Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 7

Nuclear Fission

Radiation Interactions with Matter

Spiritual Thought

"No man, having put his hand to the plough, and looking back, is fit for the kingdom of God." When difficult things are asked of us, even things contrary to the longings of our heart, remember that the loyalty we pledge to the cause of Christ is to be the supreme devotion of our lives. Although Isaiah reassures us it is available "without money and without price"—and it is—we must be prepared, using T. S. Eliot's line, to have it cost "not less than everything."

Jeffery R. Holland

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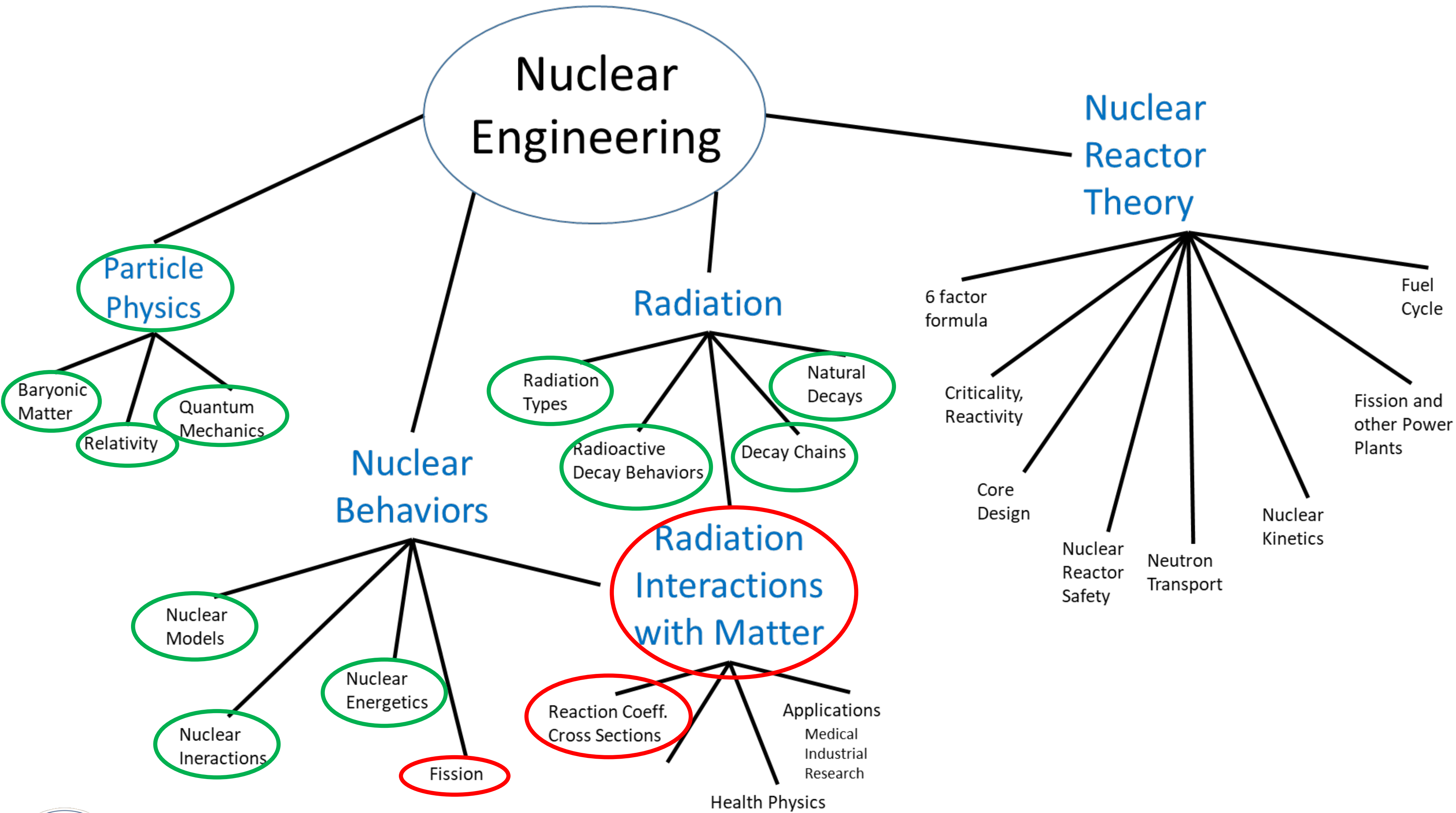
Roadmap


Nuclear Engineering
Particle Physics
Baryonic Matter
Relativity
Quantum Mechanics
Nuclear Behaviors
Nuclear Models
Nuclear Ineractions
Nuclear Energetics
Fission
Radiation
Radiation Types
Radioactive Decay Behaviors
Decay Chains
Natural Decays
Radiation Interactions with Matter
Reaction Coeff. Cross Sections
Health Physics
Applications
Medical
Industrial
Research
Nuclear Reactor Theory
6 factor formula
Criticality, Reactivity
Core Design
Nuclear Reactor Safety
Neutron Transport
Nuclear Kinetics
Fission and other Power Plants
Fuel Cycle

Objectives

- Understand mechanics and details of Fission
- Understand how radiation interacts with Matter
- Be able to calculate probabilities of interaction and radiation field intensities
- Understand both linear interaction coefficients and cross-sections
- Be able to calculate or find μ, σ, and Σ

Neutron Interactions

- Elastic scattering (n,n) – collision with no reaction and no change in total kinetic energies. Energy neutral.
- Inelastic scattering (n,n') – collisions with energy absorption by nucleus. endoergic
- Radiative capture (n,γ) – Capture of neutron by nucleus followed by γ-ray emission. exoergic.
- Charged particle reactions (n,α) – Neutron reaction to form α particles or protons. endoergic and exoergic.
- Neutron producing reactions (n,*x*n) – Reactions with a net increase in neutrons. endoergic. (n,2n) important for ^2H and ^9Be.
- Fission (n,) forms multiple products – Nucleus forms daughters. Generally exoergic.

Fission Reactions

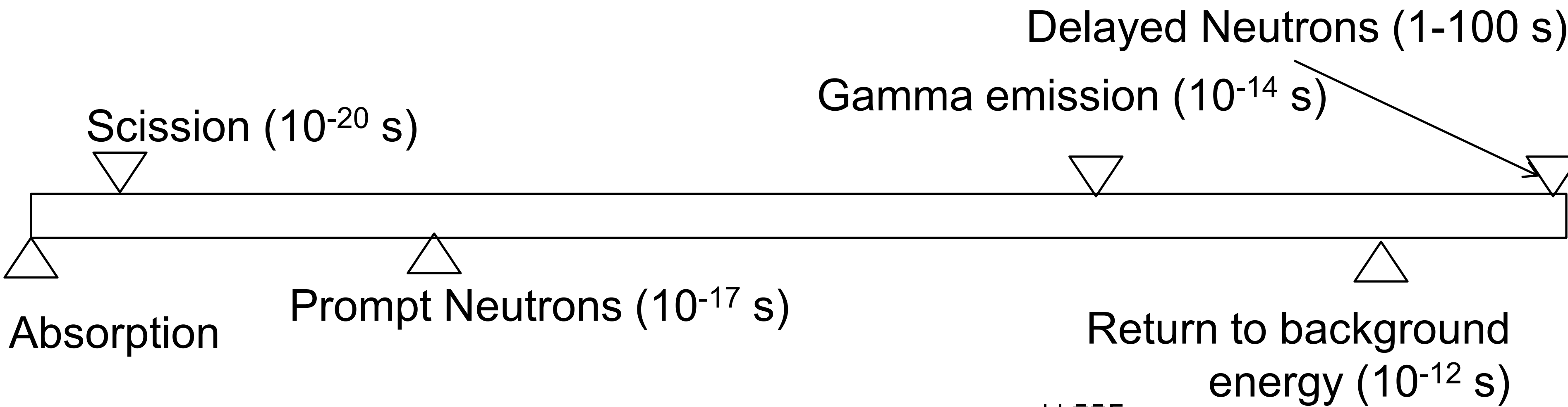
${}^{235}_{92}U$ is fissile (undergoes fission)

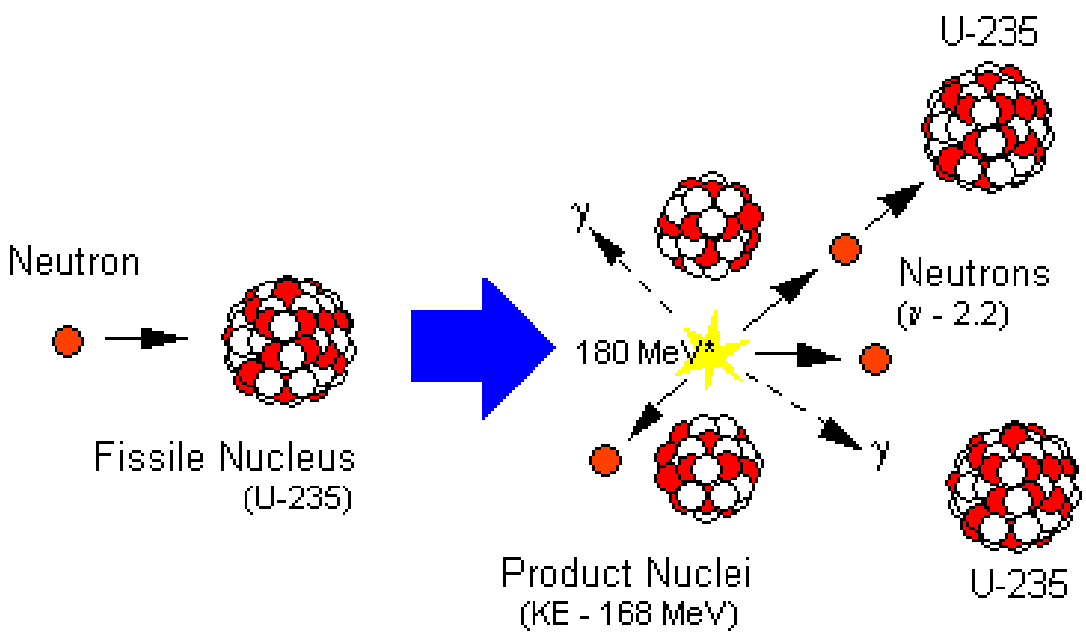
${}^{238}_{92}U$ is fertile (converts to a fissionable isotope)

