Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 19

Nuclear Reactor Theory IV

Nuclear Kinetics

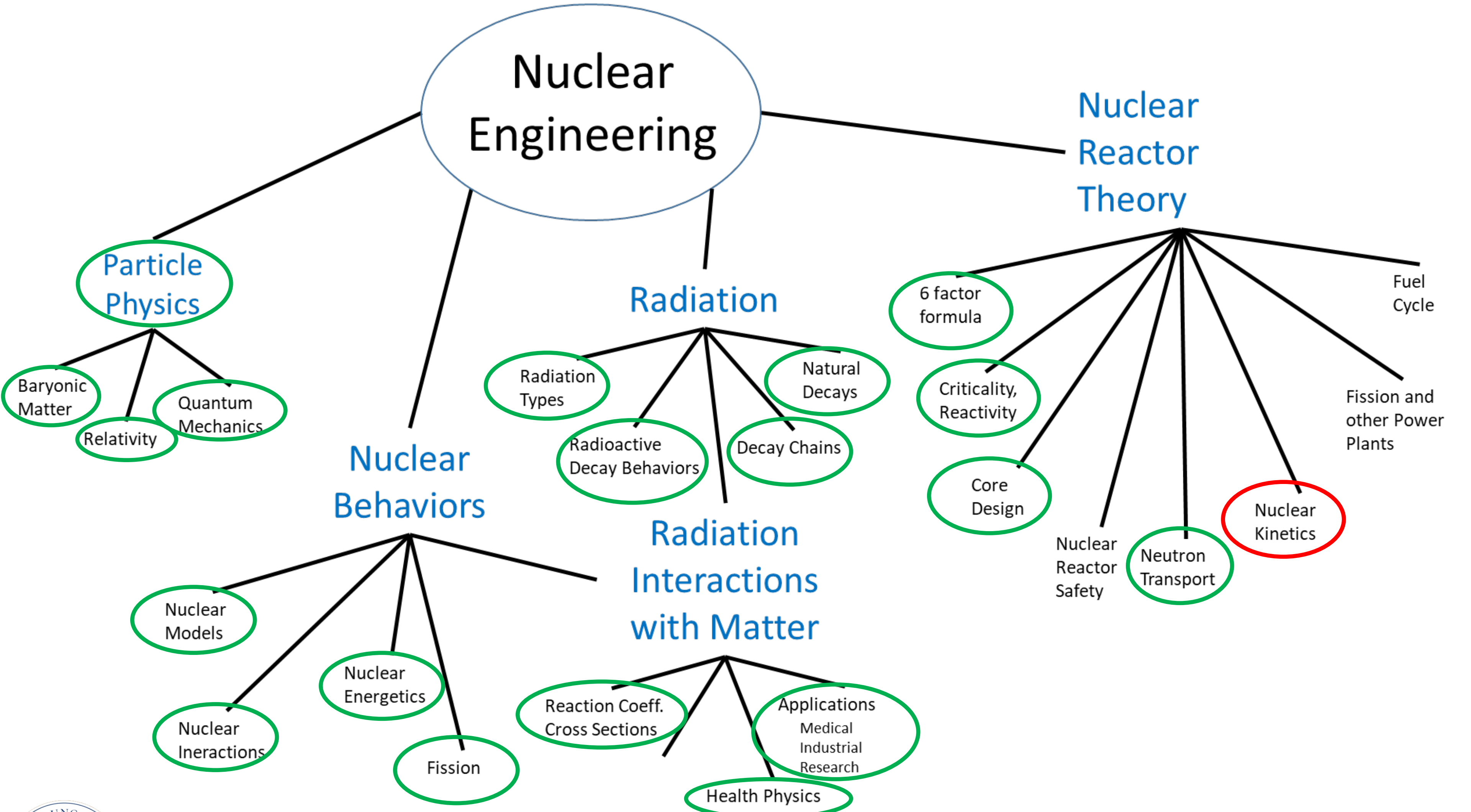
Spiritual Thought

"When we put God first, all other things fall into their proper place or drop out of our lives. Our love of the Lord will govern the claims for our affection, the demands on our time, the interests we pursue, and the order of our priorities.

We should put God ahead of *everyone else* in our lives."

