Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 9

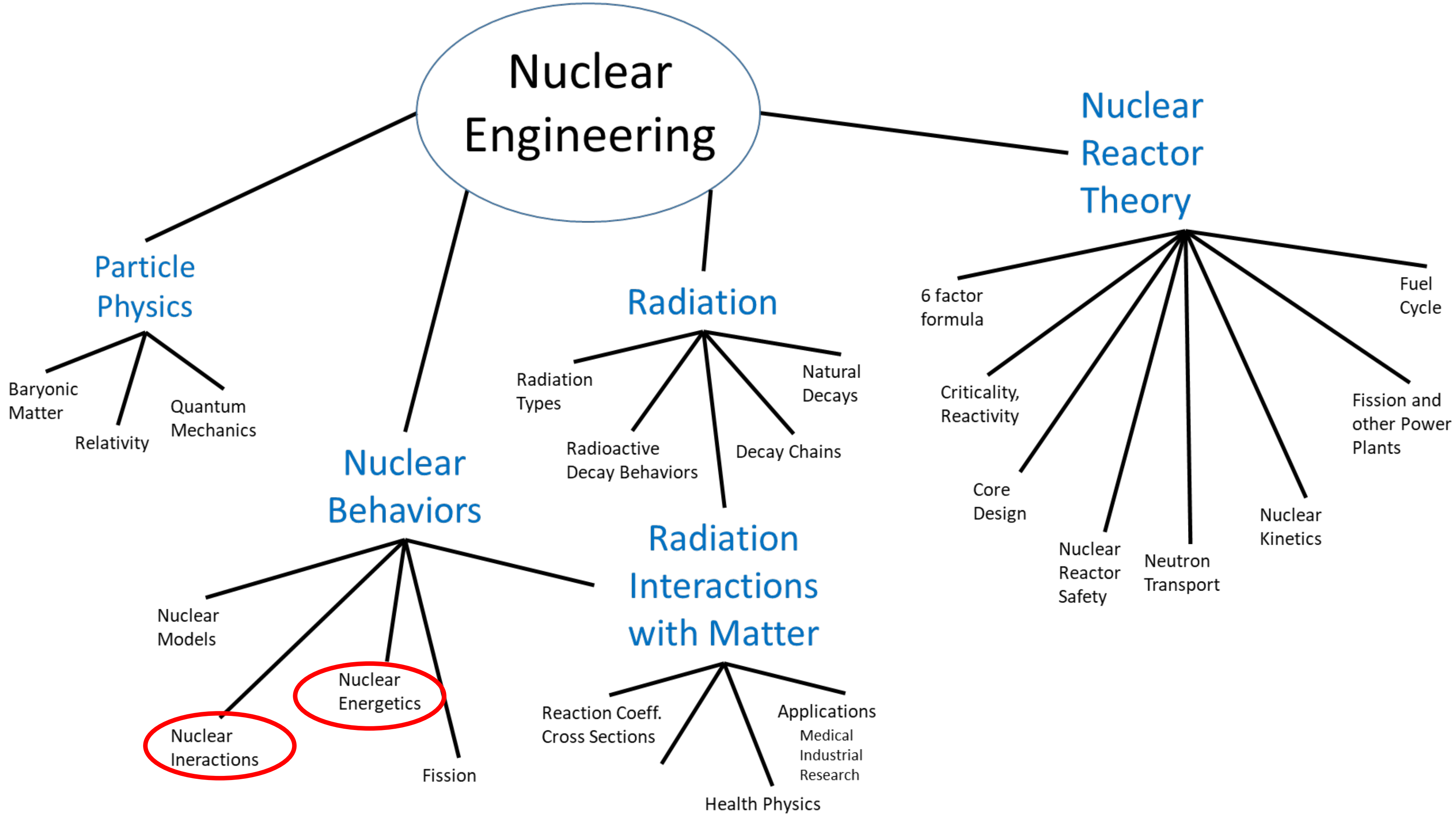
Binary Nuclear Reactions

Spiritual Thought

"You are just like us, only you don't believe in Mohamad! We believe in Jesus, but you just don't believe in Mohamad. Otherwise, we are the same!"