Possible outcomes of ${}^{235}_{92}U$ reaction with neutron

$$n + {}^{235}_{92}U \rightarrow \begin{cases} {}^{235}_{92}U + n \quad \textit{elastic scatter} \\ {}^{235}_{92}U + n + \gamma \quad \textit{inelastic scatter} \\ {}^{236}_{92}U + \gamma \quad \textit{radiative capture} \\ Y_H + Y_L + y_1 + y_2 + \cdots \quad \textit{fission} \end{cases}$$

Fission (logarithmic) Timeline





Fission Animation

Emitted/Recoverable Energy

TABLE 3.6 EMITTED AND RECOVERABLE ENERGIES FOR FISSION OF ^{235}U

Form	Emitted Energy, MeV	Recoverable Energy, MeV
Fission fragments	168	168
Fission-product decay		
β-rays	8	8
γ-rays	7	7
neutrinos	12	—
Prompt γ-rays	7	7
Fission neutrons (kinetic energy)	5	5
Capture γ-rays	—	3–12
Total	207	198–207

$$E_f = 200 \text{ MeV}$$

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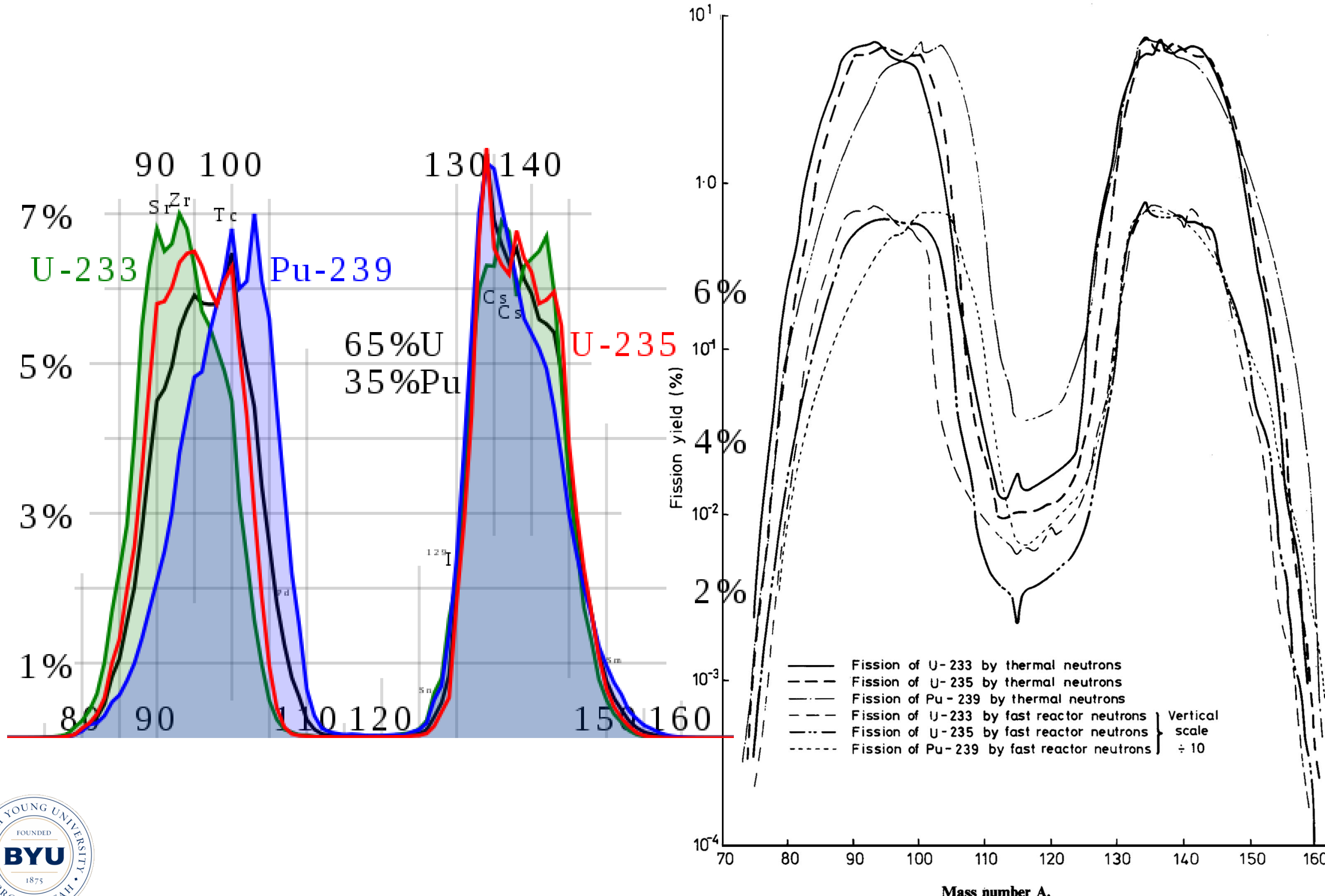
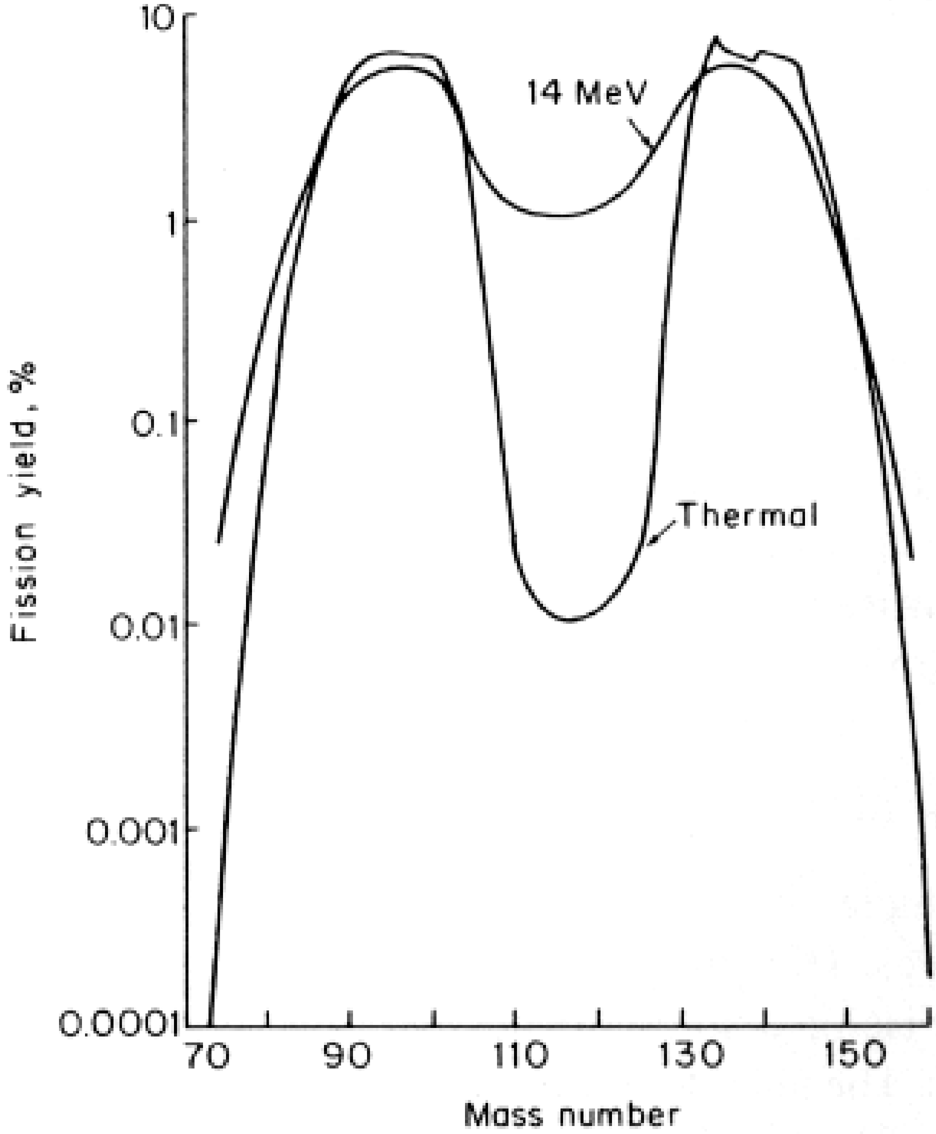
Fission Product Distribution



Product Distribution at High Energy



Delayed Neutrons

- A small fraction (<1%) of total neutron production occur seconds or minutes after scission, represented by $\boldsymbol{\beta}$ below. These delayed neutrons are essential to reactor control.
- Fast neutron emission alone is far too rapid to allow control.

Nuclide	Fast Fission		Thermal Fission	
	$\overline{\nu}$	β	$\overline{\nu}$	β
^{235}U	2.57	0.0064	2.43	0.0065
^{233}U	2.62	0.0026	2.48	0.0026
^{239}Pu	3.09	0.0020	2.87	0.0021
^{241}Pu	–	–	3.14	0.0049
^{238}U	2.79	0.0148	–	–
^{232}Th	2.44	0.0203	–	–
^{240}Pu	3.3	0.0026	–	–

Source: Keepin [1965].

