Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 7

Nuclear Decay Behaviors

Spiritual Thought

"Sooner or later, I believe that all of us experience times when the very fabric of our world tears at the seams, leaving us feeling alone, frustrated, and adrift.

It can happen to anyone. No one is immune. Everyone's situation is different, and the details of each life are unique. Nevertheless, I have learned that there is something that would take away the bitterness that may come into our lives. There is one thing we can do to make life sweeter, more joyful, even glorious.

We can be grateful!"

– Dieter F. Uchtdorf

BRIGHAM YOUNG UNIVERSITY FOUNDED BYU 1875 PROVO, UTAH

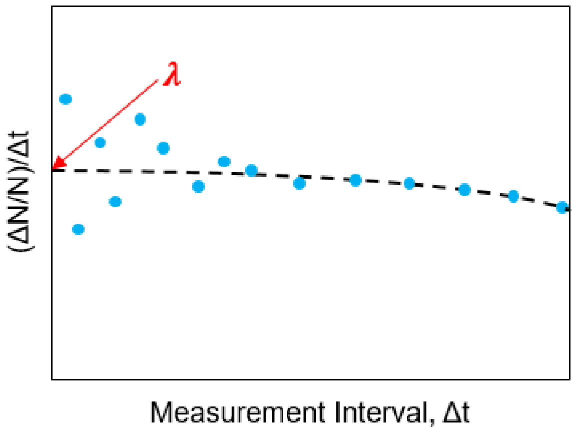
Last Time: Decay Mechanisms

- Gamma ($\boldsymbol{\gamma}$)
- Alpha ($\boldsymbol{\alpha}$)
- Beta (+/-) ($\boldsymbol{\beta}^{+}, \boldsymbol{\beta}^{-}$)
- Electron capture (EC)
- Proton (P)
- Neutron (N)
- Internal conversion (IC)

Decay Constant (λ)

N = number of atoms in the radionuclide sample
ΔN = atoms that undergo decay during Δt

$$\lambda \equiv \lim_{\Delta t \to 0} \frac{\left(\Delta N / N\right)}{\Delta t}$$



Definition: the probability that a radionuclide will decay over an infinitesimally small unit of time

$$\lambda \neq f(T, P, N_i, t)$$

Exponential Decay

Now we convert to continuous mathematics, and:

$N(t)$ = average or expected number of radionuclides at time t

$$-dN = \lambda N(t)dt$$

$$\frac{dN(t)}{dt} = -\lambda N(t)$$

$$\mathrm{N}(t) = N_o e^{-\lambda t}$$

Radioactive Decay Law

where N_o = number of radionuclides in the sample at $t = 0$

Half-Life ($T_{1/2}$)

Definition: constant representing the time it takes for a radionuclide to decay to ½ of its initial sample size

$$N(T_{1/2}) = \frac{N_o}{2} = N_o e^{-\lambda T_{1/2}}$$

$$T_{1/2} = \frac{ln2}{\lambda} \cong \frac{0.693}{\lambda}$$

$$T_{1/2} \neq f(T, P, N_i, t)$$

When solving for number of half-lives n needed for a radioactive sample to decay to a fraction ϵ of its initial value:

$$\epsilon = \frac{N(nT_{1/2})}{N_o} = \frac{1}{2^n}$$

$$n = -\frac{\ln \epsilon}{\ln 2} \cong -1.44 \ln \epsilon$$

Example

A sample contains pure $^{127}_{52}Te$ (Tellurium) which undergoes β^- decay and has a $T_{1/2}$ of 9.35 h.

(a) What is the decay constant (in h^{-1}) for this radionuclide.

