Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 16

Nuclear Reactor Theory I

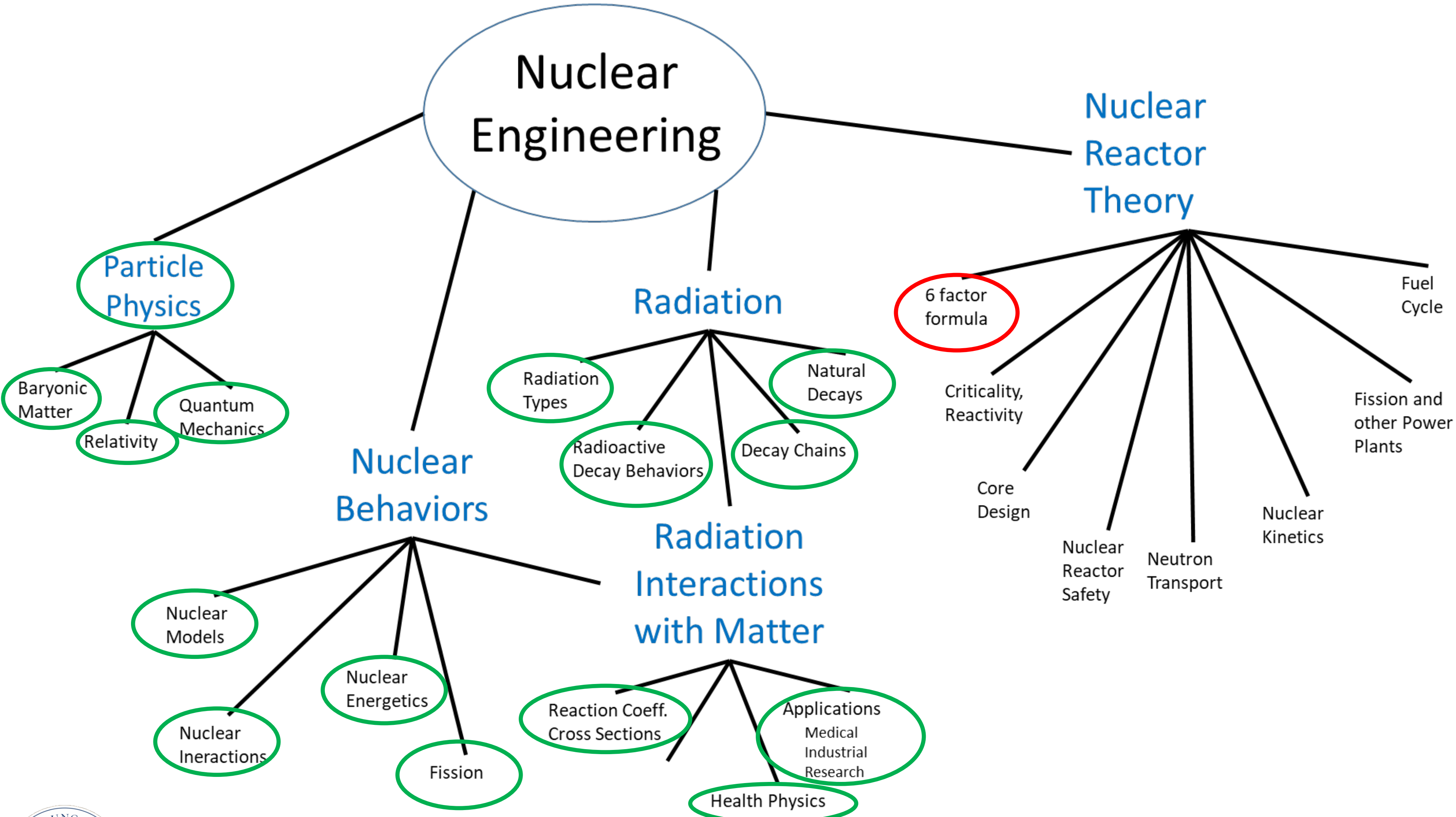
Nuclear Criticality

Spiritual Thought

“’Things will work out’ may well be President Hinckley's most repeated assurance to family, friends, and associates. ‘Keep trying’ he will say. ‘Be believing. Be happy. Don’t get discouraged. Things will work out’”