Delayed Neutron Data

TABLE 3.5 DELAYED NEUTRON DATA FOR THERMAL FISSION IN ^{235}U*

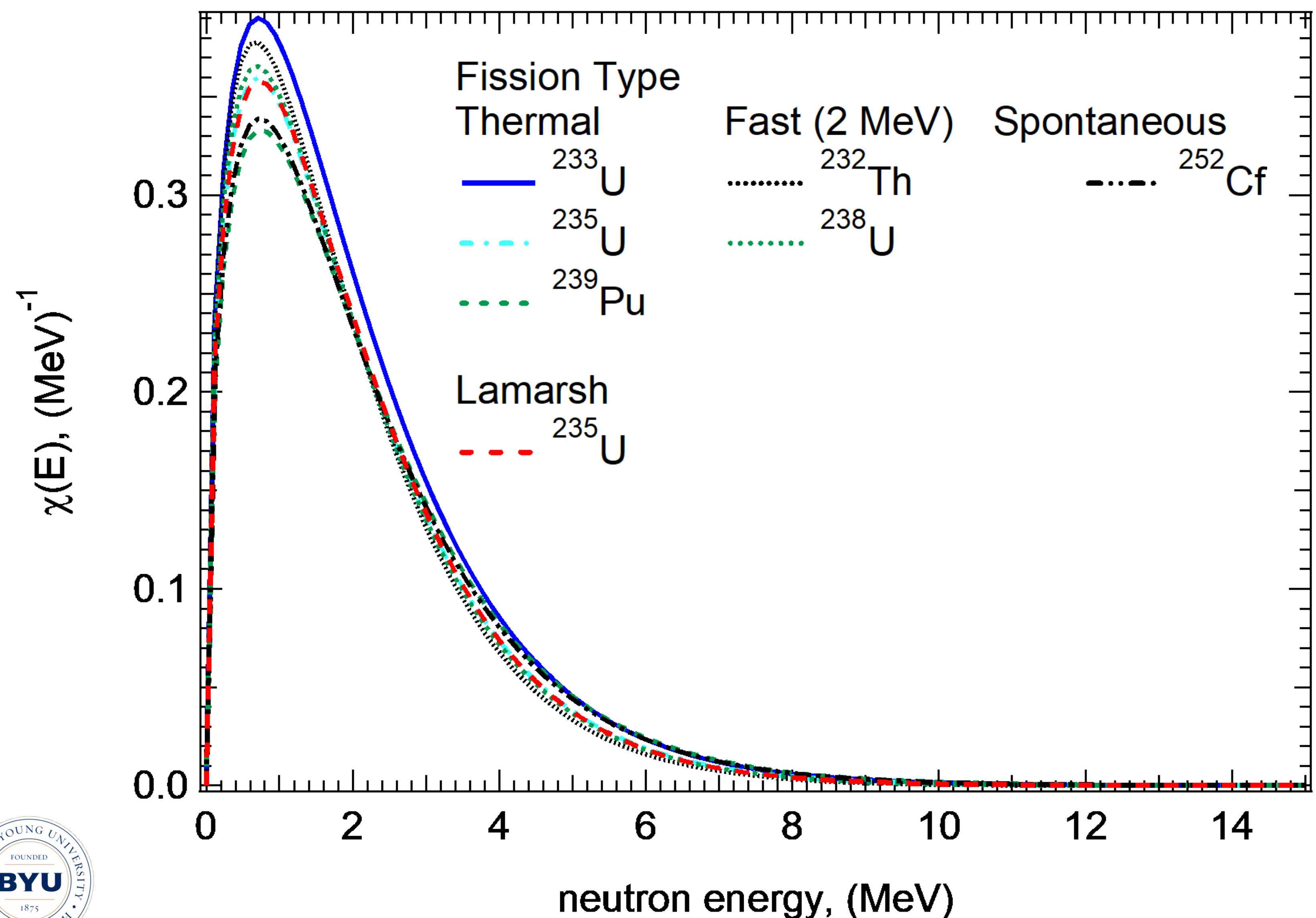
Group	Half-Life (sec)	Decay Constant (l_i, sec^{-1})	Energy (ke V)	Yield, Neutrons per Fission	Fraction (β_i)
1	55.72	0.0124	250	0.00052	0.000215
2	22.72	0.0305	560	0.00346	0.001424
3	6.22	0.111	405	0.00310	0.001274
4	2.30	0.301	450	0.00624	0.002568
5	0.610	1.14	—	0.00182	0.000748
6	0.230	3.01	—	0.00066	0.000273

Total yield: 0.0158

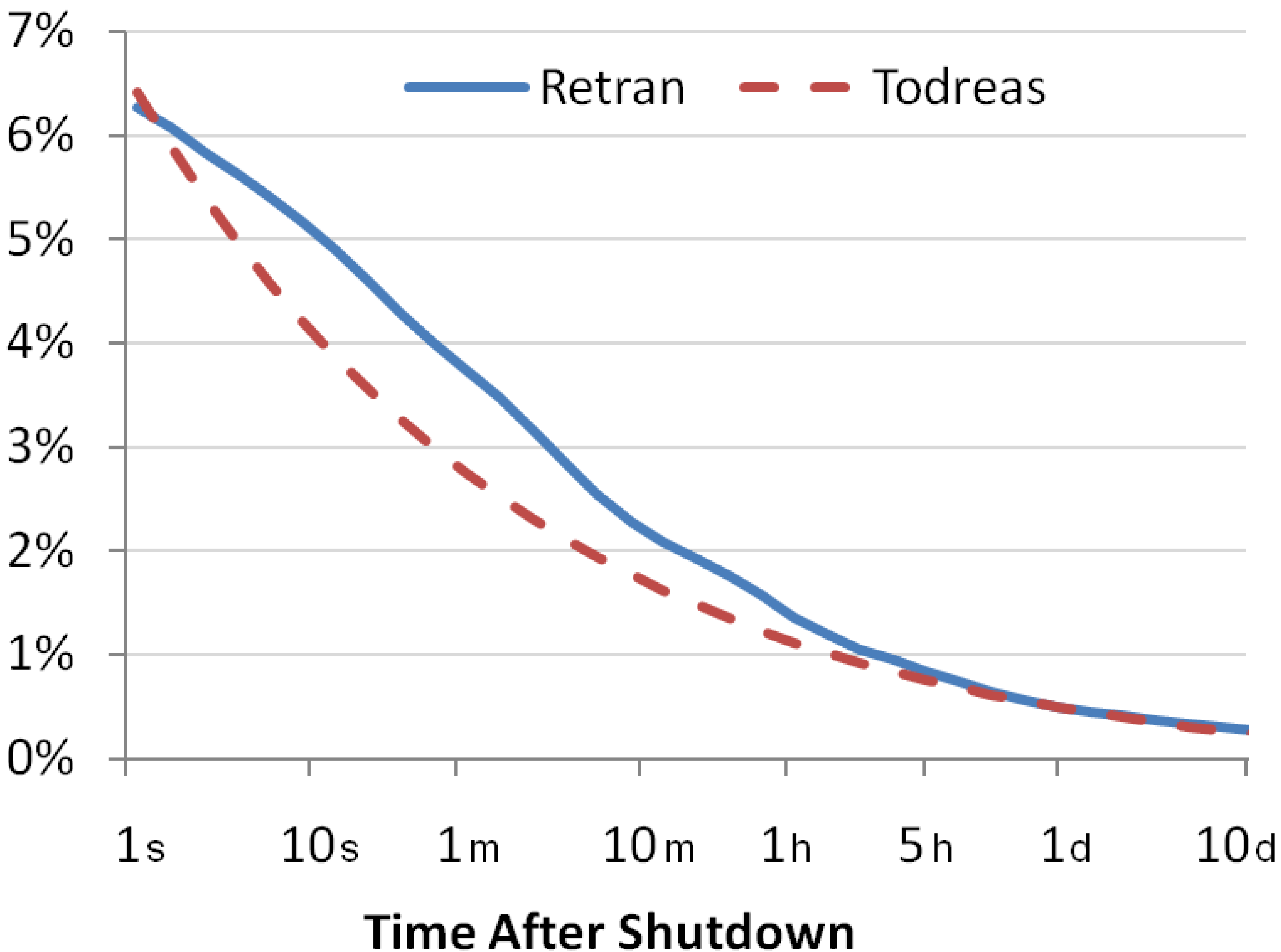
Total delayed fraction (β): 0.0065

*Based in part on G. R. Keepin, *Physics of Nuclear Kinetics*, Reading, Mass.: Addison-Wesley, 1965.

Neutron Energy Spectrum



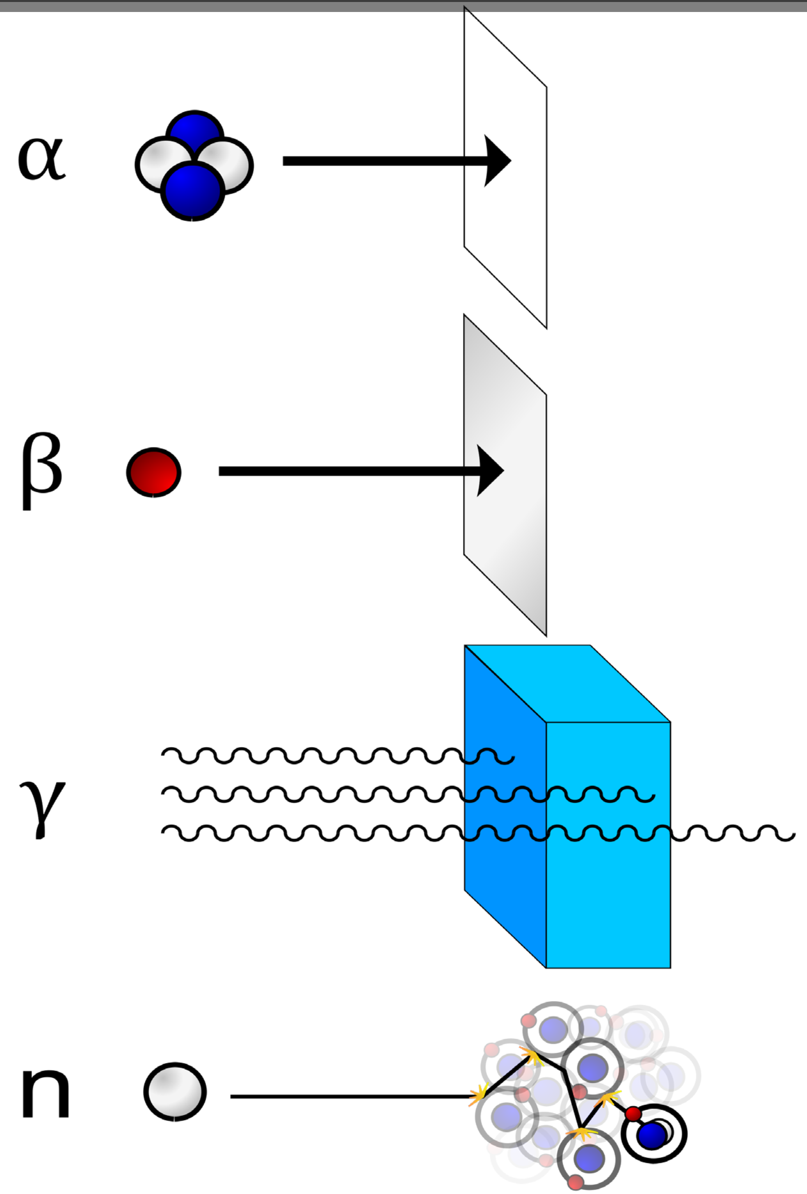
Decay Heat



Retran
Todreas
7%
6%
5%
4%
3%
2%
1%
0%
1s
10s
1m
10m
1h
5h
1d
10d
Time After Shutdown