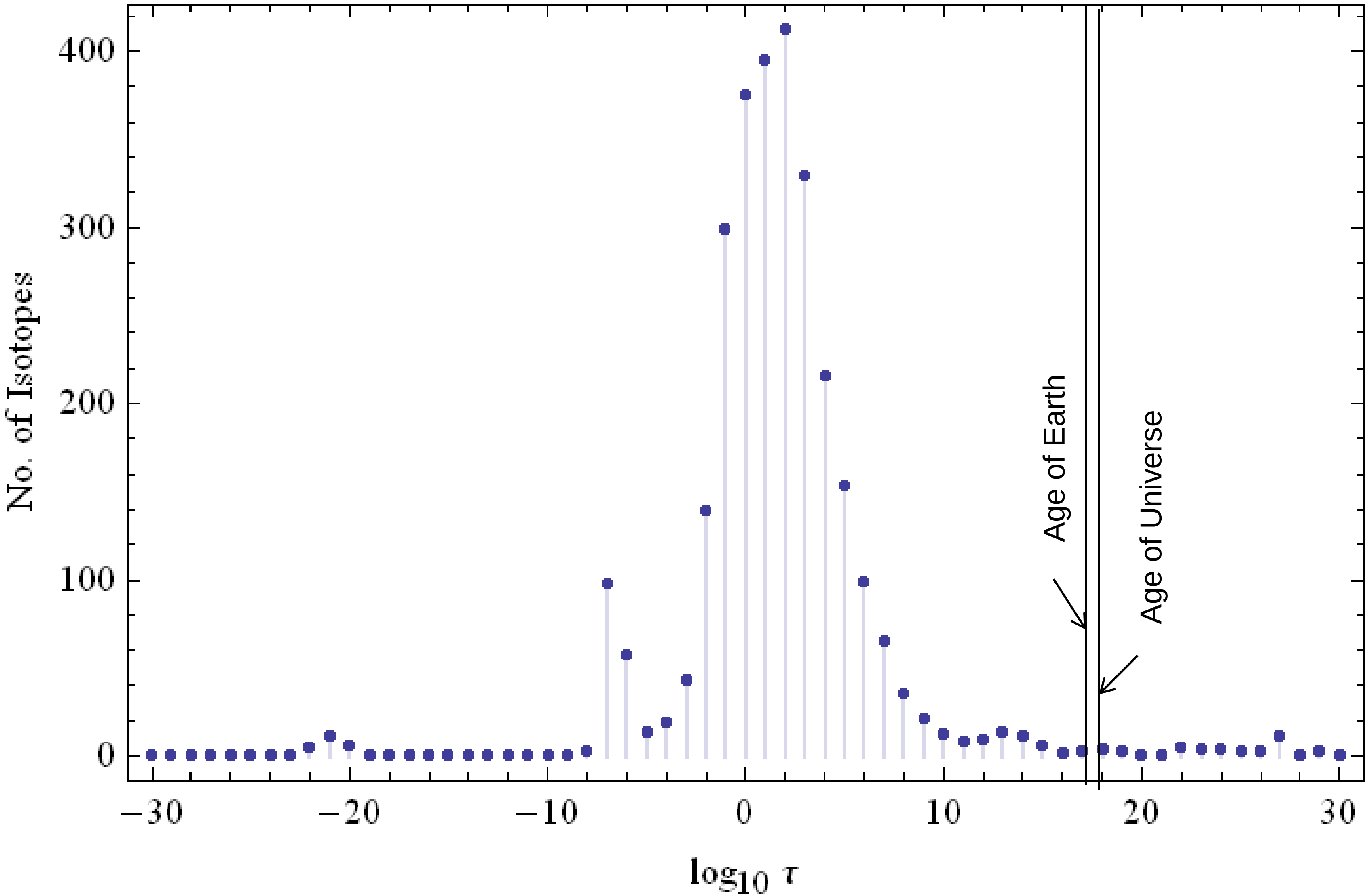
$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(9.35\ h)} = 0.0741\ h^{-1}$$

(b) How long will it take for the sample to decay to $^2/_5$ of the original number of atoms?

$$n = -\frac{\ln \epsilon}{\ln 2} = -\frac{\ln (2/5)}{\ln 2} = 1.32193$$

$$t = n * T_{1/2} = 1.32193 * (9.35\ h) = 12.36\ h$$

Half Life Histogram



Decay Probability & Avg. Lifetime

Decay Probability for a Finite Time Interval:

$$\bar{P}(t) = \frac{N(t)}{N(0)} = \exp(-\lambda t)$$

$$P(t) = 1 - \bar{P}(t) = 1 - \exp(-\lambda t)$$

$$p(t) = \lambda \exp(-\lambda t)$$

$N(t)$ = number of atoms at time t.
$\bar{P}(t)$ = probability of existence during 0-t.
$P(t)$ = decay probability in time 0-t.
$p(t)$ = probability density