Elder Jeffrey R. Holland

Roadmap



Nuclear Engineering
Particle Physics
Baryonic Matter
Relativity
Quantum Mechanics
Nuclear Behaviors
Nuclear Models
Nuclear Energetics
Nuclear Ineractions
Fission
Radiation
Radiation Types
Natural Decays
Radioactive Decay Behaviors
Decay Chains
Radiation Interactions with Matter
Reaction Coeff. Cross Sections
Applications
Medical
Industrial
Research
Health Physics
Nuclear Reactor Theory
6 factor formula
Criticality, Reactivity
Core Design
Nuclear Reactor Safety
Neutron Transport
Nuclear Kinetics
Fission and other Power Plants
Fuel Cycle

Objectives

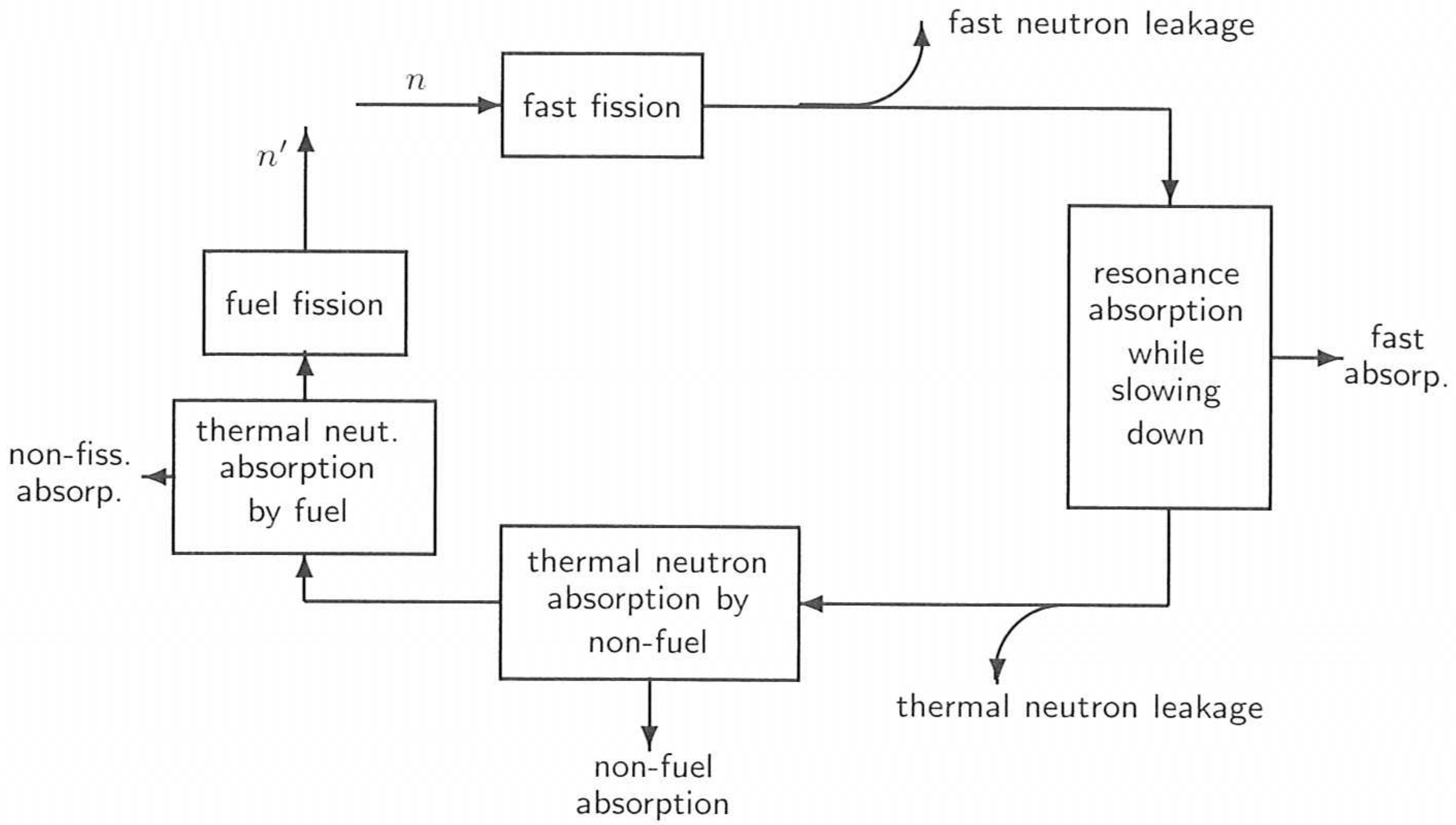
- Know the 6 factor formula (+ each factor)
- Understand (memorize) terminology for nuclear reactors
- Know the 6 factor formula (+ each factor)
- Know differences between heterogeneous and homogeneous cores
- Know the 6 factor formula (+ each factor)
- Understand General trends of 6 factors