Radiation Interaction with Matter

- World is awash with radiation
- First step to understanding impact is knowing how it interacts
- Different particles have different effects
- Derive general terms to quantify interactions



Linear Interaction Coefficient

- As a particle passes through a homogeneous material
  - Probability of interaction is constant per differential unit distance traveled
  - Empirically derived

$$\mu_i \equiv \lim_{\Delta x \to 0} \frac{P_i(\Delta x)}{\Delta x}$$

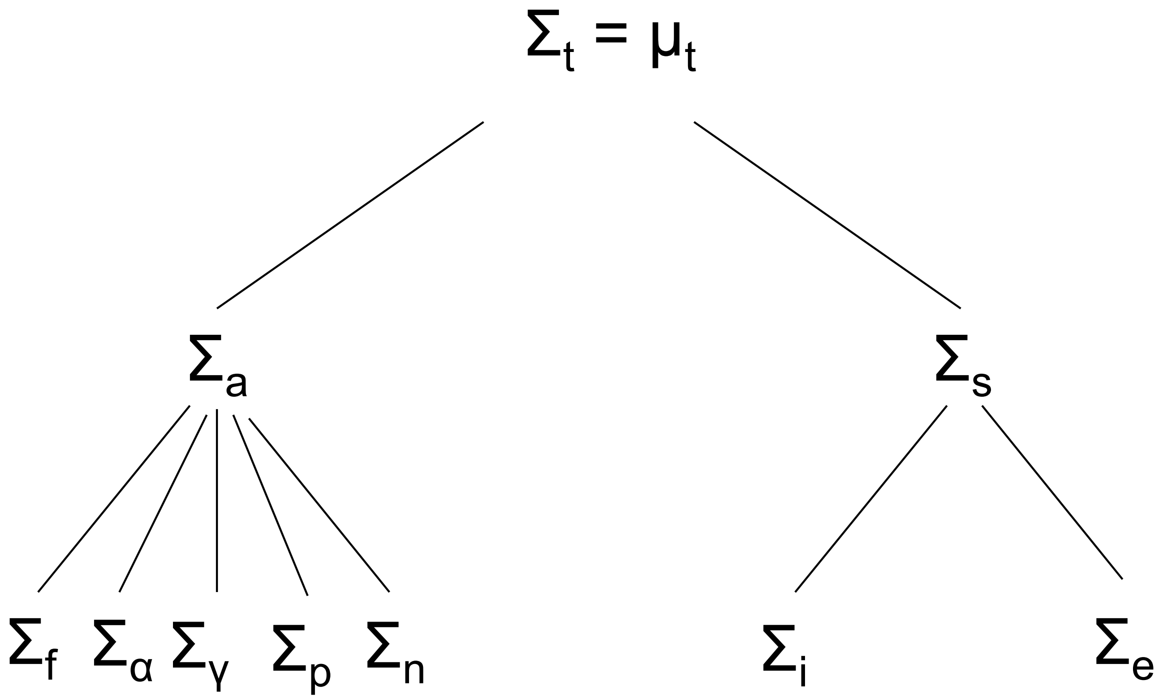
- μ_i is called the macroscopic interaction coefficient
- indicated by Σ_i (except for photons).
- Depends on
  - Particle energy
  - Reaction Type
    - Scattering, absorption, fission, etc.
      - energy-dependent macroscopic linear absorption coefficient
      - linear fission coefficient
      - linear scattering coefficient, etc.
  - Medium type

Total Probability of Interaction

- Interaction coefficients are divided into subcategories
  - i.e. total scattering coefficient, Σ_t
    - Linear scattering coefficients, Σ_s
    - Non-linear scattering coefficients
  - Total absorption coefficient, Σ_a
    - Neutron capture
    - Fission
    - Other absorbing interactions
- Total is sum of components
  - Radiation – linear attenuation coefficient
  - Neutrons – Cross Section
  - Photons – Mass Interaction Coefficient

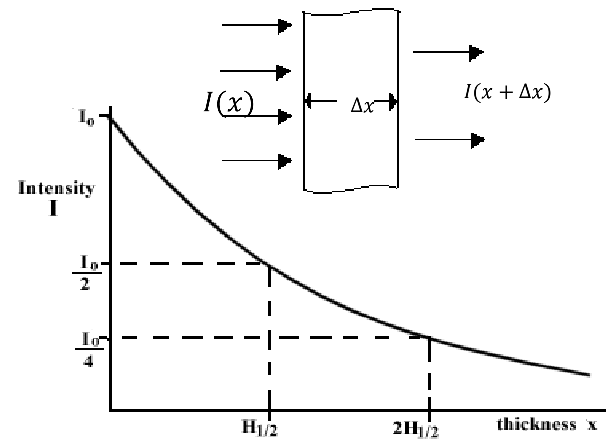
$$\mu_t(E) = \sum_i \mu_i(E)$$

Or in other words…



Interaction in Material

The fractional amount of a beam that interacts in a differential slice of a material is given by



$$\frac{I(x) - I(x+\Delta x)}{I(x)} = P(x)$$

$$\mu_t \equiv \lim_{\Delta x \to 0} \frac{P_i(\Delta x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{I(x) - I(x+\Delta x)}{\Delta x\, I(x)} = -\frac{1}{I(x)} \frac{dI(x)}{dx}$$

$$\frac{dI(x)}{dx} = -\mu_i I(x) \Rightarrow {\color{red} I(x) = I(0)\exp(-\mu_i x)}$$

Interaction Metrics

Interaction probability in distance x

$$P(x) = 1 - \frac{I(x)}{I(0)} = 1 - \exp(-\mu_i x)$$

Non-interaction probability in distance x

$$\bar{P}(x) = 1 - P(x) = \frac{I(x)}{I(0)} = \exp(-\mu_i x)$$

Average penetration distance until interaction, or mean-free-path length (assuming $\mu_i \neq \mu_i(x)$)

Prob. part gets to x

$$\bar{x} = \int_0^\infty x\, p(x) dx = \int_0^\infty x \left[\bar{P}(x) P(dx)\right] dx$$

Prob. part interacts in dx

$$= \int_0^\infty x \ \exp(-\mu_i x)\, \mu_i dx = \frac{\mu_i}{\mu_i^2} = \frac{1}{\mu_i}$$

Conceptual Interpretations

- The linear attenuation coefficient can be thought of in three ways:
  - Probability that a particle interacts in a differential length of material (does not assume constant $\boldsymbol{\mu_i}$)
  - Inverse of the mean free path of a particle (assumes constant $\boldsymbol{\mu_i}$).
  - Related to distance at which half of particles have interacted ($\boldsymbol{x_{1/2,\,i}} = \frac{\ln 2}{\mu_i}$) (assumes constant $\boldsymbol{\mu_i}$)
- Analogous to decay constants
  - Decay probabilities
  - Average lifetimes
  - Half lives.

Non-absorbing Particles

- In many cases (scattering, photons, etc.), interactions do not eliminate the particles
- The total amount of particles
  - Highly complex, calculated with large computations
  - Derive a buildup factor, B(x), that correlates complex behavior with simple expression

$$I(x) = B(x)I(0)\exp(-\mu_i x)$$

- This is especially common in calculating dose (as opposed to total particles).

Microscopic Cross Section

- Probability of interaction is proportional to the concentration of interaction sites/atoms

$$\mu_i = \sum\nolimits_i = N\sigma_i = \sigma_i \frac{\rho N_a}{A}$$

- σ_i = microscopic cross section, has units of L^2
- $\boldsymbol{N}$ = Number/atom density
- $\boldsymbol{\rho} =$ Mass density
- $\boldsymbol{N_a}$ = Avagadro's number
- $\boldsymbol{A}$ = Atomic mass of the medium

Example

- What is the power generation in a 1cm^3 section of U^{235} fuel, assuming a thermal neutron flux of 1×10^{22} neutrons/cm^2-s?

From book: $\sigma_f = 587\,b$ $N = \frac{\rho N_a}{M}$ $M = 235\ g/mol$

Appendix C $\rho = 19.1\ g/cm^3$ $N = 4.89 \times 10^{22}\ \frac{atoms}{cm^3}$

$E_r \cong 200\ MeV$

$\Sigma_f = \sigma \cdot N =$

$\text{Power} = \Sigma_f \cdot \phi \cdot E_r = \underline{9.206\ MW}$

Microscopic cross section

- The microscopic cross section
  - Independent of atomic density
  - Based strongly and complexly on particle kinetic energy
  - Play vital roles in nuclear engineering
- Behaviors are empirical!
  - (can be conceptually explained but not always quantitatively predicted by theoretical means)
- Typical unit is barns (1 barn = 1×10^{-24} cm^2)
- 1 barn is approximate physical cross section of a uranium nucleus.