Average Lifetime:

$$T_{avg} = \int_0^\infty t\, p(t)\, dt = \frac{1}{\lambda}$$

Parallel Decay Routes

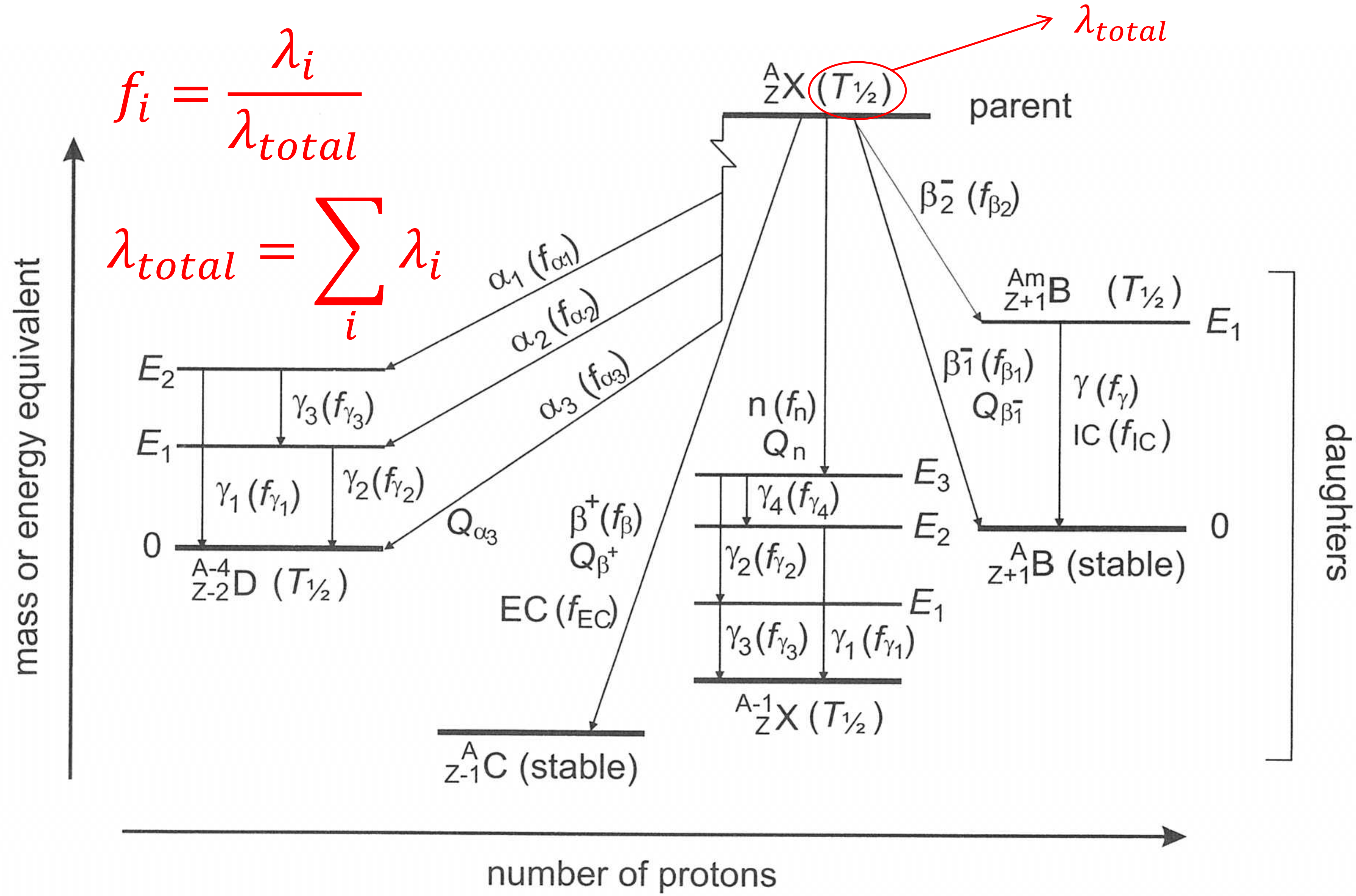
$$ {}^{64}_{29}Cu \rightarrow \begin{cases} {}^{64}_{30}Zn \ (\beta^{-} 39.0\%) = \color{red}{Path\ 1} \\ {}^{64}_{28}Ni \ (\beta^{+} 17.4\%) = \color{green}{Path\ 2} \\ {}^{64}_{28}Ni \ (EC\ 43.6\%) = \color{blue}{Path\ 3} \end{cases} $$

$$\frac{dN_{Cu}}{dt} = \sum_{1}^{3} -\lambda_i N_{Cu}(t) = -N_{Cu}(t) \sum_{1}^{3} \lambda_i = -\lambda_{tot} N_{Cu}(t)$$