Dr. Yusuf

The BIG Picture



Web Problem 2

Web Problem 2 (cont)

Dr. Emma Russell is a world-expert in cold fusion, and at the end of the movie “The Saint”, her work pays off, and Russia is saved by the miraculous demonstration of absurd amounts of power coming from cold fusion. In this particular scene, she makes a couple of bold statements and lofty claims regarding cold fusion (i.e. the low temperature fusion of 2 deuterium nuclei at low temperature). Primarily, she claims that 1) a cubic meter of seawater contains more energy than the entire oil reserves of the planet, and 2) you can drive your car for 55,000,000 miles on a gallon of heavy water (D_2O). Determine (via calculation, of course) whether or not these two claims are reasonably close to being accurate.

Definitions

- Exothermic – ($\Delta H < 0$) reaction releases heat
- Exoergic – ($Q > 0$) reaction releases energy
  - γ-rays, excited nucleous, neutrons, α particles, etc.
  - Ultimately usually produces heat, except for neutrinos
- Endothermic – ($\Delta H > 0$) consumes heat
- Endoergic – ($Q < 0$) requires energy as input
  - Rarely in the form of heat
  - Neutron/alpha particle kinetic energy, gamma energy, etc.).
- Elastic scattering – ($Q = 0$)
- Inelastic scattering $Q < 0$ ($m_Y > m_y$) because of energy absorption.

Conceptual Reaction Mechanisms

- High-energy (> 40 MeV) projectile particle
  - De Broglie wavelengths approximately the size of a nucleon
  - Reaction limited to one or small number of nucleons in periphery of target nucleus
- Lower energy projectile particle (~ few MeV)
  - Larger de Broglie wavelength, approximately size of nucleus
  - Reaction involves entire nucleus
  - Forms a compound (excited) nucleus – “virtual”
  - Compound nucleus lifetime is 10^{-13} to 10^{-15} s, much longer than the transit time of the projectile (10^{-17} to 10^{-21} s).
  - Compound nucleus decays:
    - virtual excited state to bound state (excited or not)
    - via particle emission in random direction
      - (forgets direction the exciting particle came from in COM coordinates)
  - Compound nucleus decay modes, or exit channels, independent of formation mechanism (forgets formation mode)

Major Reaction Types

- Transfer – direct transfer of nucleons between projectile and target, e.g., (d, n) and (α, d).
- Scattering – light product same as projectile
  - Elastic – no change in target energy (x, x)
  - Inelastic – target left in excited state (x, x')
- Knockout – projectile and other nucleons emitted $(n, 2n)$, (n, np) …
- Capture – projectile captured by nucleus and no light product produced (n, γ)
- Nuclear photoeffect – projectile is γ-ray and light product is nucleon, e.g., (γ, n) is an important way of generating neutrons in the lab.

Binary Reaction w/ two products

$$ {}^{A_1}_{Z_1}X_1 + {}^{A_2}_{Z_2}X_2 \rightarrow {}^{A_3}_{Z_3}X_3 + {}^{A_4}_{Z_4}X_4 $$

alternatively $$x + X \rightarrow Y + y$$

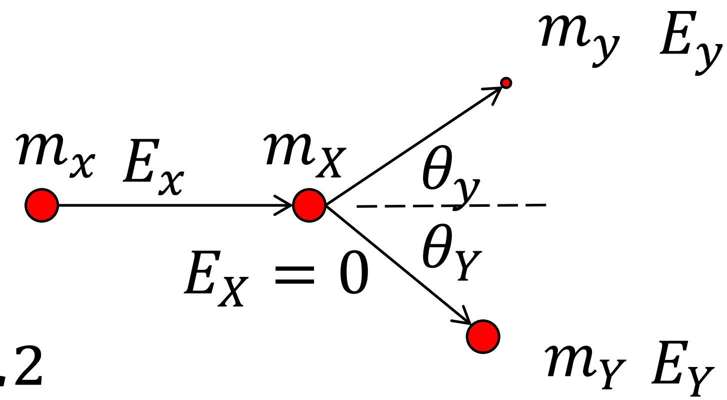
- Conservations:
  - Protons $Z_1 + Z_2 = Z_3 + Z_4$
  - Neutrons $A_1 + A_2 = A_3 + A_4$
  - Total energy
  - Linear momentum

$\left.\begin{array}{l}\text{Total energy}\\ \text{Linear momentum}\end{array}\right\}$ resulting in unique energy and momentum of products if there are only two products.

note: protons & neutrons commonly are conserved separately in these reactions, unlike spontaneous decay. Electron capture and other weak-force moderated events are an exception to conservation of protons and neutrons.