Mass Interaction Coefficient

- Photons – mass interaction coefficient
  - Interaction coefficient (macroscopic) divided by density
  - which depends only weakly on the properties of the medium (for photons)

$$\frac{\mu_i}{\rho} = \frac{\sigma_i N}{\rho} = \frac{N_a}{A}\sigma_i$$

- Homogeneous mixture properties can be determined from

$$\mu_i = \sum_j \mu_{i,j} = \sum_j N_j \sigma_{i,j} \qquad \frac{\mu_i}{\rho} = \sum_j w_j \left(\frac{\mu_i}{\rho}\right)_j$$

Cross sections for each interaction

$$\sigma_t = \sigma_e + \sigma_i + \sigma_\gamma + \sigma_f + \dots$$

total cross section

$$\sigma_a = \sigma_\gamma + \sigma_f + \sigma_\alpha + \sigma_p + \dots$$

absorption cross section

$$\sigma_s = \sigma_e + \sigma_i$$

scattering cross section

$$\sigma_t = \sigma_s + \sigma_a$$

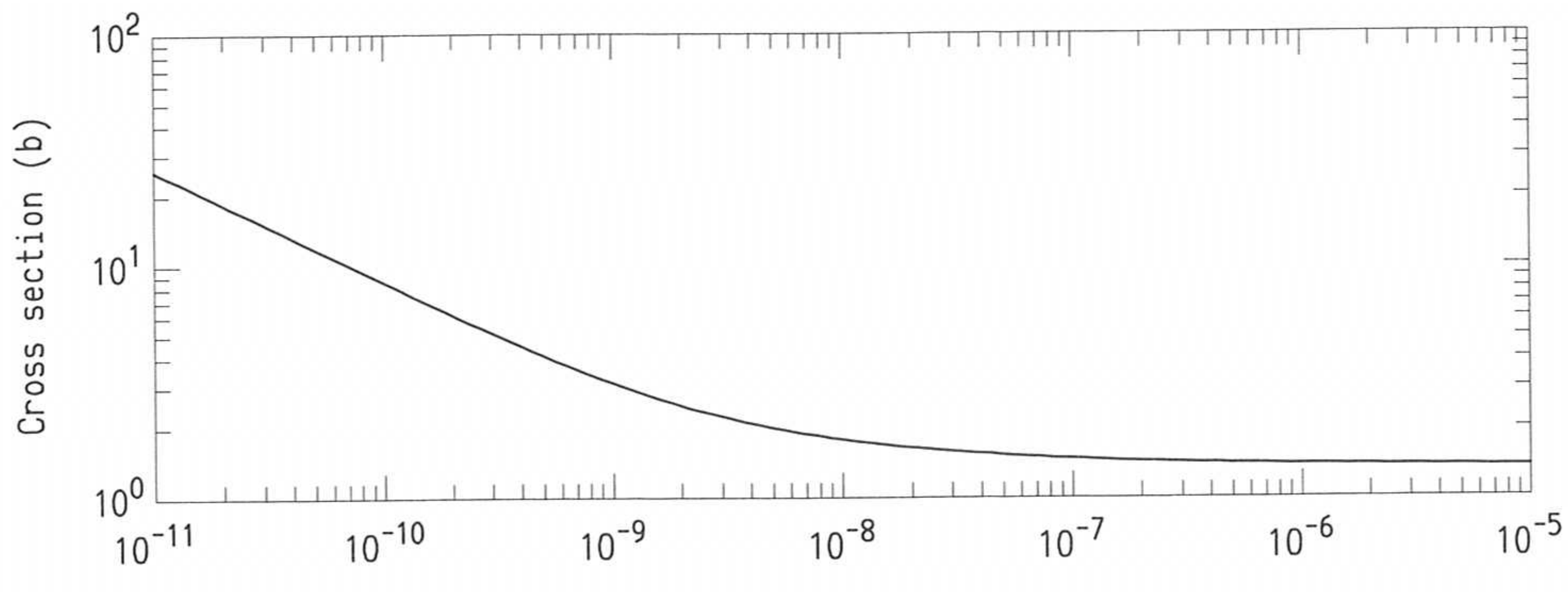
total cross section

t = total
e = elastic scattering
i = inelastic scattering
γ = radiative capture
f = fission
α= alpha (charged) particle
p = proton (charged) particle

Cross Section Trends

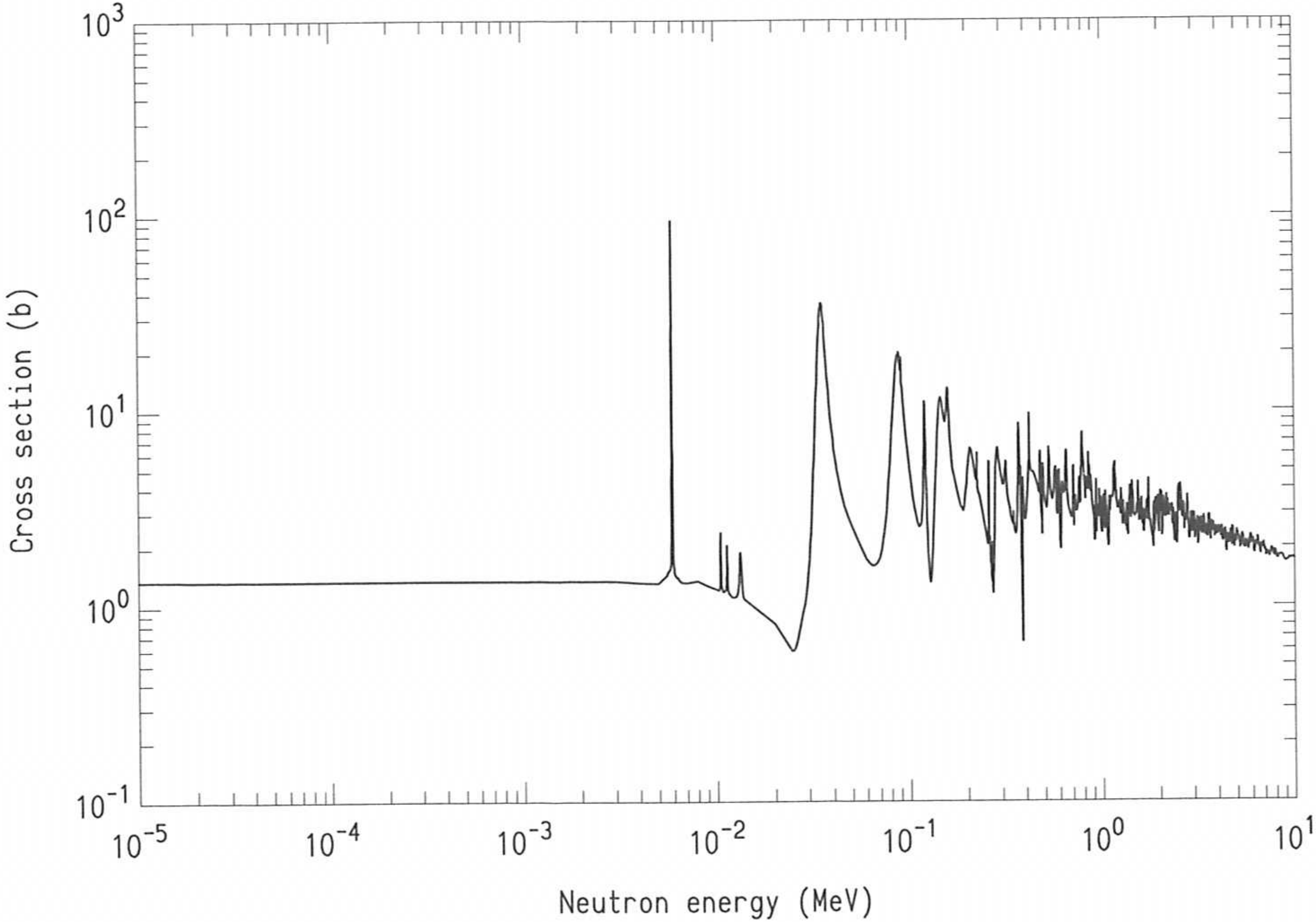
- Most Isotopes
  - Cross sections rise as neutron energy decreases.
  - Resonance regions with narrow and rapidly varying interactions that eventually are not resolvable
- Light isotopes (A < 25)
- Heavy isotopes (A > 150)
- Intermediate

Al Total Neutron Cross Section

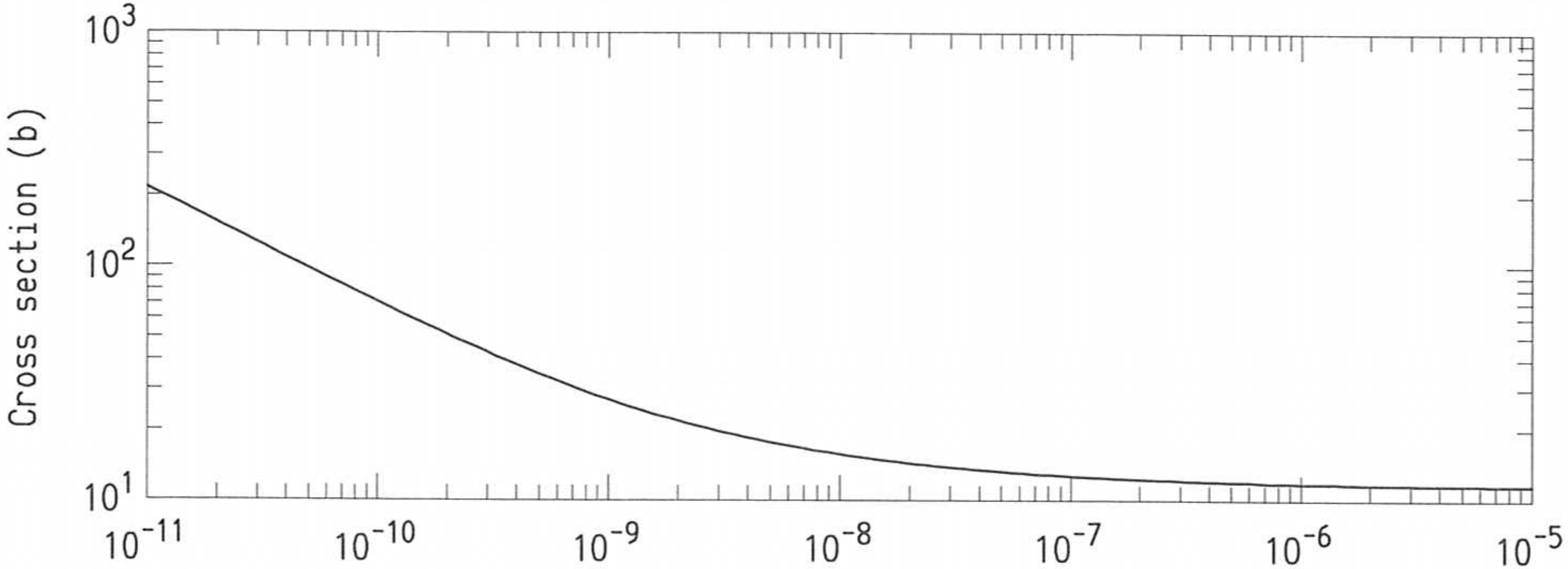


Neutron Energy (MeV)