Core Nomenclature

- Fuel – fissile material in core
- Heavy atoms – generally fissile, fertile, and fissionable material
- Moderator – collision material that slows neutrons
- Cladding – fuel containment
- Reflector – core wrapping that minimizes neutron leakage
- Blanket – material used to produce useful isotopes from core
- Shielding – neutron (and other radiation) absorbing material to protect personnel and instrumentation

Fast Neutron Life Cycle

- What happens to fast neutrons?



Multiplication Factor

$$k_{eff} \equiv \frac{neutrons\ at\ point\ in\ cycle}{neutrons\ at\ same\ point\ in\ previous\ generation}$$

$$k_{eff} = \frac{n'}{n}$$

$$k_{eff} = \epsilon p \eta f P_{NL}^{f} P_{NL}^{th}$$

$$k_{\infty} = \epsilon p \eta f$$

Fast Neutron Factors

- Fast Fission, ε
- Resonance Escape Probability, p
- thermal utilization, f
- Fission factor, η
- Fast non-leakage probability, P_{NL}^{f}
- Thermal non-leakage probability, P_{NL}^{th}
- Investigate for two reactor types:
  - Homogenous
  - Real (heterogeneous)

Six Factor Formula

$$n' \equiv n\epsilon p\eta f P_{NL}^{f} P_{NL}^{th}$$

n'	next generation neutrons
n	neutrons produced per thermal fission
ϵ	ratio of total neutrons to thermal neutrons (1.0-1.08)
p	resonance escape probability (0.8-0.9)
f	thermal utilization = $\frac{\Sigma_a^F \phi^F V^F}{\Sigma_a^F \phi^F V^F + \Sigma_a^{NF} \phi^{NF} V^{NF}}$ (0-1)
η	fission factor $= \nu \frac{\Sigma_f^F}{\Sigma_a^F}$ (2.0-2.2)
P_{NL}^{f}	non-leakage of fast neutrons = $\exp(-B_c^2 \tau)$ (near 1)
P_{NL}^{th}	non-leakage of thermal neutrons = $\frac{1}{1+L^2 B_c^2}$ (near 1)

Reactor Considerations

- Increase Power?

