any reference frame – by the first postulate) must be equal before and after the collision, that is,

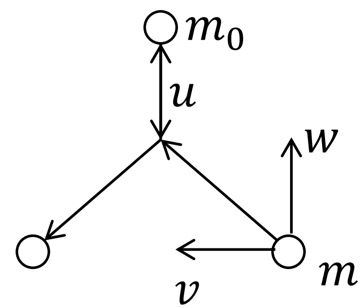
$$m_0 u - mw = -m_0 u + mw$$

$$m_0 u = mw = mu\sqrt{1 - v^2/c^2}$$

$$\Rightarrow m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

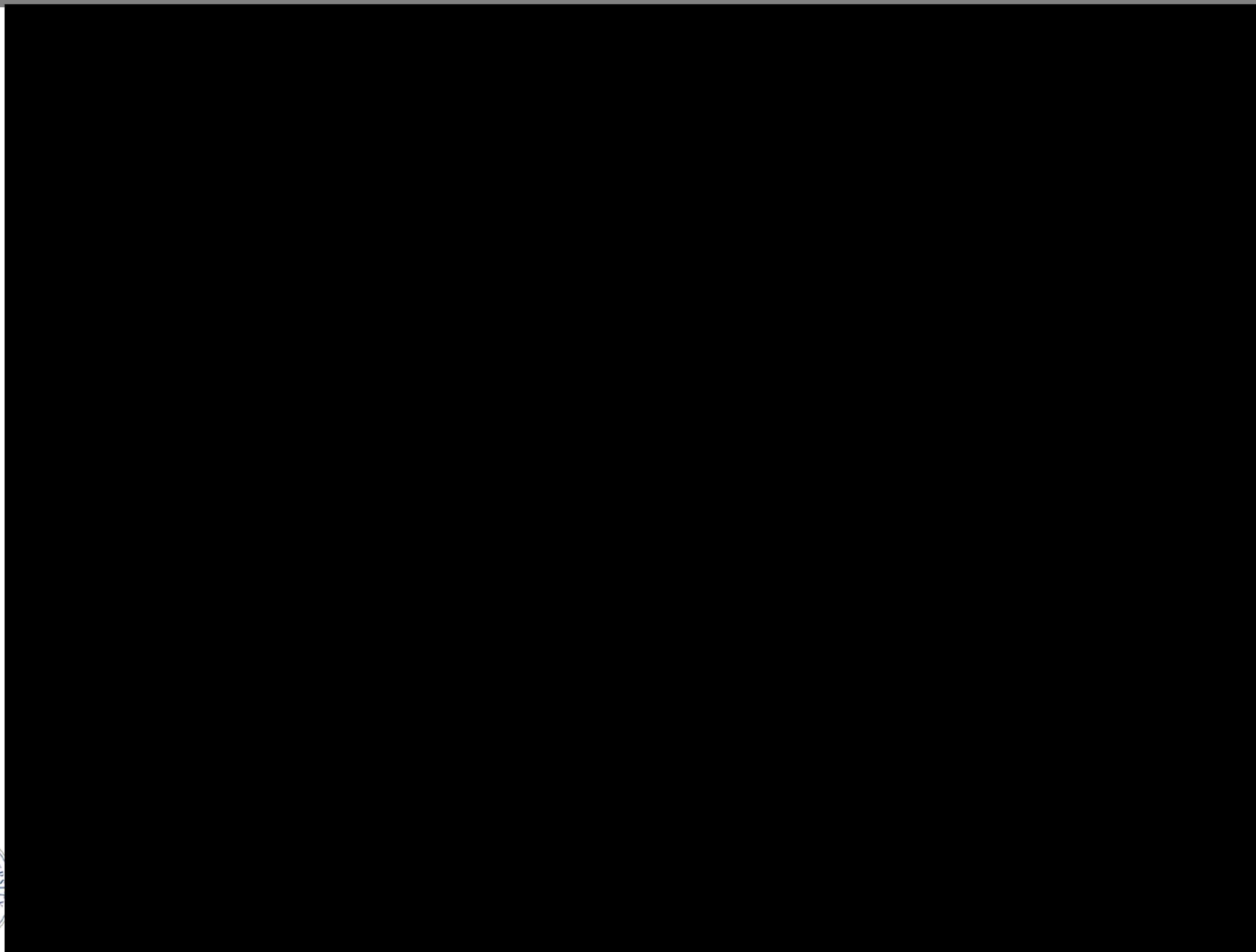


stationary perspective



moving perspective

Illustrations



What $E = mc^2$ means?