Al Total Neutron Cross Section

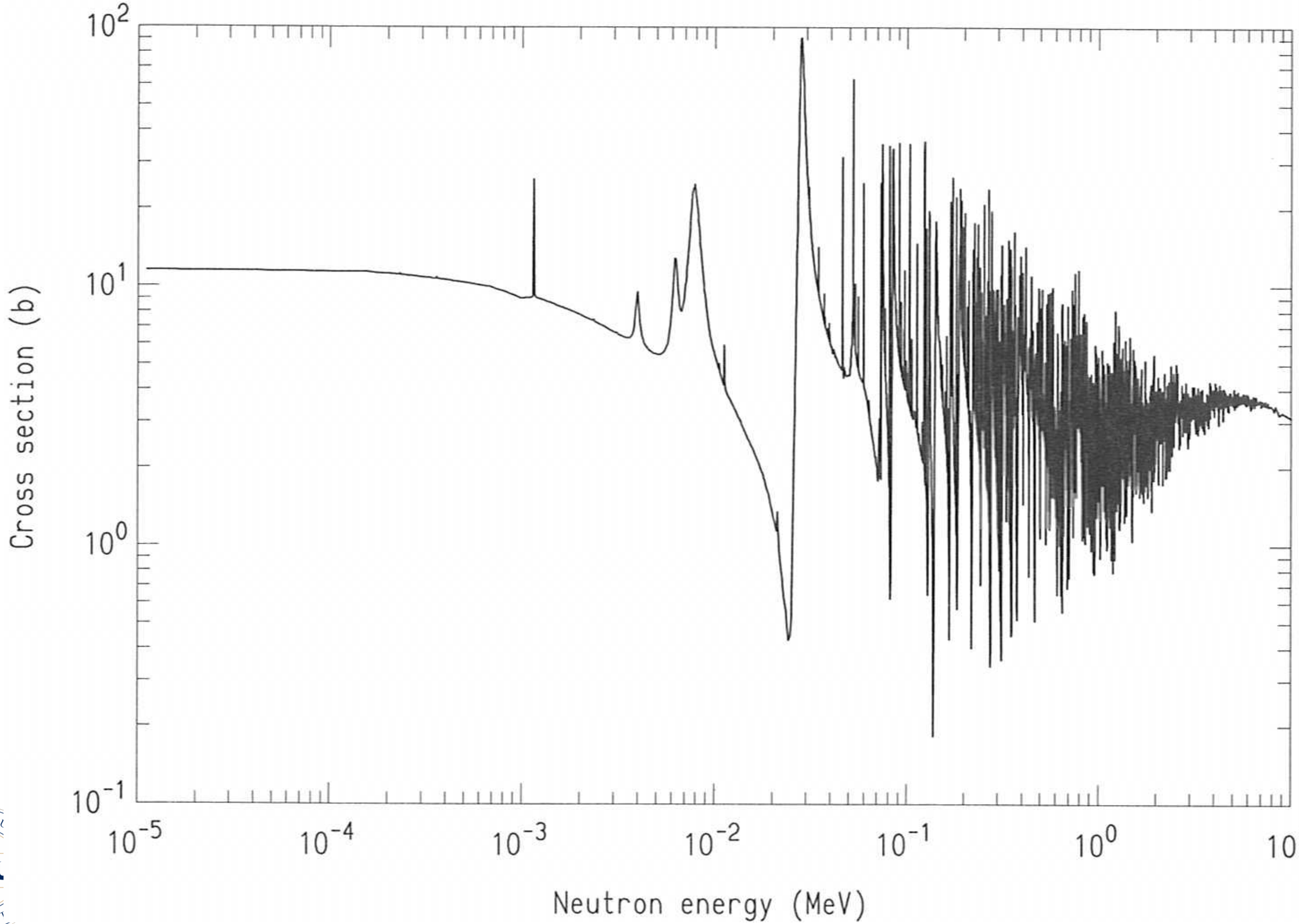


Fe total neutron cross section

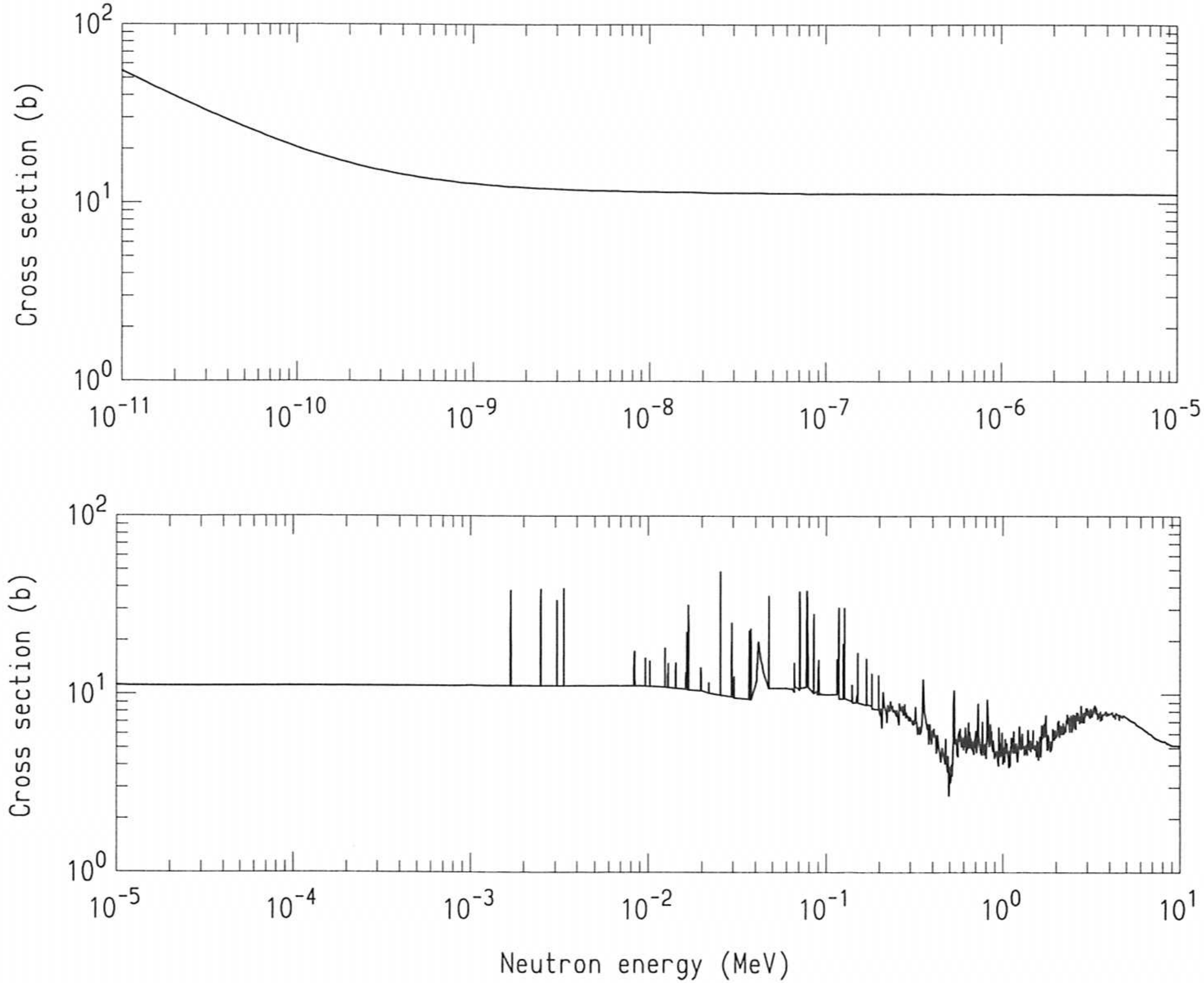


Neutron Energy (MeV)