President Ezra Taft Benson

Roadmap


Nuclear Engineering
Particle Physics
Baryonic Matter
Relativity
Quantum Mechanics
Nuclear Behaviors
Nuclear Models
Nuclear Energetics
Nuclear Ineractions
Fission
Radiation
Radiation Types
Natural Decays
Radioactive Decay Behaviors
Decay Chains
Radiation Interactions with Matter
Reaction Coeff. Cross Sections
Applications
Medical
Industrial
Research
Health Physics
Nuclear Reactor Theory
6 factor formula
Criticality, Reactivity
Core Design
Nuclear Reactor Safety
Neutron Transport
Nuclear Kinetics
Fission and other Power Plants
Fuel Cycle

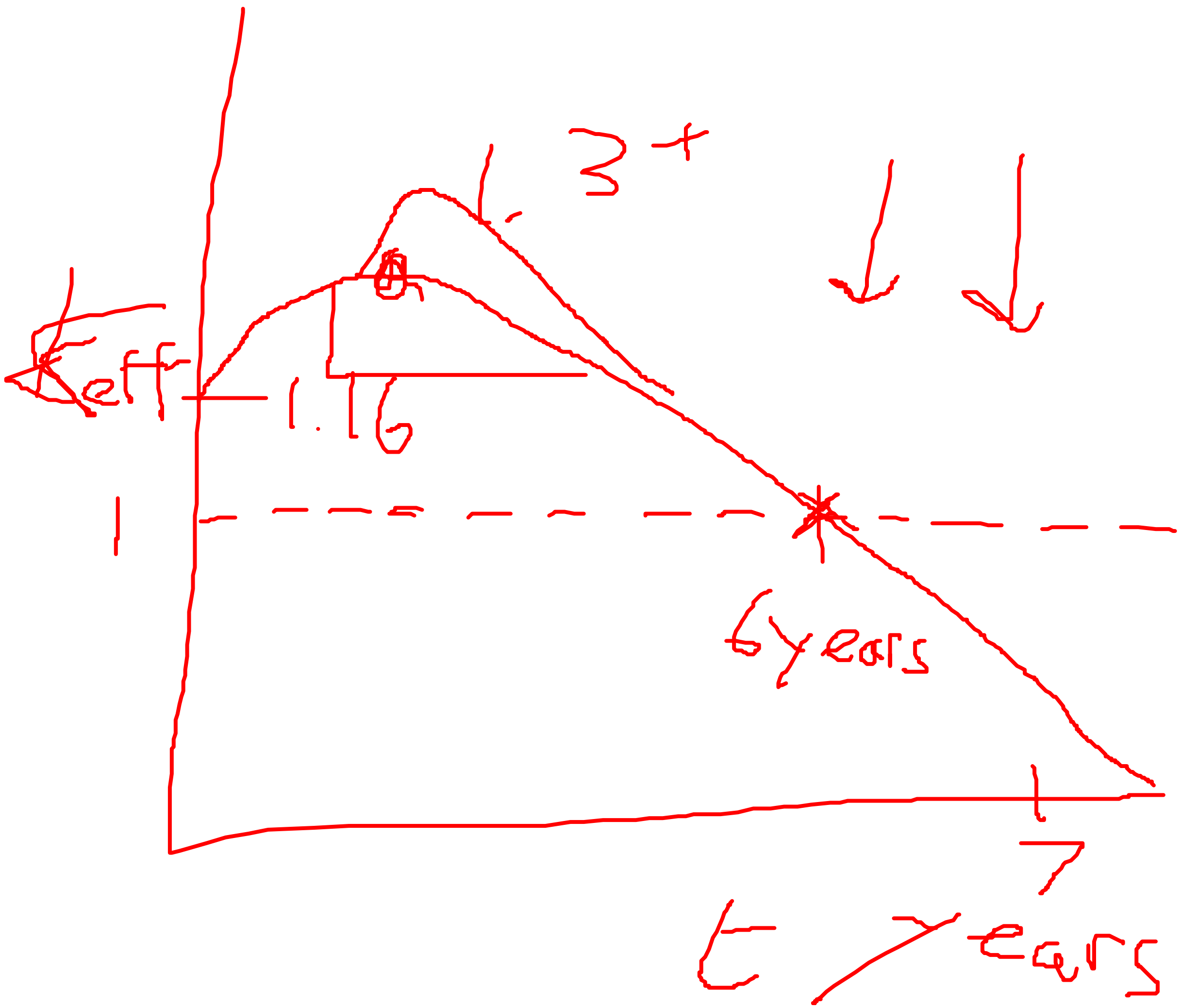
Objectives

- Understand the simple, “big picture” of kinetics
- Understand how delayed neutrons/precursors work
- Understand, recognize and do calculations relating to feedback coefficients
- Understand reactor “control strategies”
- Understand xenon peak phenomena