Reaction Kinematics

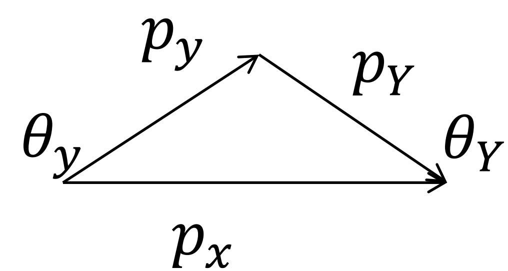
Energy Balance



$$Q = E_x + E_X - E_y - E_Y$$

$$Q = \left(m_x + m_X - m_y - m_Y\right)c^2$$

$$\approx \left(M_x + M_X - M_y - M_Y\right)c^2$$



Momentum Balance

$$\sqrt{E_y} = \sqrt{\frac{m_x m_y E_x}{\left(m_y + m_Y\right)^2}}\,\omega_y \pm \sqrt{\frac{m_x m_y E_x}{\left(m_y + m_Y\right)^2}\,\omega_y^2 + \frac{m_Y - m_x}{m_y + m_Y} E_x + \frac{m_Y Q}{m_y + m_Y}}$$

$$\omega_y \equiv \cos\theta_y$$

$$\sqrt{E_y} = a \pm \sqrt{a^2 + b}$$

Quantitative Summary

$$\sqrt{E_y} = \frac{1}{m_y + m_Y}\left(\sqrt{E_x m_x m_y}\omega_y\right.$$

$$\cos\theta_s = \frac{E_x(m_x - m_Y) - m_Y Q + \left(m_y + m_Y\right)E_y}{2\sqrt{E_x m_x m_y E_y}}$$

Quantitative Summary

$$\sqrt{E_y} = a \pm \sqrt{a^2 + b}$$

- $Q > 0, b > 0$
  - Only positive sign of $\pm$ is relevant
  - One solution
  - $\lim_{E_x \to 0} E_y = \frac{m_Y}{m_y + m_Y} Q$
  - $Q = E_y + E_Y$
- $Q > 0$
  - If $a^2 + b < 0$, no reaction
  - $a^2 + b$ can be greater than 0 if Q is sufficiently large
  - Two solutions possible – two types of particles emitted in forward and backward direction relative to complex nucleus
- Conditions that might prevent a reaction include:
  - Negative q-value $Q \leq -E_x(m_x/m_Y - 1)$ for small ω
  - Heavy projectile $m_x > m_Y(1 + Q/E_x)$ for small ω
  - Large scattering angle

Threshold Energies

- Reactions with:
  - Neg. q-values
  - positive q-values with $m_Y < m_X$
    - Still possible if the projectile provides the needed energy
    - Threshold energy
- Two flavors:
  - Greater of
    - Kinematic threshold – projectile energy must exceed that required to make up mass deficit when Q is negative, which is

$$E_x^{th} = -\frac{m_Y + m_y}{m_y + m_Y - m_x - \frac{m_x m_y}{m_Y}\sin^2\theta_y} Q \approx -\left(1 + \frac{m_x}{m_X}\right) Q$$

    - Coulomb threshold – projectile must exceed Coulombic repulsion, but products recapture this energy upon decay

$$E_x^C \approx 1.2 \frac{Z_x Z_X}{A_x^{\frac{1}{3}} + A_X^{\frac{1}{3}}}$$

Special Applications for Nuetronics

- Neutrons are incoming particles
- Energetics reveal likely behaviors
  - Statistics! 10^{17}-10^{23} particles in play
  - Describe behavior or high flux regions (nuclear fuel/core)
- Describe role of water in reactors
- Background for neutron interactions

How to Decelerate a Neutron

$$\alpha = \left(\frac{A-1}{A+1}\right)^2$$

collision parameter

$$\frac{\Delta E}{E} = \frac{1-\alpha}{2}$$

Lethargy;

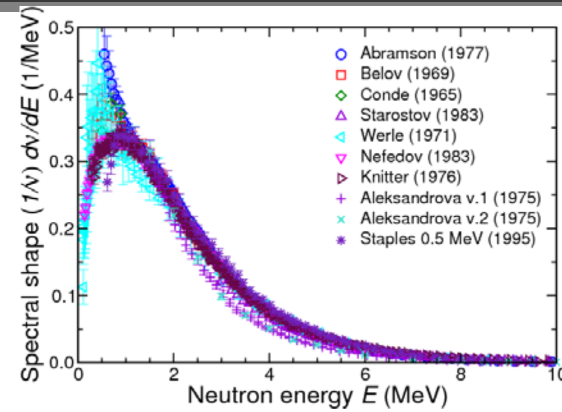
E_M is an arbitrary E, usually the highest neutron energy in the system. As neutrons decelerate, u increases.

