Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 17

Nuclear Reactor Theory II

6 Factors and Reactor Equation

Spiritual Thought

Brothers and sisters, the most powerful Being in the universe is the Father of your spirit. He knows you. He loves you with a perfect love.

God sees you not only as a mortal being on a small planet who lives for a brief season—He sees you as His child. He sees you as the being you are capable and designed to become. He wants you to know that you matter to Him.

Deiter F. Uchtdorf

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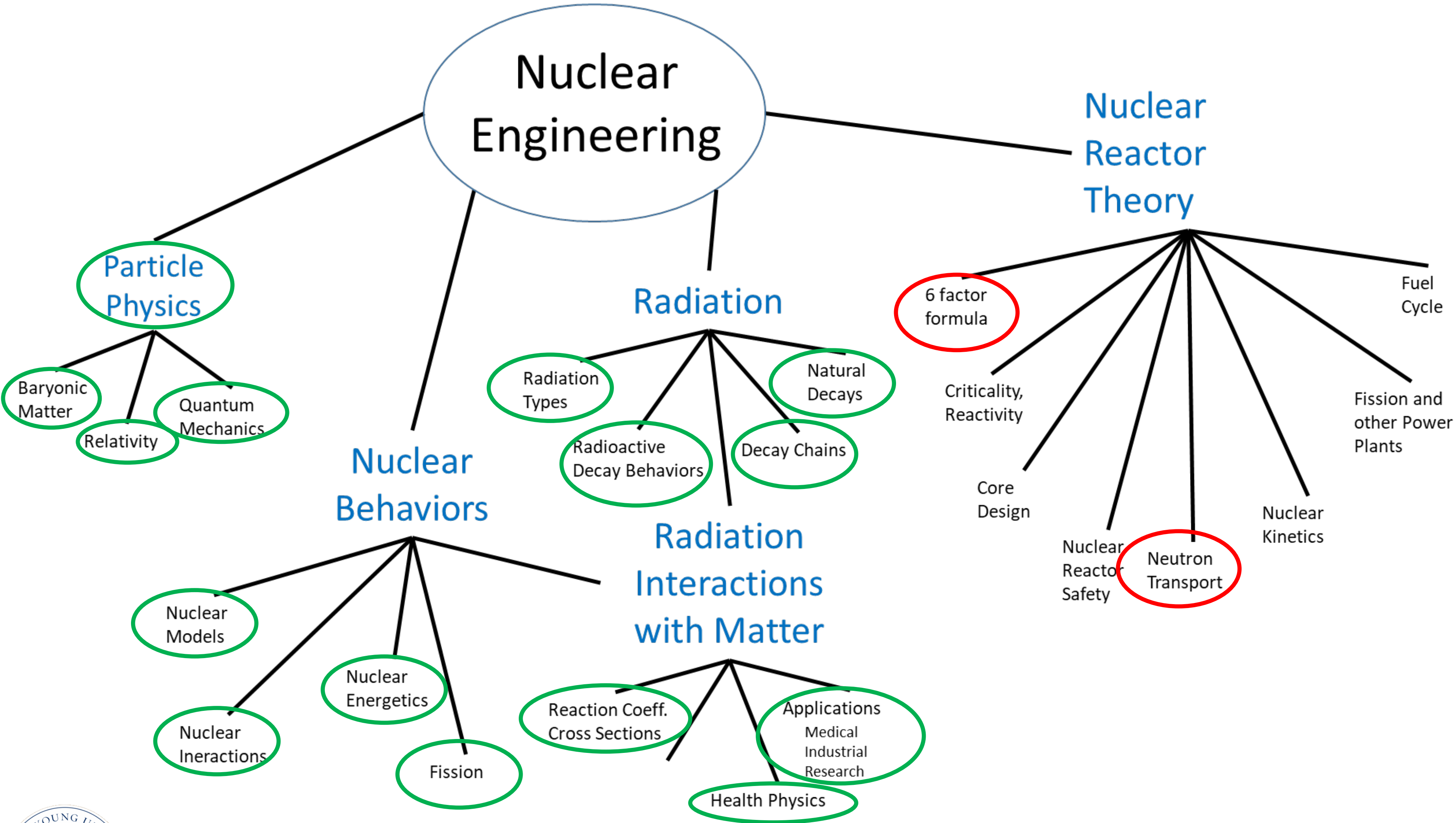
Roadmap