Power Plant Operation

- Try to achieve "steady state" operation at all times
- Set up core and fuel to provide minimal changes
- Despite this, still have three scales of change
    1. Short Term (Grid load transients)
    2. Intermediate Term (Fuel composition changes)
    3. Long Term (Burnup & Depletion)

K_{eff} vs. time curve for Power plant?



Three Time Scales (short)

- Short Time Constant (load change)
  - An abrupt change in steam demand/load.
  - A load change is seen as:
    - Reactor pressure change in the BWR
    - Reactor temperature change in a PWR.
  - Higher loads lead to higher pressures/temperatures.
- Assumptions
  - Assumes a uniform multiplicative change everywhere
    - Spatial variations in time in the reactor are not considered
  - (This method is called <u>point kinetics</u>.)

Three Time Scales (intermediate)

- Intermediate Time Constant (core composition change)
  - Changing fission product concentrations
    - Generation rates and destruction/decay rates.
  - Many fission products have measurable thermal neutron cross sections

$$f = \frac{\Sigma_a^F}{\Sigma_a^F + \Sigma_a^{NF}(V^{NF}/V^F)(\phi^{NF}/\phi^F)}$$

    - ***Change*** the value of *k* (and k_∞).
  - Point kinetics can be used if spatial variations in concentrations are negligible.
  - Otherwise, detailed spatial and temporal equations must be used!

Three Time Scales (long)

- Long Time Constant (Fuel Depletion)
  - Treated as a series of steady-state problems

$$D\nabla^2\phi - \Sigma_a\phi = -\lambda\nu\Sigma_f\phi$$

  - Two things are adjusted to maintain λ = 1 (i.e. λ = 1/*k*)
    - Material buckling
    - Reactor dimensions
    - If $\lambda \neq 1$, the equation is not valid
      - Why?
  - In operating reactor, cannot change dimensions (much)
    - *k* is adjusted slowly in time by changing chemical shim conc.
      - Chemical shim is an isotope that absorbs neutrons in the reactor
    - k is also adjusted via the control rods.
  - Managing fuel consumption is a classical example of this type of transient

Reactivity and Worth

$$\rho \equiv \frac{k_{eff} - 1}{k_{eff}} = \frac{\delta k}{k_{eff}}$$

reactivity ρ and δk

$$k(\$) \equiv \frac{\rho}{\beta}$$

β is delayed neutron fraction

worth can be measured in units of $k(\$)$ or k

- cents?
- Percent Mil?

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General Transient Problem

Mono-energetic neutrons

$$D\nabla^2\phi - \Sigma_a\phi + S = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

For a reactor, $S = \nu\Sigma_f\phi$

$$D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

$$\nabla^2\phi - B\ \ \phi \quad \frac{1}{\ \ }\frac{\partial\phi}{\ \ }$$

Prompt Neutrons

- Lifetime, l_p, is time between emission and absorption.
- Neutrons in thermal reactors:
  - Spend more time (most of l_p) in the thermal regime
  - Travel further as fast neutrons
- Average lifetime of a thermal neutron in an infinite reactor is the mean diffusion time, t_d, and is approximately the same as l_p in an infinite reactor. $l_p \approx t_d$

$$t_d = t_{dM}(1 - f)$$

TABLE 7.1 APPROXIMATE DIFFUSION TIMES FOR SEVERAL MODERATORS

Moderator	t_d, sec
H_2O	2.1×10^{-4}
D_2O*	4.3×10^{-2}
Be	3.9×10^{-3}
Graphite	0.017

*With 0.25% H_2O impurity.