Fe total neutron cross section

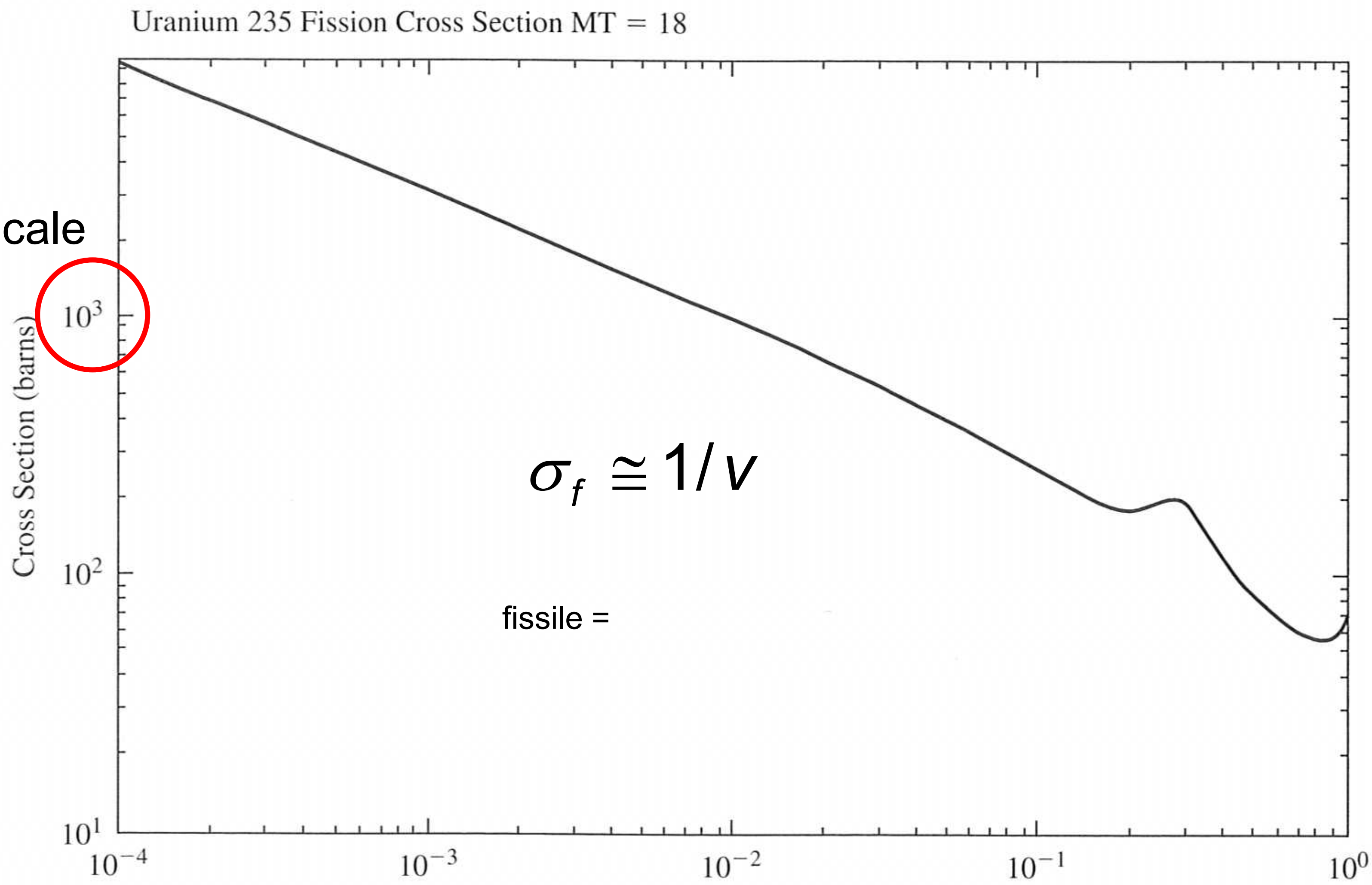


Lead Total Neutron Cross Section



Fission Cross Sections ^{235}U

note scale



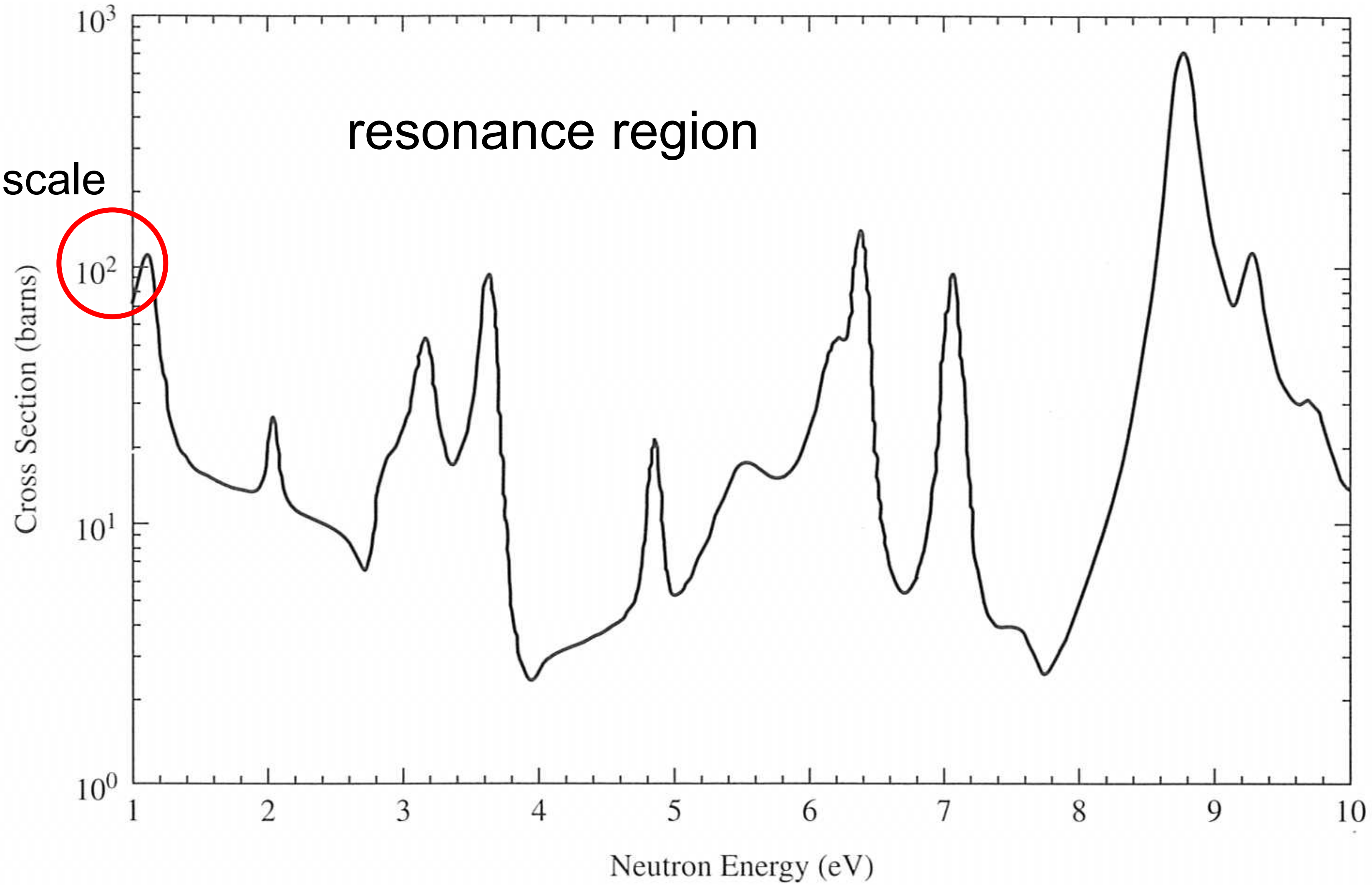
$$\sigma_f \cong 1/v$$

fissile =

Fission Cross Sections ^{235}U

