Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 5

Nuclear Energetics

Spiritual Thought

"I add my voice to these wise and inspired brethren and say to you that one of the most important things you can do as priesthood leaders is to immerse yourselves in the scriptures. Search them diligently. Feast upon the words of Christ. Learn the doctrine. Master the principles that are found therein. There are few other efforts that will bring greater dividends to your calling. There are few other ways to gain greater inspiration as you serve.

But that alone, as valuable as it is, is not enough. You must also bend your efforts and your activities to stimulating meaningful scripture study among the members of the Church."

President Ezra Taft Benson

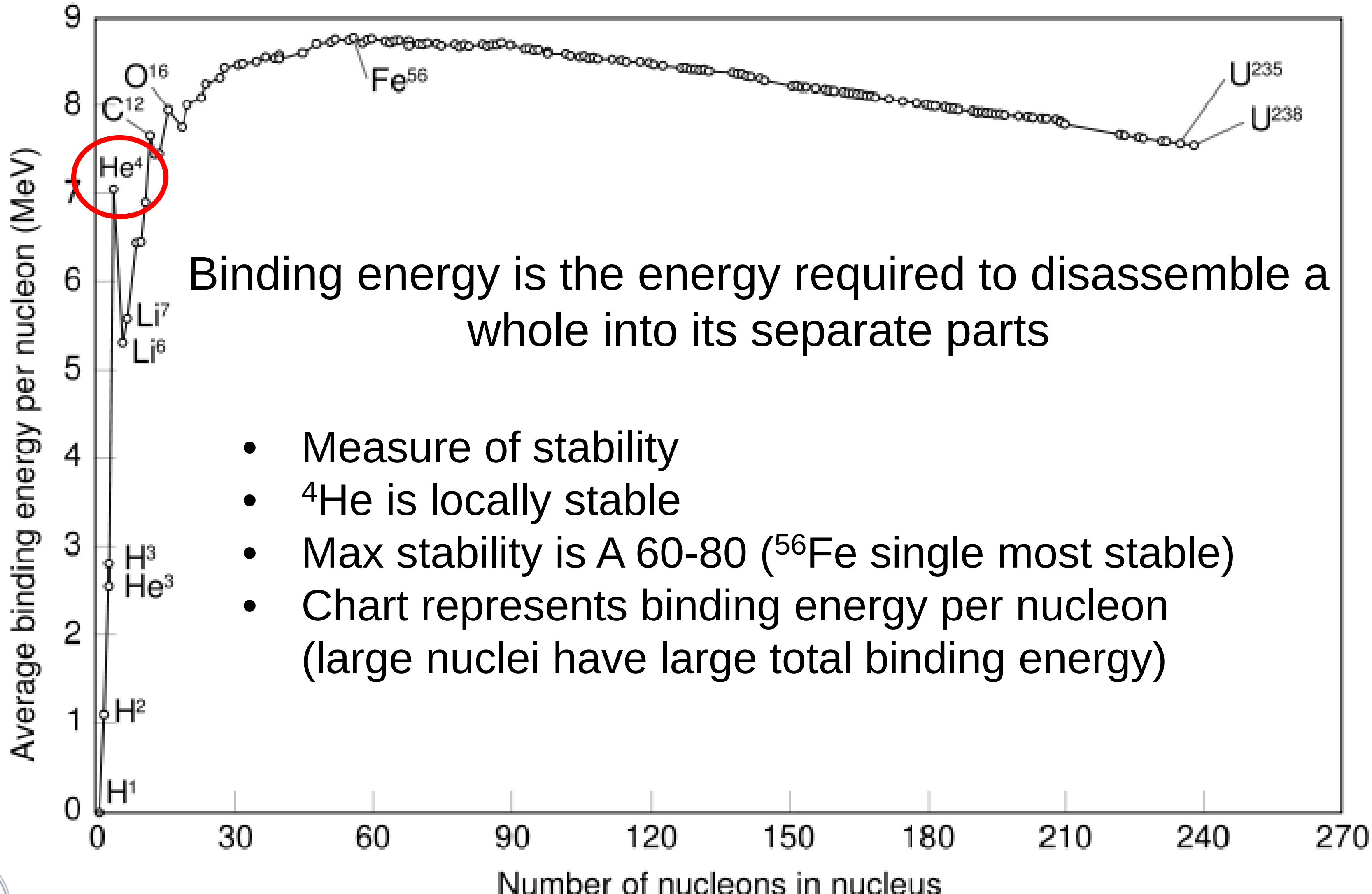
Big Picture

- Introduced subatomic particles
- Explored Quantum Mechanics and impacts
- Defined Nucleus
  - Made of protons and neutrons
  - Some mass is converted to energy
  - This holds nucleus together (binding energy)
- Nuclear Energetics
  - Q Value