Parallel Decay Routes

$$f_i = \frac{\lambda_i}{\lambda_{total}}$$

$$\lambda_{total} = \sum_i \lambda_i$$



Example

$$^{64}_{29}Cu \rightarrow \begin{cases} ^{64}_{30}Zn \ (\beta^- 39.0\%) = Path\ 1 \\ ^{64}_{28}Ni \ (\beta^+ 17.4\%) = Path\ 2 \\ ^{64}_{28}Ni \ (EC\ 43.6\%) = Path\ 3 \end{cases}$$

If $^{64}_{29}Cu$ has an overall $T_{1/2}$ of 12.7 h, calculate the decay constants for the three decay modes in h^{-1}.

$$\lambda_{tot} = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(12.7\ h)} = 0.05458\ h^{-1}$$

$$\lambda_{\beta^-} = f_{\beta^-}\lambda_{tot} = 0.390\ \lambda_{tot} = 0.0213\ h^{-1}$$
$$\lambda_{\beta^+} = f_{\beta^+}\lambda_{tot} = 0.174\ \lambda_{tot} = 0.0095\ h^{-1}$$
$$\lambda_{EC} = f_{EC}\lambda_{tot} = 0.436\ \lambda_{tot} = 0.0238\ h^{-1}$$

Activity

Definition: number of decays or transmutations per unit of time that occur within the sample

Units: **Becquerel (Bq)** = 1 transformation / sec
Curie (Ci) = $3.7*10^{10}$ Bq

$$A(t) \equiv -\frac{dN_i}{dt} = \lambda N_i(t) = A_0 \exp(-\lambda t)$$

Specific Activity = activity per unit mass

$$\hat{A}(t) = \frac{A(t)}{m(t)} = \frac{\lambda N_a}{M}$$

Nuclides undergoing single decay mechanisms exhibit decreasing activity and constant specific activity with time.

Series – Decay with Production

When the decay of a radionuclide is accompanied by the creation of new atoms of the radionuclide:

$$\frac{dN_i\,(t)}{dt} = -\lambda N_i\,(t) + Q_i(t)$$

$$N_i(t) = N_{i,0}e^{-\lambda t} + \int_0^t Q_i(t')e^{-\lambda(t-t')}dt'$$

For Q = constant: $N_i(t) = N_{i,0}\exp(-\lambda t) + \frac{Q_{i,0}}{\lambda}[1 - \exp(-\lambda t)]$

And as $t = \infty$: $N_i^{eq} = \frac{Q_{i,0}}{\lambda}$

General Decay Chain

Decay chain: when a parent radionuclide decays into a daughter, that then decays into another daughter, as so on, until a stable daughter is formed

$$X_1 \xrightarrow{\lambda_1} X_2 \xrightarrow{\lambda_2} \dots X_i \xrightarrow{\lambda_i} \dots \xrightarrow{\lambda_{n-1}} X_n \text{(stable)}$$

$$\frac{dN_1(t)}{dt} = -\lambda_1 N_1(t)$$

$$\frac{dN_2(t)}{dt} = \lambda_1 N_1(t) - \lambda_2 N_2(t)$$

$$\vdots$$

$$\frac{dN_i(t)}{dt} = \lambda_{i-1} N_{i-1}(t) - \lambda_i N_i(t)$$

$$\vdots$$

$$\frac{dN_n(t)}{dt} = \lambda_{n-1} N_{n-1}(t)$$

General Decay Chain

For the case when only radionuclides of the parent X_1 are initially present ($N_1(0) \neq 0$ and $N_i(0) = 0, i > 1$):

$$A_j(t) = \lambda_j N_j(t) = N_1(0)\left(C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t} + \cdots + C_j e^{-\lambda_j t}\right)$$

$$A_j(t) = N_1(0) \sum_{m=1}^{j} C_m e^{-\lambda_m t}$$

$$C_m = \frac{\prod_{i=1}^{j} \lambda_i}{\prod_{\substack{i=1 \\ i\neq m}}^{j} (\lambda_i - \lambda_m)} = \frac{\lambda_1 \lambda_2 \lambda_3 \cdots \lambda_j}{(\lambda_1 - \lambda_m)(\lambda_2 - \lambda_m)(\lambda_3 - \lambda_m)\cdots(\lambda_j - \lambda_m)}$$

Secular Equilibrium

At long times compared to the half lives of the daughters (but short compared to the chain head), the activities of all species are the same.