- In fast reactors, prompt neutron lifetimes are much shorter, on the order of 10^{-7} seconds

Simple Kinetics Model

$$\Delta n(t) \equiv \ell_p \frac{dn(t)}{dt} = (k_{eff} - 1)n(t)$$

$$\frac{dn(t)}{dt} = \frac{k_{eff} - 1}{\ell_p} n(t)$$

$$\Rightarrow n(t) = n(0)\exp\left(\frac{k_{eff} - 1}{\ell_p} t\right)$$

- For ^{235}U
  - ℓ_p= 2.1x10^{-4} s
  - $k_{eff} - 1 = 0.001$
  - and $t = 1s$,
- n/n^0 = 117 (22,027 if $\ell' = 10^{-4}$ as in text)
- Far too rapid to control!!!

Reactors with delayed neutrons

Delayed neutrons: β is fraction of total neutrons

$$\bar{\ell}_p = (1-\beta)\ell_p + \beta(\ell_p + \tau) \approx \ell_p + \beta\tau$$

τ is lifetime of delayed neutrons = $\frac{T_{1/2}}{\ln 2} \approx 12.8\ s$

For $\delta k \ll \beta$

$$\frac{n(t)}{n_0} = \exp\left(\frac{k_{eff}-1}{\bar{\ell}_p}\right) = \exp\left(\frac{t}{T}\right)$$

$$T = \frac{\bar{\ell}_p}{k_{dff}-1} = \frac{\beta\tau}{\delta k}$$

- For ^{235}U, T = 83 s, k_{eff}-1 = 0.001,
- n/n^0 = 1.012
- This can be controlled!

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Delayed Neutrons

TABLE 3.5 DELAYED NEUTRON DATA FOR THERMAL FISSION IN ^{235}U*

Group	Half-Life (sec)	Decay Constant (l_i, sec^{-1})	Energy (ke V)	Yield, Neutrons per Fission	Fraction (β_i)
1	55.72	0.0124	250	0.00052	0.000215
2	22.72	0.0305	560	0.00346	0.001424
3	6.22	0.111	405	0.00310	0.001274
4	2.30	0.301	450	0.00624	0.002568
5	0.610	1.14	—	0.00182	0.000748
6	0.230	3.01	—	0.00066	0.000273
					Total yield: 0.0158
					Total delayed fraction (β): 0.0065

*Based in part on G. R. Keepin, *Physics of Nuclear Kinetics*, Reading, Mass.: Addison-Wesley, 1965.

For 1-group model, $T_{\frac{1}{2}}$ for ^{235}U is about 8.87 s and τ is about 12.8 s.

Delayed Neutron Fractions

Group	^{235}U		^{233}U		^{239}Pu	
	Half-life (s)	fraction β_i	Half-life (s)	fraction β_i	Half-life (s)	fraction β_i
1	55.7	0.00021	55.0	0.00022	54.3	0.00007
2	22.7	0.00142	20.6	0.00078	23.0	0.00063
3	6.22	0.00127	5.00	0.00066	5.60	0.00044
4	2.30	0.00257	2.13	0.00072	2.13	0.00068
5	0.610	0.00075	0.615	0.00013	0.618	0.00018
6	0.230	0.00027	0.277	0.00009	0.257	0.00009
total	-	0.0065	-	0.0026	-	0.0021

Point Kinetic Equations

$$\frac{dn(t)}{dt} = \frac{\rho - \beta}{l} n(t) + \sum_{i=1}^{G} \lambda_i C_i(t) + S(t)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta}{l} n(t) - \lambda_i C_i(t), \mathrm{i} = 1, \dots, \mathrm{G}$$

Reactivity Equation Solutions

$$\phi_T = \underbrace{A_1 \exp(\omega_1 t)}_{\substack{dominant\ term \\ as\ t\to\infty}} + \underbrace{A_2 \exp(\omega_2 t)}_{approaches\ 0\ rapdily}$$

General solution for single group of delayed neutrons

$$T = \frac{1}{\omega_1}$$

Definition of reactor or stable period

$$\boxed{\phi_T = \exp\left(\frac{t}{T}\right)}$$

General solution for single group of delayed neutrons

$$\rho = \frac{\omega l_p}{1+\omega l_p} + \frac{\omega}{1+\omega l_p}\sum_{i=1}^{6}\frac{\beta_i}{\omega+\lambda_i}$$