Mass and Energy-mathematically equivalent

- Physically equivalent
  - Different manifestations of same thing
  - Adding energy adds mass
    - Higher temperature = more mass
    - Imperceptible at traditional ranges
- Waves - mass, gravitation & momentum
  - Light, electrical waves, kinetic energy, potential energy, and thermal energy
- Mass annihilation – large energy releases
  - Star Trek, not nuclear power

Momentum, KE: Classical & Relativistic

- Classical Mechanics: p = mv
  - $T = \frac{mv^2}{2} = \frac{p^2}{2m} = mc^2 - m_o c^2$
  - $p = \sqrt{2mT}$
- Relativistic Mechanics:
  - $m = \frac{m_0}{\sqrt{1-v^2/c^2}} \rightarrow m_o{}^2 = m^2\left(\frac{c^2-v^2}{c^2}\right)$

Using relativistic mass

$$p^2 \equiv (mv)^2 = \frac{m_0^2 v^2}{1-\frac{v^2}{c^2}} = \frac{1}{c^2}\left[\left(mc^2\right)^2 - \left(m_0 c^2\right)^2\right] = \frac{1}{c^2}\left(T^2 + 2Tm_0c^2\right)$$

Resulting in

$$p = \frac{\sqrt{T^2 + 2Tm_0c^2}}{c} \quad T = c\sqrt{p^2 + m_0^2c^2} - m_0c^2$$

Wave Momentum/Mass

- Waves have mass/momentum ($m = h\nu/c^2$)
- Mass comes only from speed
- $\lambda = \frac{c}{\nu}$
- $E = h\nu$
  - $E = mc^2$
- $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h\nu}{\lambda}$

Note difference between frequency, given the symbol ν, and velocity, given the similar symbol v. Since light always travels at velocity c, it is rare to have v in an equation about light, though common to have ν in such equations.

- H = planck's constant
  - = 6.626×10^{-34} J·s

Summary

quantity	real (relativistic)	classical
time (t)	$\gamma\, t_0$	t_0
length (ℓ)	ℓ_0/γ	ℓ_0
mass (m)	γm_0	m_0
momentum (p)	$mv = \gamma m_0 v$	$m_0 v$
kinetic energy (T)	$(m - m_0)c^2$ $= (\gamma - 1)m_0 c^2$	$1/2 m_0 v^2$

Lorentz factor $$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Particle Wave Duality (I)

- Waves have particle properties
  - quanta or photons
  - Photoelectric effect, Compton Scattering
- Particles have wave properties
  - De Broglie wave-length
  - Electron scattering
  - $\lambda = \frac{h}{p} = \frac{hc}{\sqrt{T^2+2Tm_oc^2}}$

Particle Wave Duality (II)

- One property (wave or particle) usually dominates
  - Waves:
    - Large wavelengths (λ >10^{-6} m) wave properties
    - Small wavelengths (λ <10^{-8} m) particle properties
  - Particles: $\lambda = \frac{hc}{\sqrt{T^2+2Tm_oc^2}}$
    - At very high speed, relativistic behavior
      - $T^2 \gg 2Tm_oc^2$
    - If λ<10^{-10} m, particle behaves as particle

Particle Wave Duality (III)

- Neutron behavior?
  - Low E:
    - E~10^{-6} eV
    - $\lambda = 2.86\times10^{-8}$ m
    - comparable to atom spacing
    - scatter off multiple atoms.
  - High E:
    - E~1 MeV
    - $\lambda = 2.86\times10^{-14}$ m
    - comparable to nucleus size
    - scatter off nucleus.