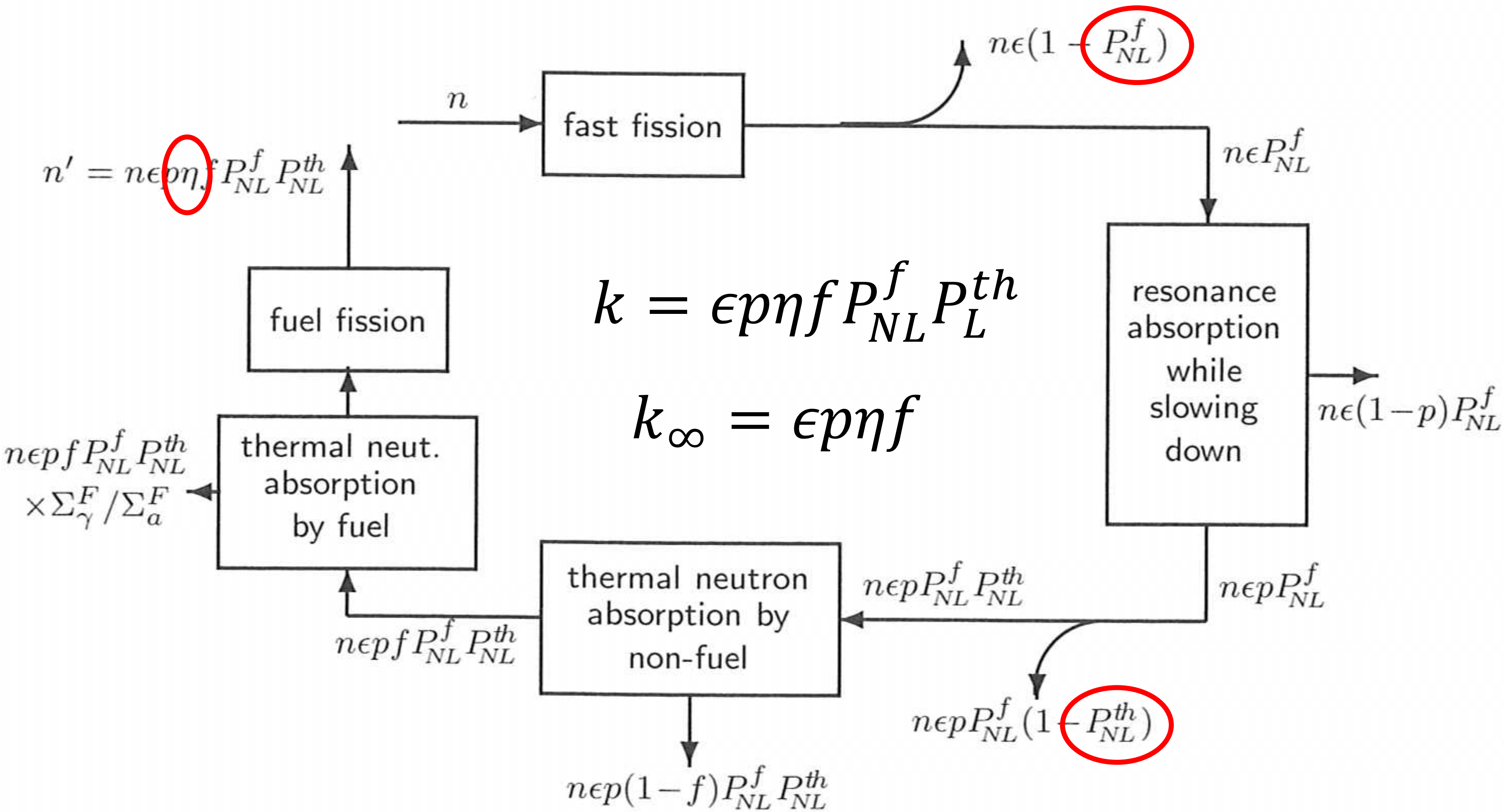
$$k_{eff} > 1$$

- Decrease Power?

$$k_{eff} < 1$$

- Most reactors have K_{eff}>1, but cancel excess out with absorptive “poisons”, which are removed with time.
- Most reactors designed to avoid prompt supercriticality
- If k_{eff} increases, “feedback” effects resist increase
- What if we want to change amount of fuel or moderator?
  - Impacts various “six factor” parameters
  - Changes k_{eff}