$$N_1\lambda_1 = N_2\lambda_2 = \cdots = N_j\lambda_j$$

$$A_o = A_1 = A_2 = \cdots = A_j$$

Species with short half lives (large λ) have low concentrations, but concentrations can be estimated from species with longer half lives, in particular from the head of the chain.

$$N_1\lambda_1 = N_k\lambda_k$$

$$\frac{N_1}{(T_{1/2})_1} = \frac{N_k}{(T_{1/2})_k}$$

Example

Secular Equilibrium Example Problem

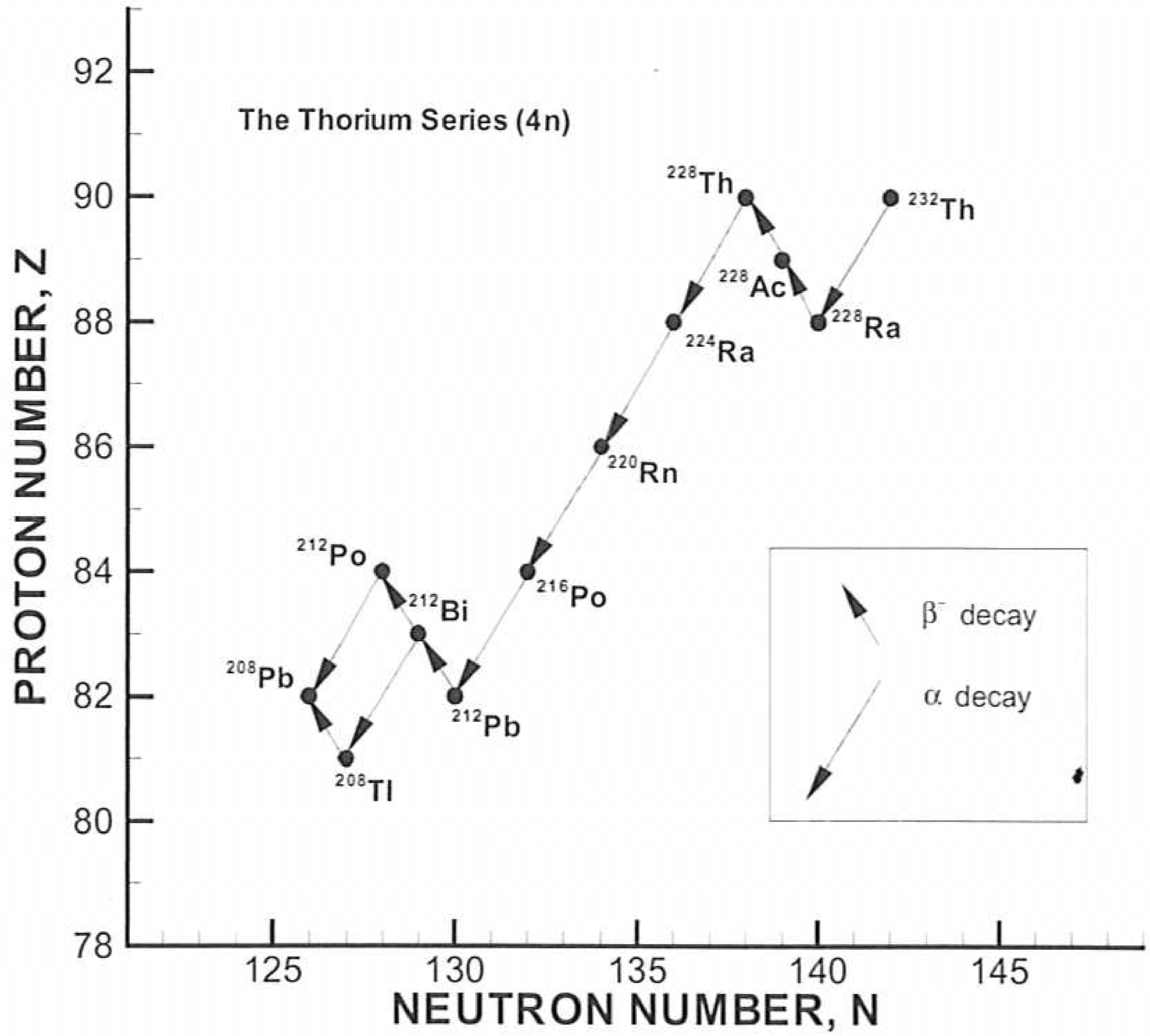
Natural Radionuclides

- 65 natural isotopes
- Cosmogenically produced
  - ^{3}H, ^{7}Be, ^{14}C
  - H and C used for dating. All three source of radioactivity in air samples.
- Primordial Isotopes
  - Singly occurring (17) including ^{40}K and ^{87}Rb (parts of humans)
  - Decay Products with Z > 83 form from ^{232}Th, ^{235}U, or ^{238}U via α and β emission.