$$u = \ln\frac{E_M}{E}$$

$$\xi = \Delta u = 1 - \frac{(A-1)^2}{2A}\ln\frac{A+1}{A-1} = 1 + \frac{\alpha}{1-\alpha}\ln\alpha \cong \frac{2}{A+\frac{2}{3}}$$

$$\lim_{A\to 1}\xi = 1$$

Neutron Energies



- Fission neutrons
  - Distribution of speeds
  - 2 MeV typical
  - Often interested "slowing" neutrons
  - Collisions required to slow from energy E_1 to E_2 is given by:

$$n = \frac{1}{\xi} \ln \frac{E_1}{E_2}$$

- Thermal neutrons:
  - equilibrated with the vibrating atomic nuclei at room temperature (293 K)
  - Average energy of 0.025 eV (2200 m/s)
  - Maxwellian distribution of speeds
  - likely to lose OR GAIN energy from medium nuclei
  - Readily produce fissions in U^{235}, U^{233}, Pu^{239}

Capture and Absorption

- Decelerating Neutrons from fission energies (2-5 MeV) to thermal energies (0.025 eV)
  - Requires many collisions
  - Smaller Nuclides
  - Risk of “capture”
- Capture occurs in “resonance energy regions” (fuel)
- Also could be absorbed by the “moderator” (water)
- Can calculate probability of capture or absorption
  - Resonance integral
  - Absorption cross-sections

Collision parameters

Atom	A	α	ξ	n
H	1	0.000	1.000	18.2
H_2O	1, 16		0.920	19.8
D	2	0.111	0.725	25.1
D_2O	2, 16		0.509	35.8
He	4	0.360	0.425	42.8
Be	9	0.640	0.207	88.1
B	11	0.694	0.171	106.3
C	12	0.716	0.158	115.3
O	16	0.779	0.120	151.7
Na	23	0.840	0.084	215.4
Fe	56	0.931	0.035	515.6
^{238}U	238	0.983	0.008	2171.6

n values here assume a neutron slowing from 2 MeV to 0.025 eV

Neutron Interactions

- Elastic scattering (n,n) – collision with no reaction and no change in total kinetic energies. Energy neutral.
- Inelastic scattering (n,n’) – collisions with energy absorption by nucleus. endoergic
- Radiative capture (n,γ) – Capture of neutron by nucleus followed by γ-ray emission. exoergic.
- Charged particle reactions (n,α) – Neutron reaction to form α particles or protons. endoergic and exoergic.
- Neutron producing reactions (n,*x*n) – Reactions with a net increase in neutrons. endoergic. (n,2n) important for ^{2}H and ^{9}Be.
- Fission (n,) forms multiple products – Nucleus forms daughters. Generally exoergic.

Neutrons Eventually Are Captured

$$n + {}_{Z}^{A}X \rightarrow \left[{}_{Z}^{A+1}X\right]^{*} \rightarrow \left[{}_{Z}^{A+1}X\right] + \gamma$$

Control rods

$$n + {}_{5}^{10}B \rightarrow \left[{}_{5}^{11}B\right]^{*} \rightarrow \left[{}_{3}^{7}Li\right] + \gamma + \alpha$$

“Fertile” isotopes form “fissile” isotopes through neutron absorption

$$n + {}_{90}^{232}Th \rightarrow \left[{}_{90}^{233}Th\right]^{*} \rightarrow \left[{}_{90}^{233}Th\right] + \gamma \underset{22\ m}{\overset{\beta^{-}}{\rightarrow}} {}_{91}^{233}Pa \underset{27\ d}{\overset{\beta^{-}}{\rightarrow}} {}_{92}^{233}U$$

$$n + {}_{92}^{238}U \rightarrow \left[{}_{92}^{239}U\right]^{*} \rightarrow \left[{}_{92}^{239}U\right] + \gamma \underset{24\ m}{\overset{\beta^{-}}{\rightarrow}} {}_{93}^{239}Np \underset{56\ h}{\overset{\beta^{-}}{\rightarrow}} {}_{94}^{239}Pu$$