Quantitative Neutron Cycle



$$k = \epsilon p \eta f P^f_{NL} P^{th}_{L}$$

$$k_\infty = \epsilon p \eta f$$

Moving forward: Assume a Homogenous Reactor

Thermal Fission Factor, η

For a homogenous reactor: $\phi_F = \phi_{NF}$

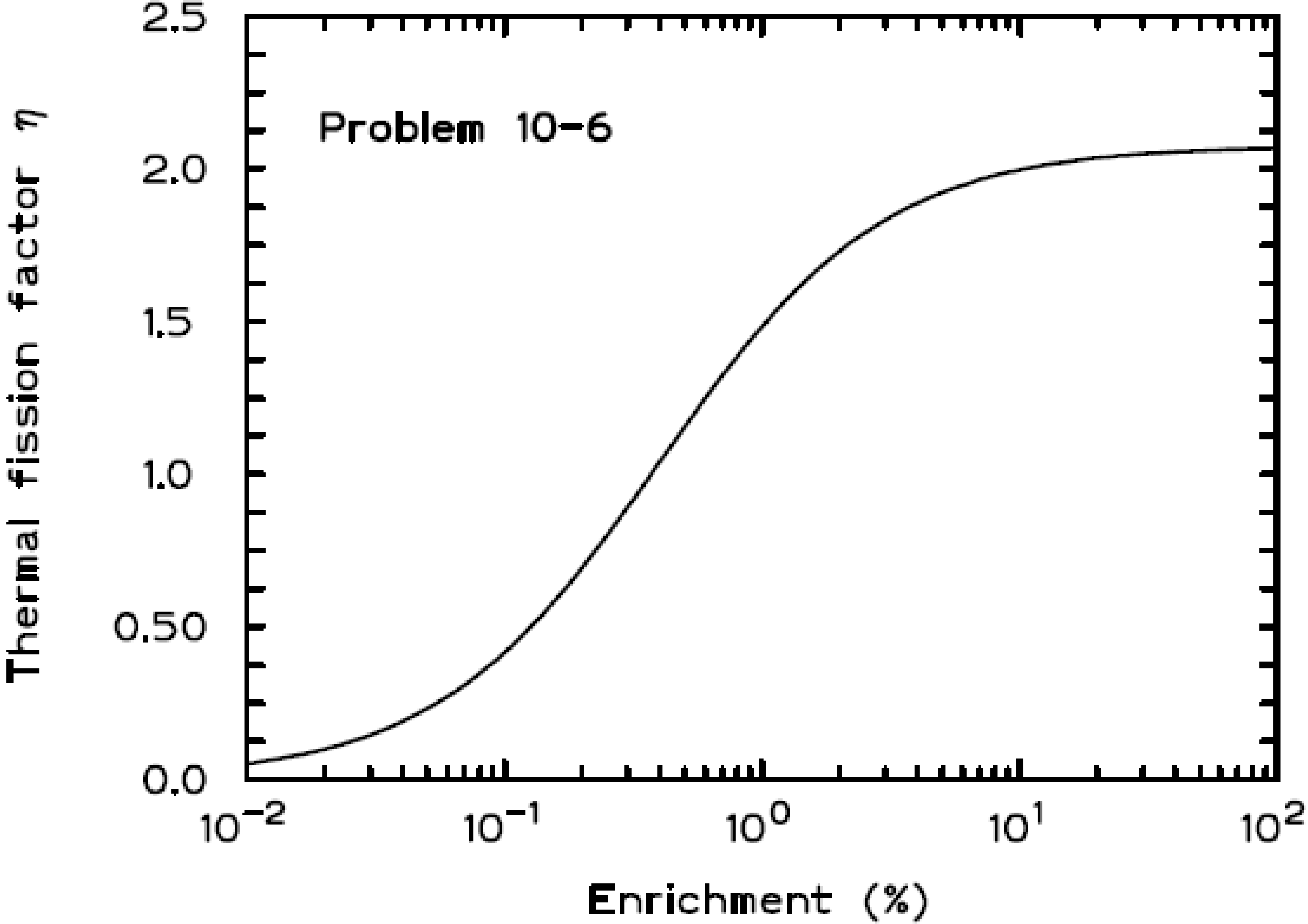
$$\eta \equiv \nu \frac{\Sigma_f}{\Sigma_a} = \nu \frac{\sigma_f^{235} N^{235}}{\sigma_a^{235} N^{235} + \sigma_a^{238} N^{238}} = \frac{\nu^{235} \sigma_f^{235}}{\sigma_a^{235} + \sigma_a^{238} \left(\frac{N^{238}}{N^{235}} \right)}$$