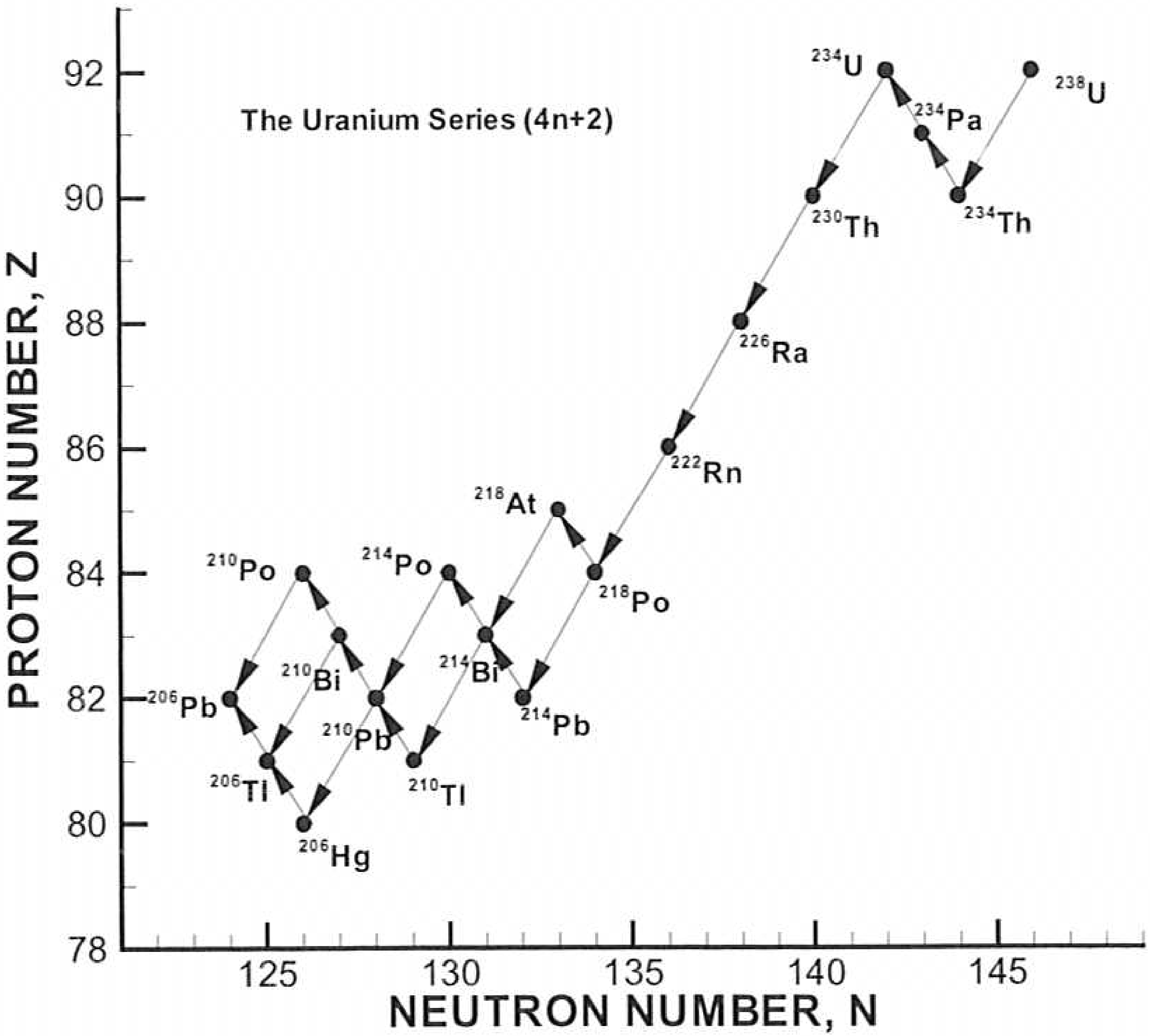
isotope	decay	half-life (years)	isotopic abund. (%)	decay product
^{40}K	β^-, β^-	1.27 E09	0.0117	^{40}Ca, ^{40}Ar
^{50}V	ε, β^-	1.4 E17	0.250	^{50}Ti, ^{50}Cr
^{87}Rb	β^-	4.88 E10	27.83	^{87}Sr
^{113}Cd	β^-	7.7 E15	12.22	^{113}In
^{115}In	β^-	4.4 E14	95.71	^{115}Sn
^{123}Te	ε	6 E14	0.89	^{123}Sb
^{138}La	ε, β^-	1.05 E11	0.090	^{138}Ba, ^{138}Ce
^{144}Nd	α	2.38 E15	23.80	^{140}Ce
^{147}Sm	α	1.06 E11	14.99	^{143}Nd
^{148}Sm	α	7. E15	11.24	^{144}Nd
^{152}Gd	α	1.1 E14	0.20	^{148}Sm
^{176}Lu	β^-	3.75 E10	2.59	^{176}Hf
^{174}Hf	α	2.0 E15	0.16	^{170}Yb
^{180m}Ta	ε, β^-	>1.2 E15	0.012	^{180}Hf
^{187}Re	β^-	4.12 E10	62.60	^{187}Os
^{186}Os	α	2. E15	1.59	^{182}W
^{190}Pt	α	6.5 E11	0.014	^{186}Os
^{232}Th	α	1.40 E10	100.	^{208}Pb
^{235}U	α	7.04 E08	0.720	^{207}Pb