Nuclear Engineering
Particle Physics
Baryonic Matter
Relativity
Quantum Mechanics
Nuclear Behaviors
Nuclear Models
Nuclear Ineractions
Nuclear Energetics
Fission
Radiation
Radiation Types
Radioactive Decay Behaviors
Natural Decays
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Applications
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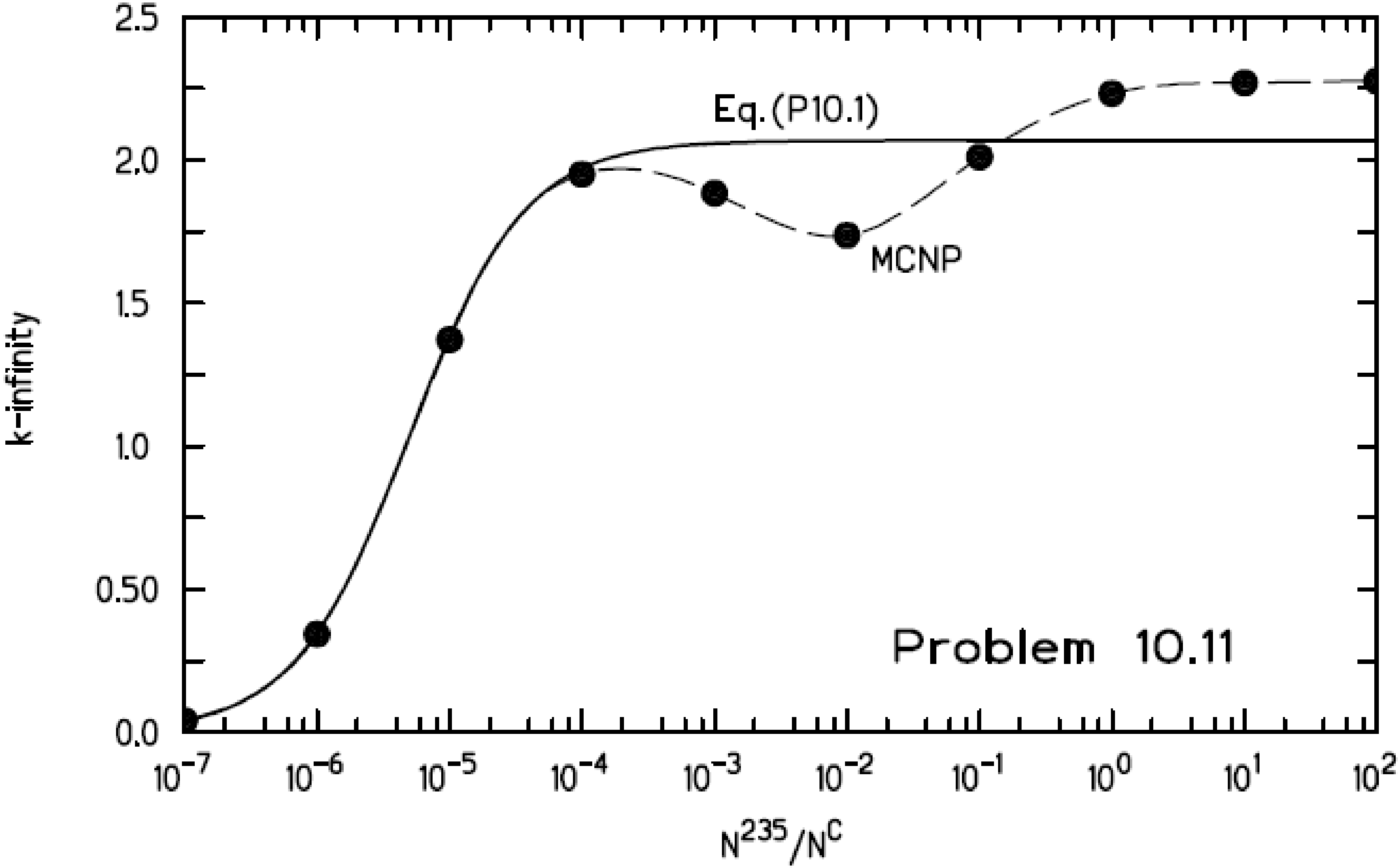
Research
Health Physics
Nuclear Reactor Theory
6 factor formula
Criticality, Reactivity
Core Design
Nuclear Reactor Safety
Neutron Transport
Nuclear Kinetics
Fission and other Power Plants
Fuel Cycle