Nuclear Energetics

- Study of mass/energy changes in nucleus
  - Reactions
  - Stability
  - Mass Defect
  - Binding Energy/Separation Energy
- Foundational to understanding radioactive decay

Binding Energy



Binding energy is the energy required to disassemble a whole into its separate parts

- Measure of stability
- ^{4}He is locally stable
- Max stability is A 60-80 (^{56}Fe single most stable)
- Chart represents binding energy per nucleon (large nuclei have large total binding energy)

Reaction Terminology

- Chemistry
  - Exothermic reactions
    - Generate heat
    - Negative heat of reaction
  - Endothermic reactions
    - Consume heat
    - Positive heat of reaction
- Nuclear chemistry
  - Exothermic = Exoergic
    - Positive Q-values
  - Endothermic = Endoergic
    - Negative Q-values

Not “thermal”, because it’s not traditional heat transfer; atomic scale with wave emission and kinetic energy

Mass Defect/Binding Energy

- $E=mc^2$ -----> $\Delta E = \Delta mc^2$
  - Even for macroscopic effects, but tiny
    - 10^{-8} % for formation of CO_2 molecule
- Δm = mass defect
    - m = nuclear, M = atomic → How to define M?
    - $M({}^{A}_{Z}X) = m({}^{A}_{Z}X) + Zm_e - \frac{BE_{Ze}}{c^2}$ → How to define Δm?
  - $\Delta m = \frac{BE}{c^2} = Zm_p + (A-Z)m_n - m({}^{A}_{Z}X)$ Nuclear values
- Binding Energy
  - $BE({}^{A}_{Z}X) = [ZM({}^{1}_{1}H) + (A-Z)m_n - M({}^{A}_{Z}X)]c^2$

Put in terms of atomic values

- Separation Energy
  - $Sn({}^{A}_{Z}X) = BE({}^{A}_{Z}X) - BE({}^{A-1}_{Z}X)$

Reactions

- Nuclear reactions
  - 1 (decay), 2, or 3 (rare) particles
  - Sometimes written like Chemical reactions:

$$ {}^{4}_{2}He + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + {}^{1}_{1}H $$

- For single reactions this is common
- For binary nuclear reactions a more compact nomenclature is typical,

$$ {}^{14}_{7}N(\alpha, p)\,{}^{17}_{8}O $$

Lightest nuclides in parentheses

Note: this is the first nuclear reaction detected, by Rutherford

Nuclear Conservation

- Chemical reactions
  - – conserve enthalpy, elements, and total mass.
- Nuclear reactions
  - – Don’t conserve any of these
  - – Do conserve
    - Total energy (mass + kinetic/radiative energy)
    - Nucleons (protons + neutrons)
    - Electrical charge.
  - – Note ***sum*** of protons and neutrons (nucleons) is conserved.
  - – Electrons <u>**NOT**</u> conserved; charge is.

Parallel reactions

- Chemistry – multiple possible reactions
  - Large quantities of reactants
  - Branching ratios, equilibrium, etc.
- Nuclear – single nuclei at a time
  - Statistical probabilities for various outcomes
- Example, neutron interacting with sulfur
  - $^{32}_{16}S(n,n)^{32}_{16}S$ α%
  - $^{32}_{16}S(n,n')^{32}_{16}S^{*}$ β% where β=0% if E_n<XX
  - $^{32}_{16}S(n,p)^{32}_{15}P$ δ% where δ =0% if E_n<YY
  - $^{32}_{16}S(n,\gamma)^{33}_{16}S$ (1-α-β-δ)%

Q-value

$$Q = KE_{products} - KE_{reactants}$$

$$\frac{Q}{c^2} = reactant\ rest\ mass - product\ rest\ mass$$

note that this is reactants – products, the opposite convention as is used in chemistry

- 3 special challenges:
  - Charge balancing
  - Reactions with changes in the number of protons
  - Excited-state rather than ground-state isotopes
- The next several slides cover these