Isolated natural radionuclides

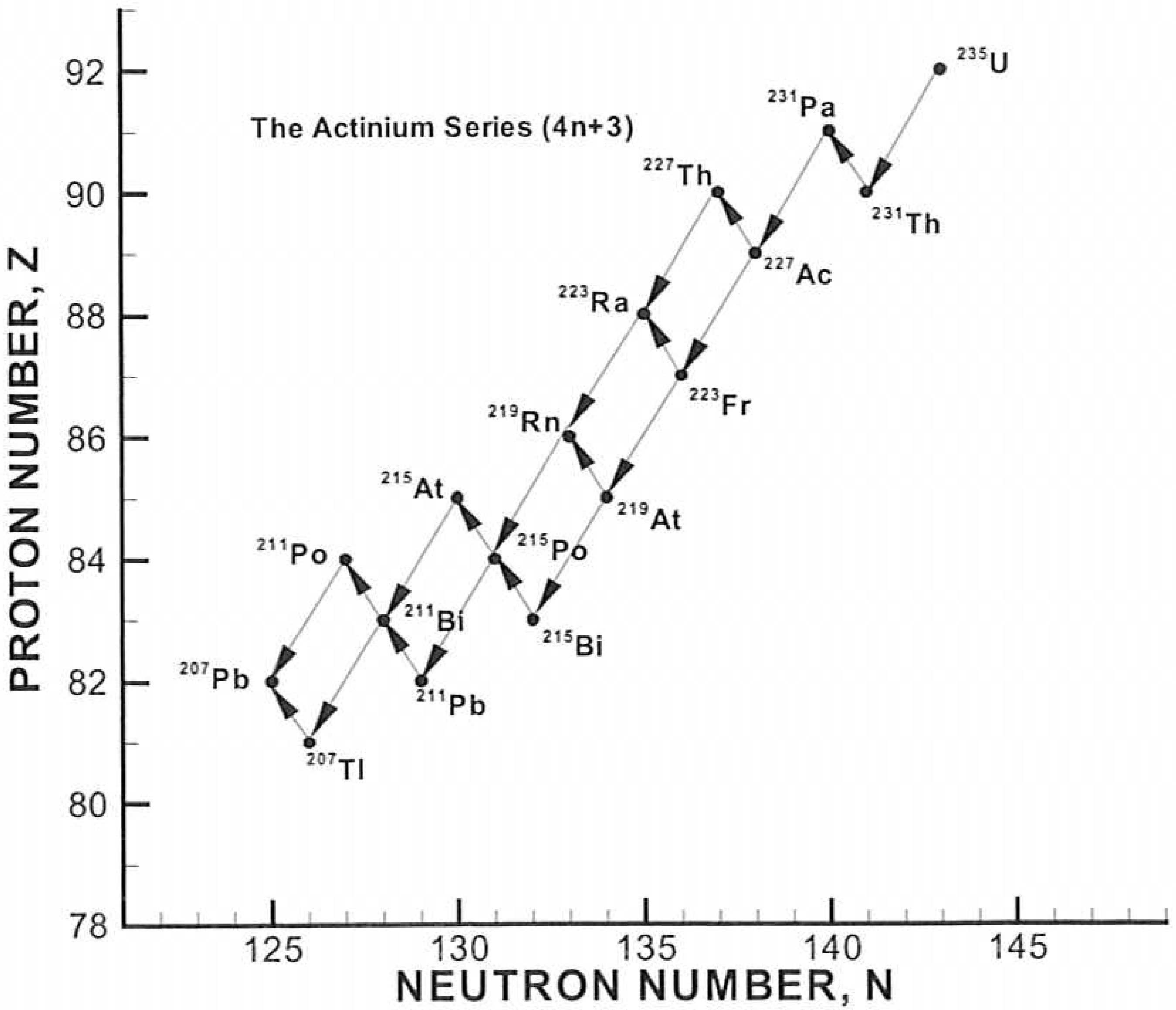
Thorium Decay Chain (4n)