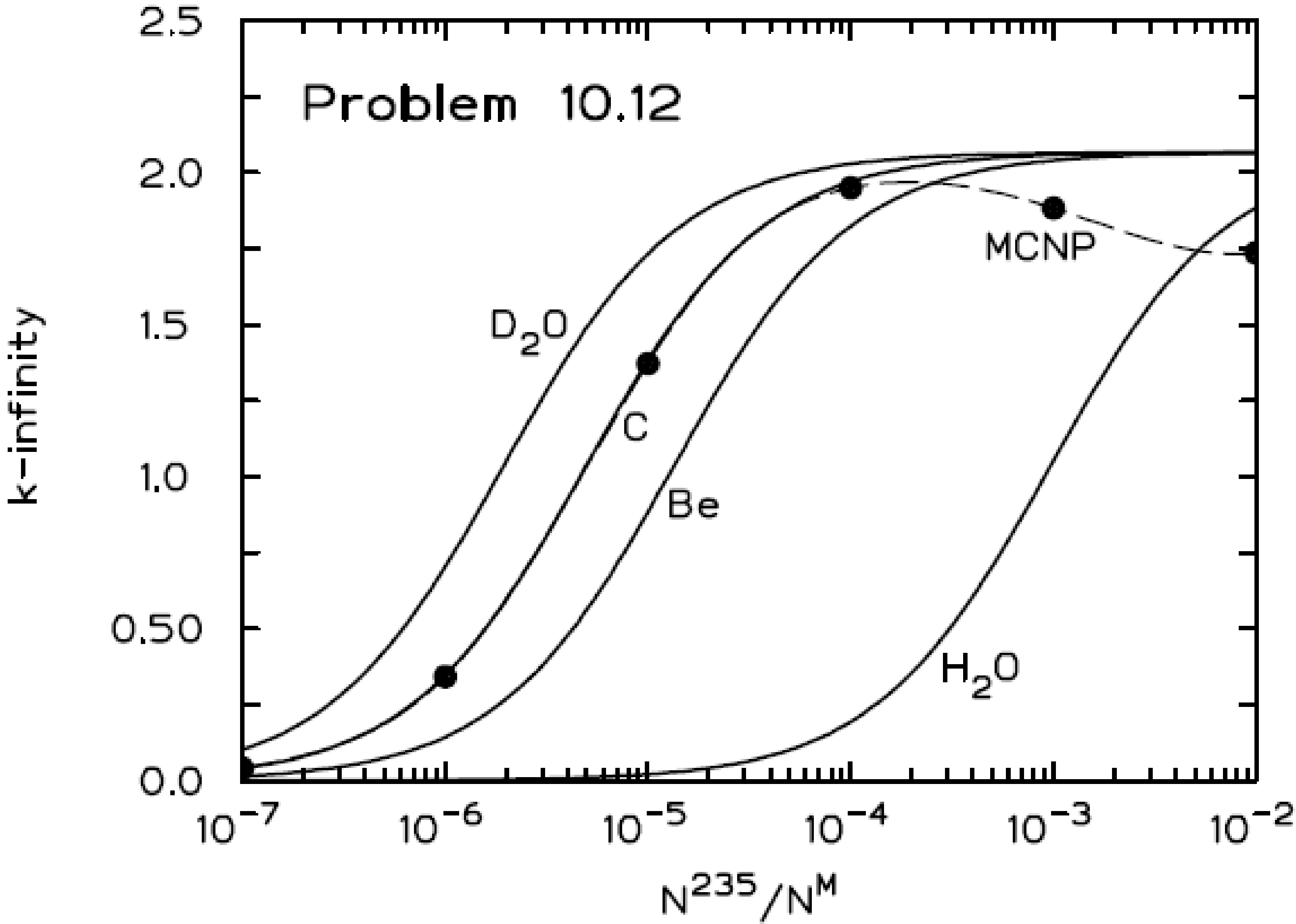
Objectives

- Know the 6 factor formula (+ each factor)
- Know differences between heterogeneous and homogeneous cores
- Know the 6 factor formula (+ each factor)
- Understand General trends of 6 factors
- Know the 1 group reactor equation
- Be familiar with neutron diffusion theory

k_∞ variation with fuel:modifier ratio (HM/H)



k_∞ variation with (HM/H)



Neutron Cycle Parameters

Natural uranium and moderator in homogeneous reactor:

Moderator	$(N^M/N^U)_{opt}$	ϵ	η	f	p	k_∞
H_2O	1.64	1.057	1.322	0.873	0.723	0.882
D_2O	272	1.000	1.322	0.954	0.914	1.153
Be	181	1.000	1.322	0.818	0.702	0.759
C	453	1.000	1.322	0.830	0.718	0.787

Heavy water moderation allows homogeneous reactor operation with natural uranium. CANDU reactors take advantage of this in principle – though no reactor is a homogeneous reactor.