Simple Q-value Calculations

- No:
  - proton-neutron transformations (β emissions),
  - excited states,
  - other special cases
- For example, the reaction of lithium with deuterium

$$ {}^{6}_{3}Li\left({}^{2}_{1}H, {}^{4}_{2}He\right){}^{4}_{2}He $$

$$ Q = M_{{}^{6}_{3}Li} + M_{{}^{2}_{1}H} - M_{{}^{4}_{2}He} - M_{{}^{4}_{2}He} $$
$$ = 6.015122 + 2.014101 - 2(4.002603) = $$
$$ = 0.02402\, u \left(931.5 \frac{MeV}{u}\right) = 22.37\ MeV $$

Be sure to conserve charge

- Often, reactions notations don't include electrons
  - $^{16}_{8}O(n,p)^{16}_{7}N$ or, more explicitly, $^{16}_{8}O(n,p)^{16}_{7}N + {}^{0}_{-1}e$

can be represented as

$$^{16}_{8}O\left(n, {}^{1}_{1}H\right)^{16}_{7}N$$

- So, to conserve charge, non-ionized particles used.

$$\frac{Q}{c^2} = M\left(^{16}_{8}O\right) + m_n - M\left(^{16}_{7}N\right) - M\left(^{1}_{1}H\right)$$
$$= 15.994915 + 1.0086649156 - 16.006101$$
$$- 1.007825 = -0.01035/c^2 \text{ u} = -9.637 \text{ MeV}$$

Note: Q-values commonly involve mass differences in the 3rd to 4th decimal point.

When Z changes

- Changes in the proton number (Z) require special step
  - Include neutrino or anti-neutrino
  - Sometimes a positron (antimatter electron with positive charge).

$$p\left(p, {}_{+1}^{0}e\ \nu\right)d \quad \text{or} \quad {}_{1}^{1}p + {}_{1}^{1}p \rightarrow {}_{1}^{2}d + {}_{+1}^{0}e + \nu$$

- Drives deuterium formation in the sun
  - Q-value is computed by adding two electrons to each side
  - Form two hydrogens on the left
  - Form deuterium and an extra electron on the right

$${}_{1}^{1}H\left({}_{1}^{1}H, \nu\, {}_{+1}^{0}e\, {}_{-1}^{0}e\right){}_{1}^{2}H$$

$$\frac{Q}{c^2} = 2M\left({}_{1}^{1}H\right) - M\left({}_{1}^{2}H\right) - 2m_e - m_\nu$$

=0.420 MeV

m_ν is unknown but known to be very small (negligible) compared to m_e

Excited Nuclei

- Some reactions produce stable (long decay time) excited-state nuclei
- Excited states are designated by an asterisk
- For these, the energy difference between the ground and excited states must be known, (E^*)

$$^{10}_{5}B(n,\alpha)^{7}_{3}Li^*$$

$$\frac{Q}{c^2} = m_n + M\left(^{10}_{5}B\right) - M\left(^{7}_{3}Li^*\right) - M\left(^{4}_{2}He\right)$$
$$= m_n + M\left(^{10}_{5}B\right) - M\left(^{7}_{3}Li\right) - M\left(^{4}_{2}He\right) - E^*$$
$$m_n + M\left(^{10}_{5}B\right) - M\left(^{7}_{3}Li\right) - M\left(^{4}_{2}He\right) - Q_{^{7}_{3}Li^* \rightarrow ^{7}_{3}Li}$$
$$= m_n + M\left(^{10}_{5}B\right) - M\left(^{7}_{3}Li\right) - M\left(^{4}_{2}He\right) - 0.48MeV$$

Nuclear Fusion vs. Nuclear Fission

- Nuclear Fusion:
  - $\left({}^{2}_{1}H\right) + \left({}^{2}_{1}H\right) \rightarrow \left({}^{4}_{2}He\right)$
  - Energy released?
    - 23.85 MeV
- Nuclear Fission
  - $\left({}^{235}_{92}U\right) \rightarrow \left({}^{117}_{46}Pd\right) + \left({}^{117}_{50}Sn\right) + n$
  - Energy released?
    - ~210 MeV
- Why fusion?

Some notes

- Text provides isotopic masses for most isotopes.
- The official masses (in the US) are updated by NIST at http://www.nist.gov/pml/data/comp.cfm.