Reactivity equation for six group model

Power Changes

$$T = \frac{\beta\tau}{\delta k} = \frac{\beta\tau}{keff \cdot \rho} = \frac{\tau}{keff \cdot k(\$)} \sim \frac{\tau}{k(\$)}$$

- T = Reactor Period (units of time)
  - Time required to increase reactor power (or neutron flux) by 2.72
- Integrate solution with specific values (initial time power to final time/power

$$\frac{t}{T} = ln\left[\frac{P(t)}{P(0)}\right]$$

Reactivity Equation Solutions

$$T = \frac{\bar{l}}{k_{eff} - 1} = \frac{\bar{l}}{\delta k} = \frac{\beta\tau}{\delta k} =$$

Reactor period - The time required for a neutron population to change by a factor of e

$$k_{eff} = 1 + \delta k = 1 + \frac{\beta\tau}{T}$$

$\tau =$ Lifetime of delayed neutrons ~12.8s (U235)

$$T = \frac{\beta\tau}{\delta k} = \frac{\beta\tau}{k_{eff} - 1} = \frac{\beta\tau}{k_{eff}\rho} = \frac{\tau}{k_{eff}\rho(\$)} \approx \frac{\tau}{\rho(\$)}$$

$$\phi(t) = exp\left(\frac{t}{T}\right)$$

Remember, Flux is proportional to power….

$$C \cdot P(t) = exp\left(\frac{t}{T}\right) \qquad \frac{t}{T} = ln\left(\frac{P(t) \cdot C}{P(0) \cdot C}\right) \qquad \frac{P(t)}{P(0)} = exp\left(\frac{t}{T}\right)$$

Example

- Following a reactor scram in which all the control rods are inserted into a power reactor, how long is it before the reactor power decreases to 0.0001 of the steady-state power prior to shutdown? (Assume a reactor period of -80 s)