A Few Parameters

$$\nu = neutrons / fission$$

Total (prompt and delayed) neutrons produced per fission

$$\alpha = \frac{\sigma_\gamma}{\sigma_f}$$

Capture to fission ratio

$$\eta = \nu \frac{\sigma_f}{\sigma_a} = \nu \frac{\sigma_f}{\sigma_\gamma + \sigma_f} = \frac{\nu}{1+\alpha}$$

Neutrons released per absorption (> 1 converter, > 2 breeder)

$$k = \frac{neutrons\ after\ one\ generation}{original\ neutrons}$$

Multiplication factor

$$C = \frac{fissile\ atoms\ produced}{fissile\ atoms\ consumed}$$

conversion/breeding ratio (>1 breeder reactor)

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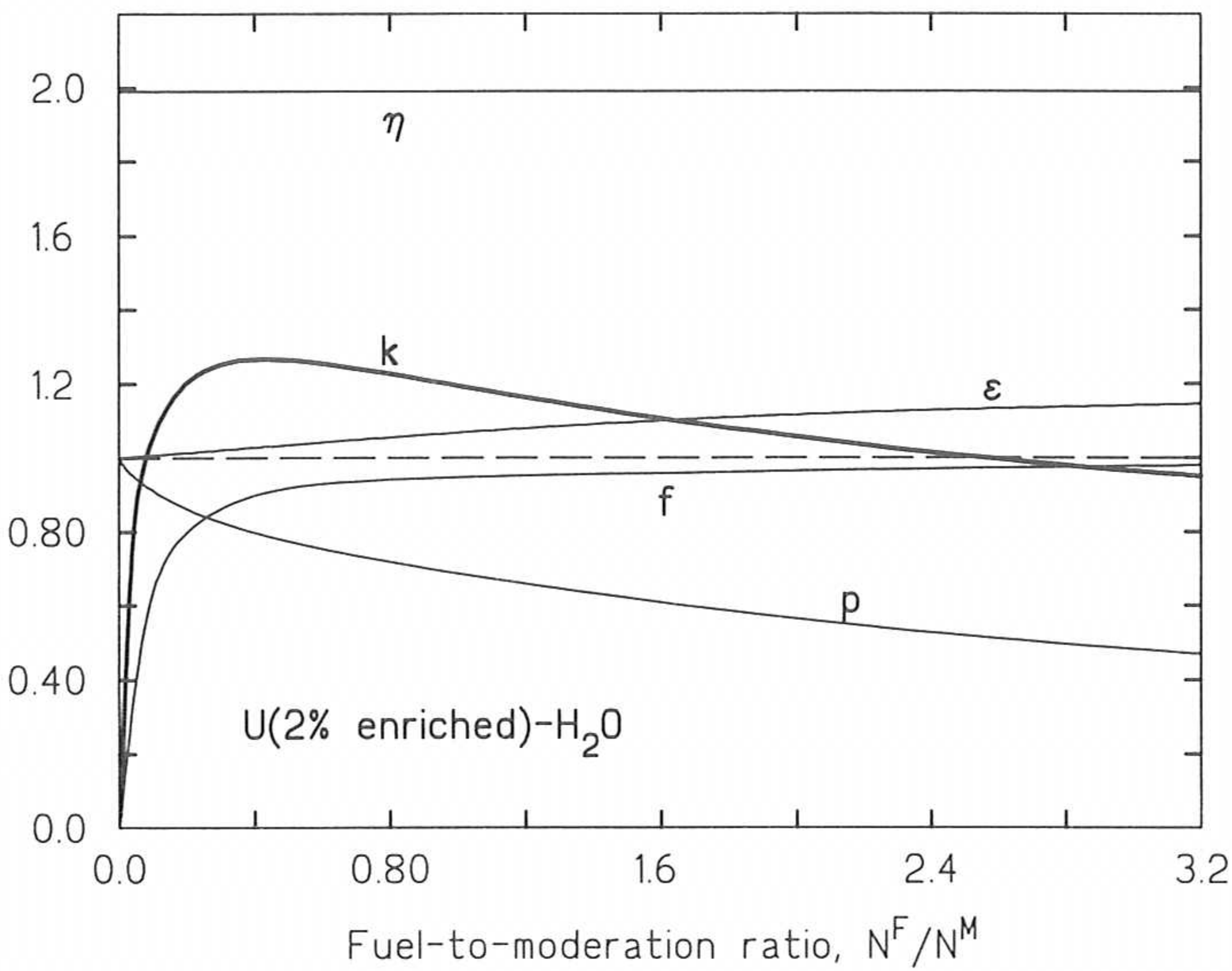
Typical Parameter Variation