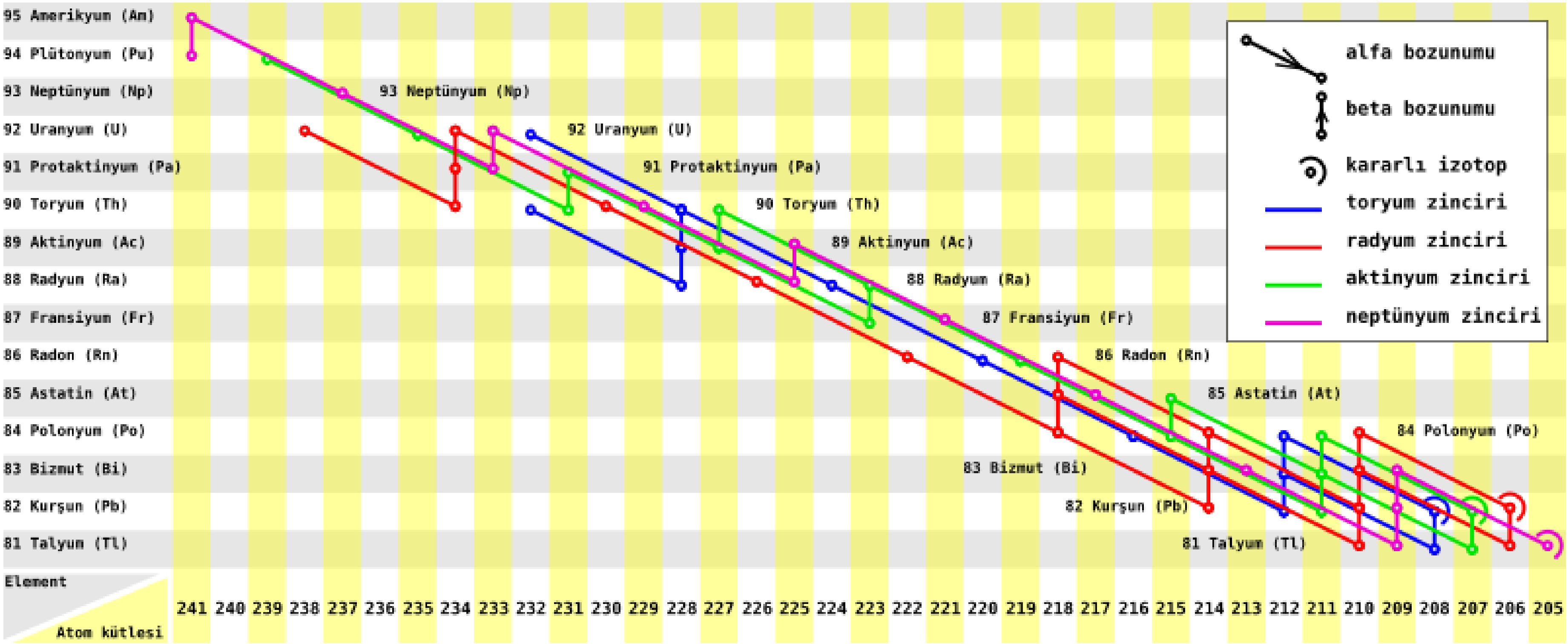
Uranium Decay Chain (4n+2)



Actinium Decay Chain (4n+3)



4 Possible decay chains



Mass number given by 4n (^{232}Th), 4n+2 (^{238}U or ^{234}Np) , and 4n+3 (^{235}U or ^{239}Pu) are near secular equilibrium. 4n+1 (^{241}Ac/Pu) has no step slow enough. Radioisotopes from it have long since decayed and are not found in nature.