$$T = -80\,s \qquad \frac{P(t)}{P(0)} = .0001$$

$$t = T \ln\left(\frac{P(t)}{P(0)}\right)$$

$$\boxed{t = 736.8\ sec}$$

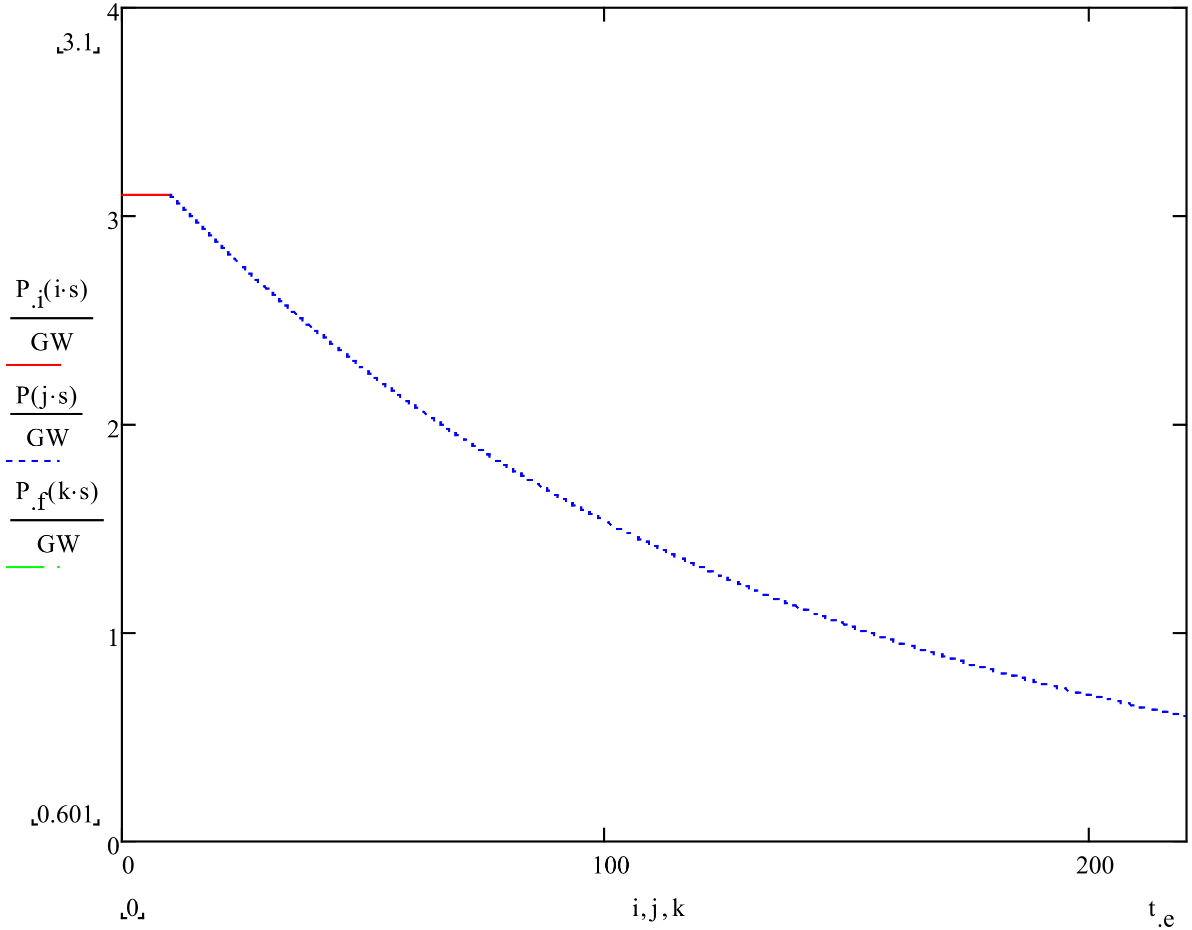
1-level Model Parameters

	β	$T_{\frac{1}{2},d}(s)$	$\tau_d(s)$
^{232}Th	0.0203	6.98	10.07
^{233}U	0.0026	12.4	17.89
^{235}U	0.0064	8.82	12.72
^{238}U	0.0148	5.32	7.68
^{239}Pu	0.002	7.81	11.27
^{241}Pu	0.0054	104.1	150.18
^{241}Am	0.0013	10	14.43
^{243}Am	0.0024	10	14.43
^{242}Cm	0.0004	10	14.43