Homogeneous reactor, 2% enrichment

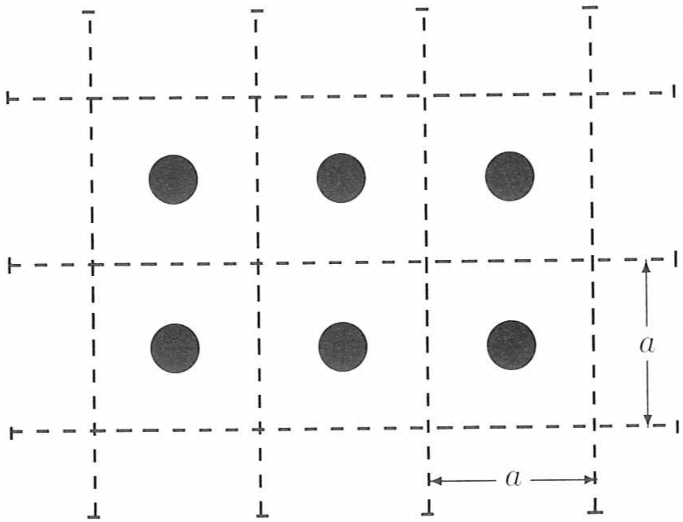
Example – Homogeneous, ^{239}Pu

- Pure ^{239}Pu fuel and reactor is a water-moderated($\frac{N^{NF}}{N^F}$=1), water-cooled bare infinite cylinder. How large of radius for criticality?
  - $\epsilon p \eta f P_{NL}^{f} P_{L}^{th} = 1 = k_{eff}$
  - $\epsilon p \approx 1$ (pure fuel makes p close to 1 and ϵ slightly greater than 1)
  - $\eta = 2.11$ (Fuel property – Table 10.1)
  - $f = (749 + 271) / \left[(749 + 271) + 2\left(0.333 + \frac{0.00019}{2}\right)\left(\frac{N^{NF}}{N^F}\right) \right]$ = 0.999
  - $P_{NL}^{th} = \frac{1}{L^2 B_c^2} = \frac{1}{L_M^2 (1-f) B_c^2} = \frac{1}{2.85^2 (0.001) \left(\frac{2.405}{R}\right)^2}$
  - $P_{NL}^{f} = \exp(-B_c^2 \tau) = \exp\left(-\left(\frac{2.405}{R}\right)^2 27\right)$
  - $\epsilon p \eta f P_{NL}^{f} P_{L}^{th} = 1 = 1(2.11) f \frac{\exp\left(-\left(\frac{2.405}{R}\right)^2 27\right)}{2.85^2 (1-f) \left(\frac{2.405}{R}\right)^2}$ = $68.8 R^2 \frac{N^F}{N^{NF}} exp(-\frac{156.6}{R^2})$

Heterogeneous vs. homogeneous

- Heterogeneous cores change the reactor parameters :
  - $\boldsymbol{p}$ resonance escape probability
    - increases significantly
    - neutrons slow primarily in the moderator
    - no (or controlled amounts of) highly absorbing nuclides.
  - $\boldsymbol{\epsilon}$ fast fission
    - Increases slightly
    - fast neutrons are primarily surrounded by fissionable and fissile nuclides
  - $\boldsymbol{f}$ thermal utilization at fixed fuel loading ($\boldsymbol{N^F}/\boldsymbol{N^{NF}}$)
    - Lower in heterogeneous reactor
    - Thermal neutron flux in fuel rod is less than that in moderator
  - $\boldsymbol{\eta}$ thermal fission factor
    - unchanged
    - depends only on the type of fuel
  - P_{NL}^{f}, P_{NL}^{th} Leakage probabilities
    - Unchanged
    - Depend primarily on reactor shape and size