$$e = \frac{N^{235}}{N^{235} + N^{238}}$$

$$\Rightarrow \frac{N^{238}}{N^{235}} = \frac{1 - e}{e}$$

$$\eta = \frac{\nu^{235} \sigma_f^{235}}{\sigma_a^{235} + \sigma_a^{238} \left(\frac{1 - e}{e} \right)}$$

η variation with fuel enrichment



Conceptual Definitions

- L = ½ distance thermal neutron travels from point of thermalization to absorption
- τ = 1/6 mean squared distance from initial (birth) point to thermalization point
- Neutrons
  - Travel further in fast spectrum
    - 30 vs. 6 cm in light water
  - Spend far more time as thermal neutrons
- Probability of non-leakage
  - Approaches 100% as reactor dimensions become infinite
  - Both fast and thermal neutrons.

Non-leakage probabilities

$$P_{NL}^{th} = \frac{1}{1 + L^2 B_c^2}$$

L thermal diff. length

D thermal diff. coefficient

Σ_a absorption cross-section

f fuel utilization factor

$$L^2 \equiv \frac{D}{\Sigma_a} = \frac{D^M}{\Sigma_a^M} \frac{\Sigma_a^M}{\Sigma_a^M + \Sigma_a^F} = L_M^2 \left(1 - \frac{\Sigma_a^F}{\Sigma_a^M + \Sigma_a^F}\right) = L_M^2 (1 - f)$$

- B_c critical buckling – comes from reactor geometry
  - Table 10.6
  - Derived from Neutron Diffusion Equation
  - Balance of size and geometry of reactor

Non-leakage probabilities

$$P_{NL}^{f} = \exp(-B_c^2 \tau)$$

τ Fermi age

$$\tau \approx \tau_M$$

B_c critical buckling – comes from reactor geometry (Table 10.6)

Critical Bare Reactor Summary

geometry	Buckling (B_g^2)	Flux	A	$\Omega = \dfrac{\phi_{max}}{\phi_{av}}$
plate – 1D	$\left(\dfrac{\pi}{a}\right)^2$	$A\cos\dfrac{\pi x}{a}$	$1.57P/aE_R\Sigma_f$	1.57
plate – 3D	$\left(\dfrac{\pi}{a}\right)^2+\left(\dfrac{\pi}{b}\right)^2+\left(\dfrac{\pi}{c}\right)^2$	$A\cos\dfrac{\pi x}{a}\cos\dfrac{\pi y}{b}\cos\dfrac{\pi z}{c}$	$3.85P/VE_R\Sigma_f$	3.88
cylinder – 1D	$\left(\dfrac{2.405}{R}\right)^2$	$A\,J_0\left(\dfrac{2.405}{R}\right)$	$0.738P/R^2E_R\Sigma_f$	2.32
cylinder – 3D	$\left(\dfrac{2.405}{R}\right)^2+\left(\dfrac{\pi}{H}\right)^2$	$A\,J_0\left(\dfrac{2.405}{R}\right)\cos\dfrac{\pi z}{H}$	$3.63P/VE_R\Sigma_f$	3.64
sphere	$\left(\dfrac{\pi}{R}\right)^2$	$\dfrac{A}{r}\cos\dfrac{\pi r}{R}$	$P/4R^2E_R\Sigma_f$	3.29