Source: Laboratoire de Physique Subatomique et de Cosmologie

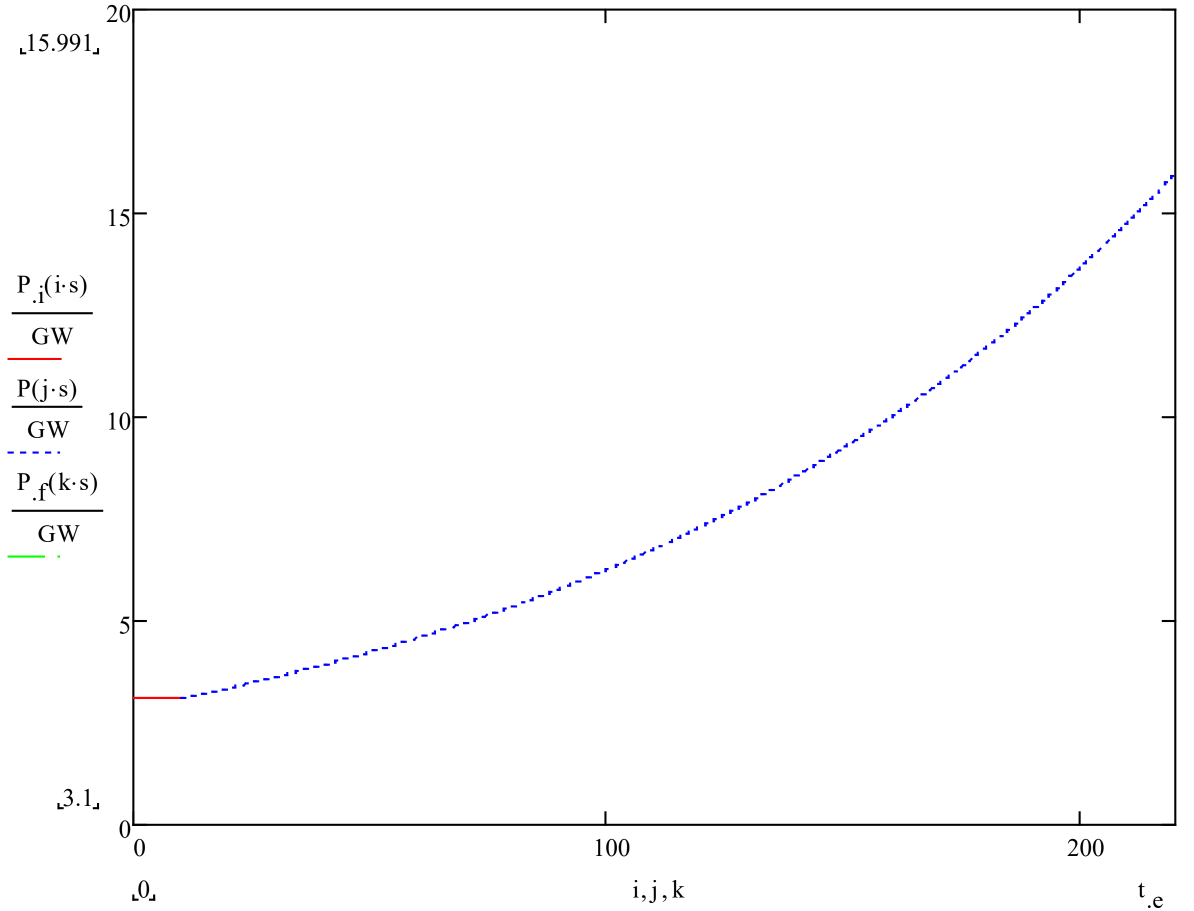
Exploration 1

- What if we add -$0.1 to AP1000 core?