Dependence on Design



Fuel with carbon moderator

local optimum – heterogeneous cores allow even graphite-moderated reactors to operate on natural uranium

Pitch a (cm)	ϵ	η	f	p	k_∞
12	1.027	1.322	0.972	0.742	0.979
16	1.027	1.322	0.947	0.848	1.090
20	1.027	1.322	0.916	0.900	1.120
21	1.027	1.322	0.907	0.909	1.121
22	1.027	1.322	0.898	0.917	1.119
26	1.027	1.322	0.860	0.940	1.098
30	1.027	1.322	0.818	0.955	1.060

One-group Reactor Equation

Mono-energetic neutrons (Neutron Balance)

$$D\nabla^2\phi - \Sigma_a\phi + s = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

v is neutron speed

For reactor, $s = \nu\Sigma_f\phi$

ν is neutrons/fission

In eigenfunction form and at steady state

$$D\nabla^2\phi - \Sigma_a\phi + \frac{\nu}{k}\Sigma_f\phi = 0$$

$$\Rightarrow \nabla^2\phi - \frac{\Sigma_a - \frac{\nu}{k}\Sigma_\mathrm{f}}{D}\phi = \nabla^2\phi + B^2\phi = 0$$

Material Buckling

$$\nabla^2\phi = -B^2\phi$$ eigenfunction form of reactor equation

$$\nabla^2\phi + B^2\phi = 0$$ one-group, steady-state, reactor equation

$$B^2 = \frac{\frac{\nu}{k}\Sigma_f - \Sigma_a}{D}$$ material buckling = (neutron generation – absorption)/diffusion

$$k = \frac{\nu\Sigma_f\phi}{DB^2\phi + \Sigma_a\phi} = \frac{\nu\Sigma_f}{DB^2 + \Sigma_a}$$ reactor multiplication factor

multiplication factor = neutron generation rate/(leakage + absorption)

Fuel utilization and k_∞

$$s = \eta \Sigma_{aF} \phi = \eta \frac{\Sigma_{aF}}{\Sigma_a} \Sigma_a \phi = \eta f \Sigma_a \phi$$

$$f = \frac{\Sigma_{aF}}{\Sigma_a}$$

f = fuel utilization factor – neutrons absorbed by fuel / (those absorbed by fuel + by other means – coolant, moderator, etc.)

$$k_\infty = \frac{\eta f \Sigma_a \phi}{\Sigma_a \phi} = \eta f$$

k_∞ = k-value for infinite (no overall leakage) reactor – a material property

η = fission neutrons generated per absorbed neutron

Operating Critical Reactor Equation

$k = 1$ Reactor operating at steady state

$$-DB^2\phi - \Sigma_a\phi + \frac{k_\infty}{k}\Sigma_a\phi = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

$$-DB^2\phi + (k_\infty - 1)\Sigma_a\phi = 0$$

$$-B^2\phi + (k_\infty - 1)\frac{\Sigma_a}{D}\phi = -B^2\phi + \frac{(k_\infty - 1)}{L^2}\phi = 0$$

$$L^2 = \frac{D}{\Sigma_a}$$ One-group diffusion area

$$B^2 = \frac{k_\infty - 1}{L^2}$$ One-group buckling

Perspective

- Previous equations show
  - how to solve for neutron flux profile $\boldsymbol{\phi}$ as a function of space
  - How to determine critical reactor dimensions
    - $\boldsymbol{B^2} = \frac{\boldsymbol{k_\infty - 1}}{\boldsymbol{L^2}} = \frac{\boldsymbol{(k_\infty - 1)\Sigma_a}}{\boldsymbol{D}}$, for a bare reactor ($B^2_g = B^2_{mat}$)
- First find solutions to the reactor equations
  - 1D, 2D, or 3D
  - Then find dimensions for a critical reactor
- Assumptions:
  - Bare, homogeneous reactors
  - Constant (special and temporal) properties
  - None are valid but, but help to develop insight into reactor operations
- Because source terms are proportional to the flux, the generally inhomogeneous differential equations are now homogeneous equations.