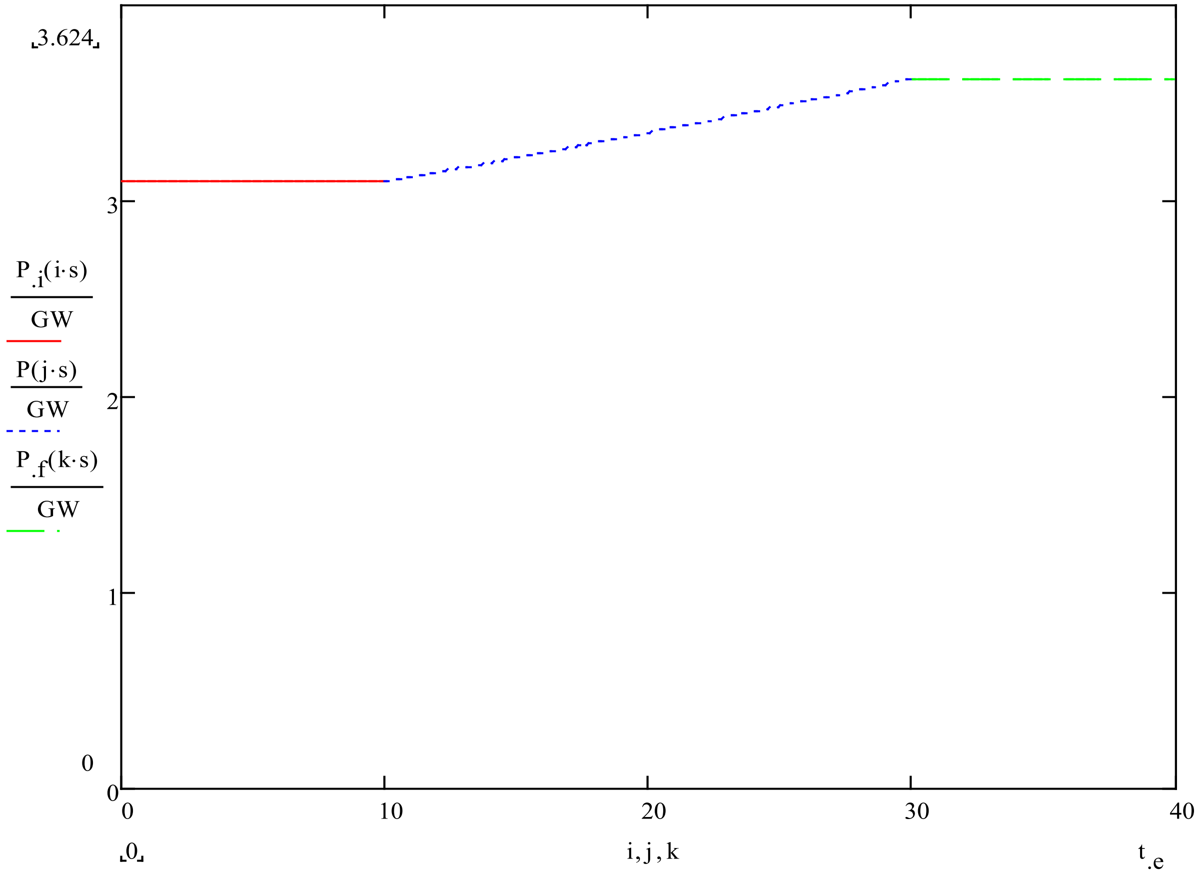
Exploration 2

- What if we add $0.1 to AP1000 core?

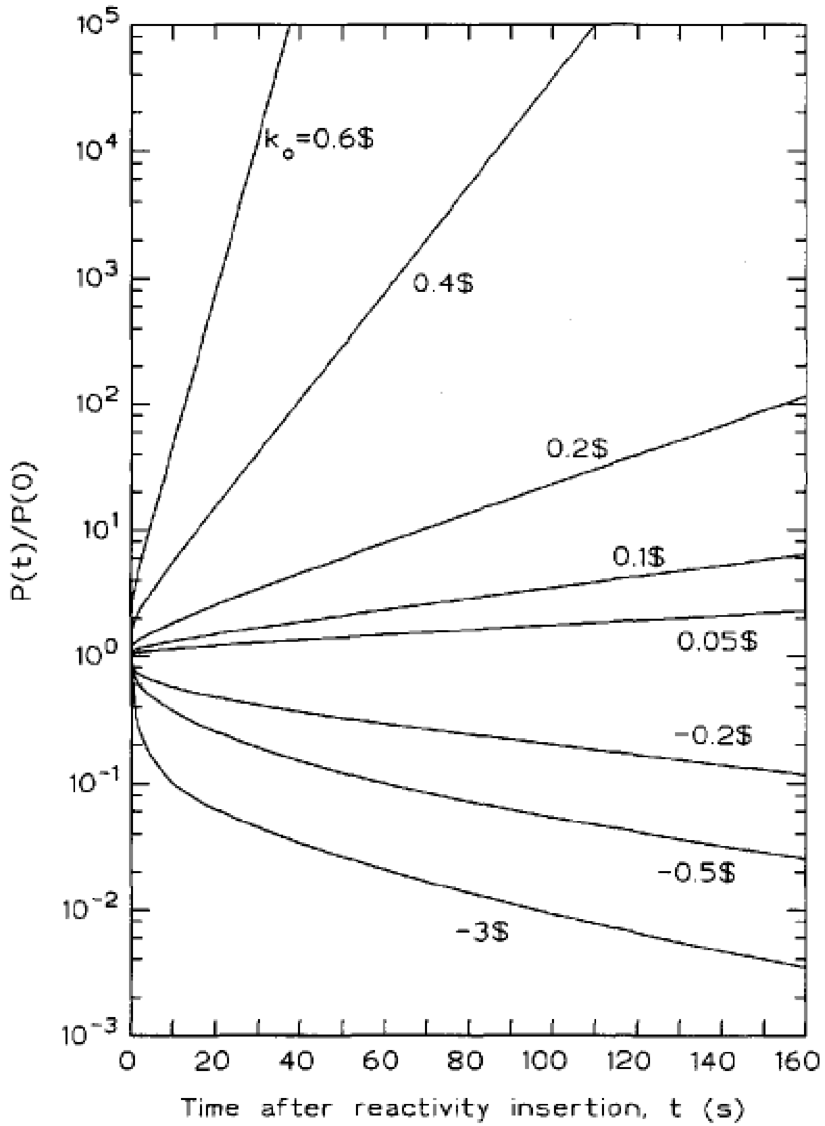


Exploration 3

- What if we add \$0.1 to AP1000 core, then after 10 seconds we add -\$0.1?



Reality



10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
P(t)/P(0)
k_o=0.6$
0.4$
0.2$
0.1$
0.05$
−0.2$
−0.5$
−3$
0
20
40
60
80
100
120
140
160
Time after reactivity insertion, t (s)