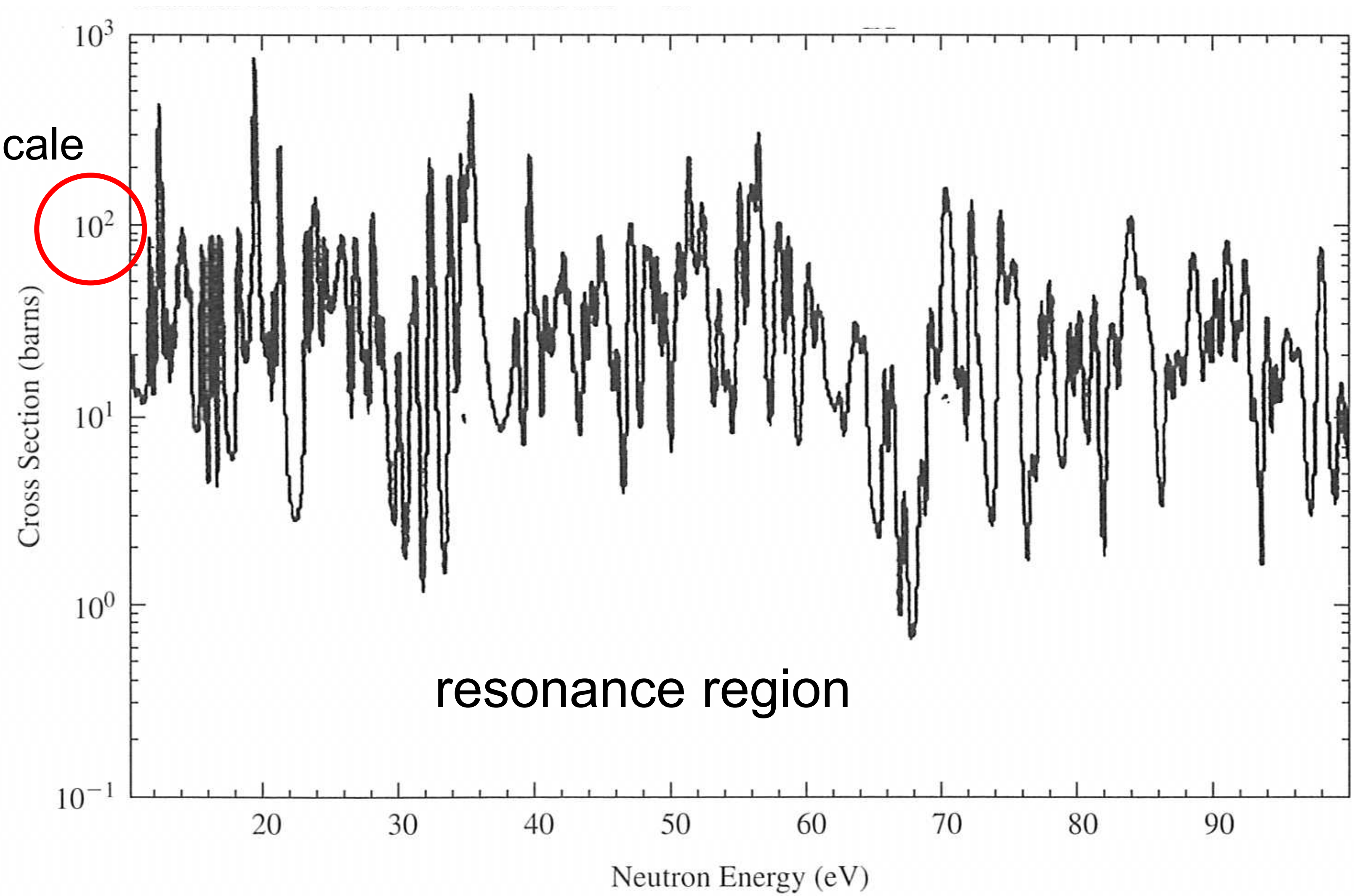
note scale

resonance region

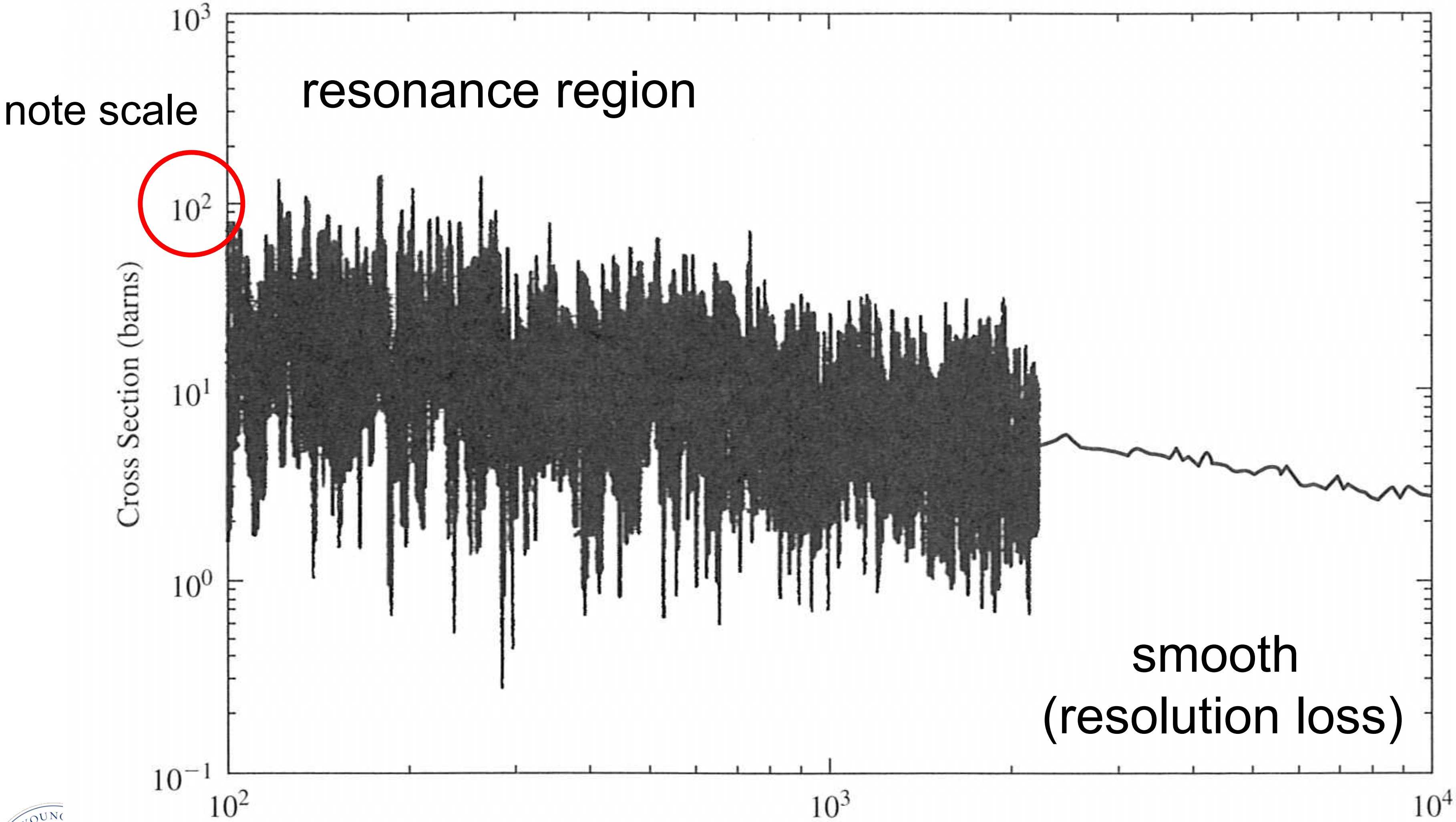


Fission Cross Sections ^{235}U

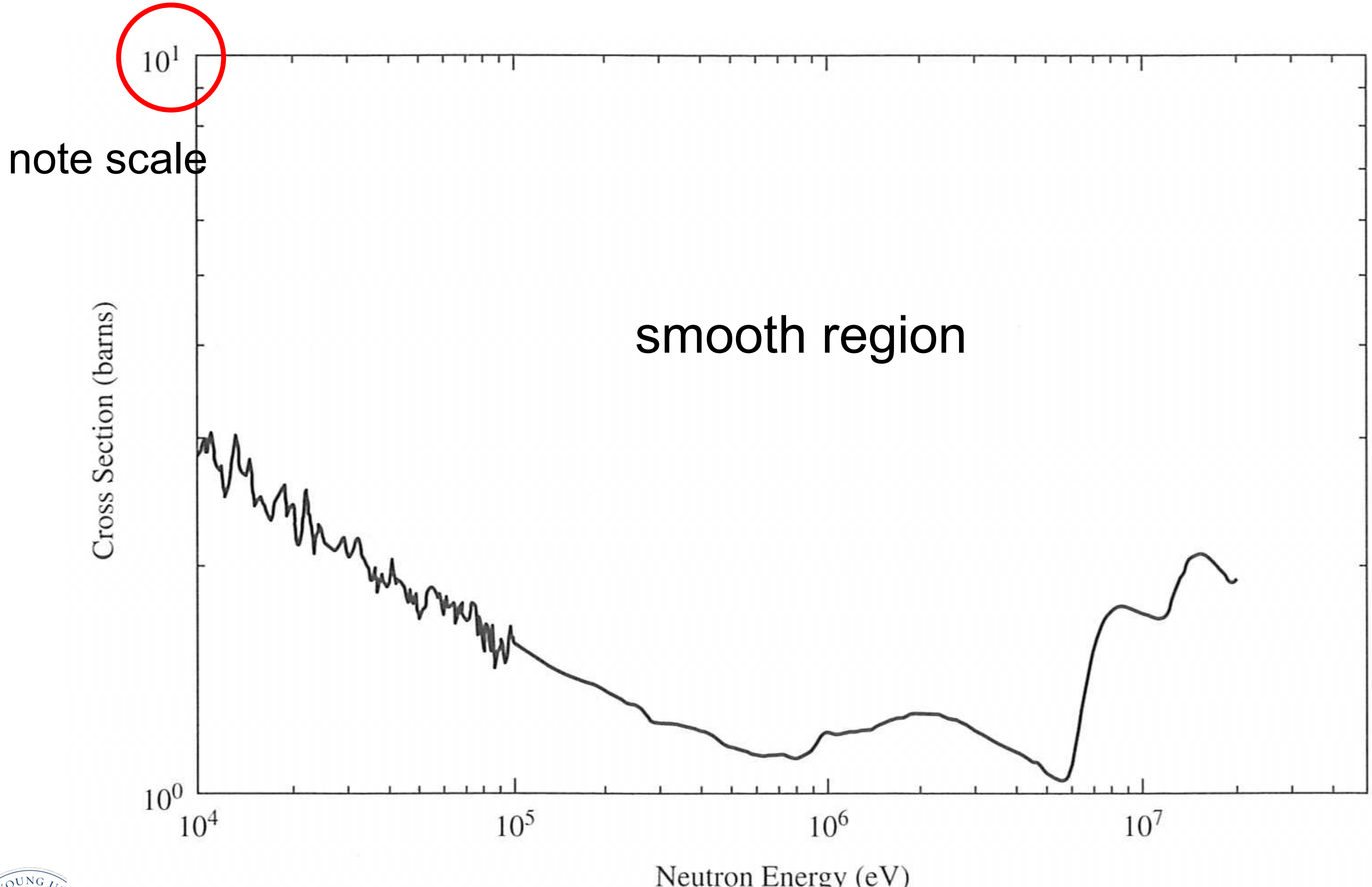
note scale



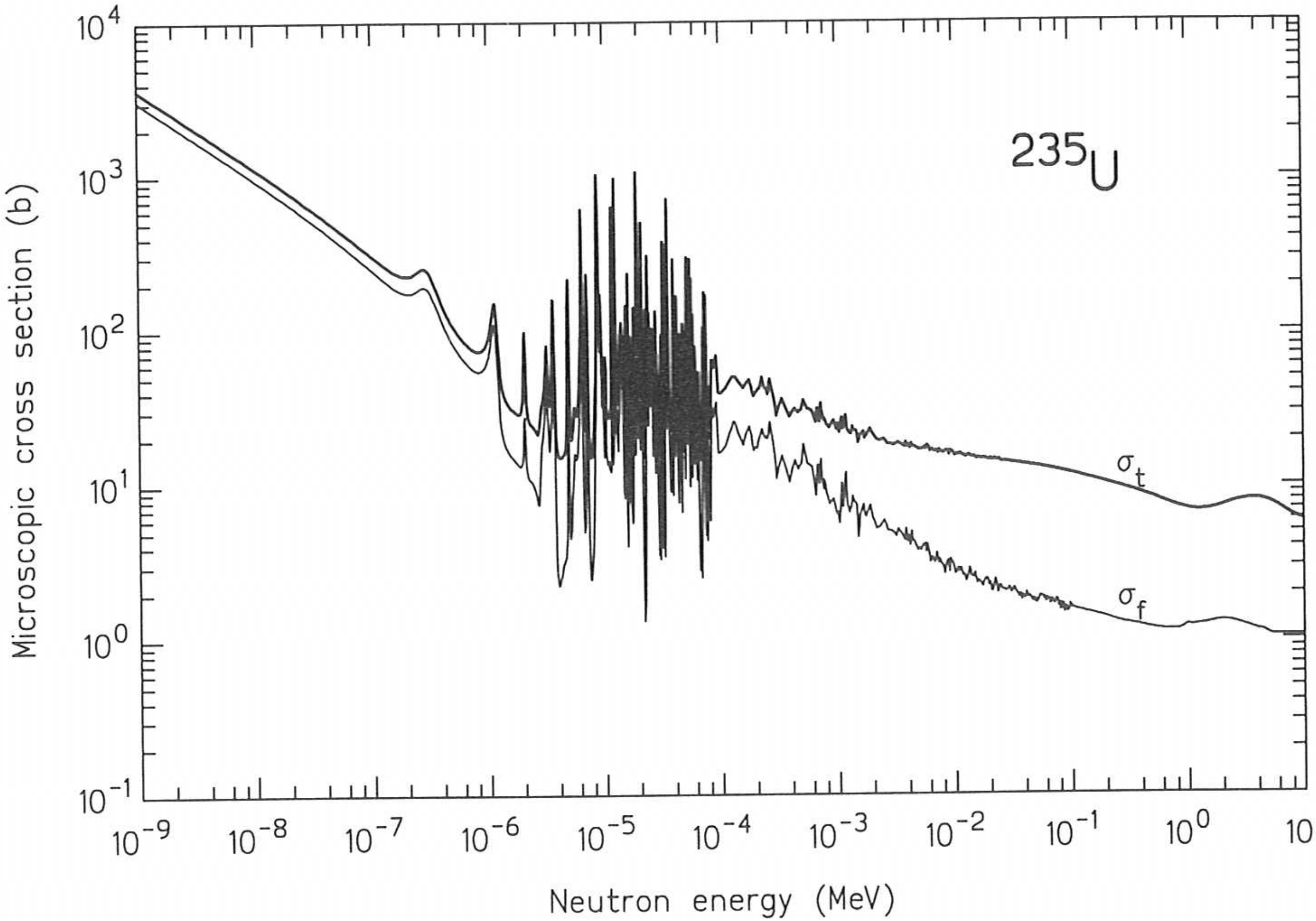
Fission Cross Sections ^{235}U



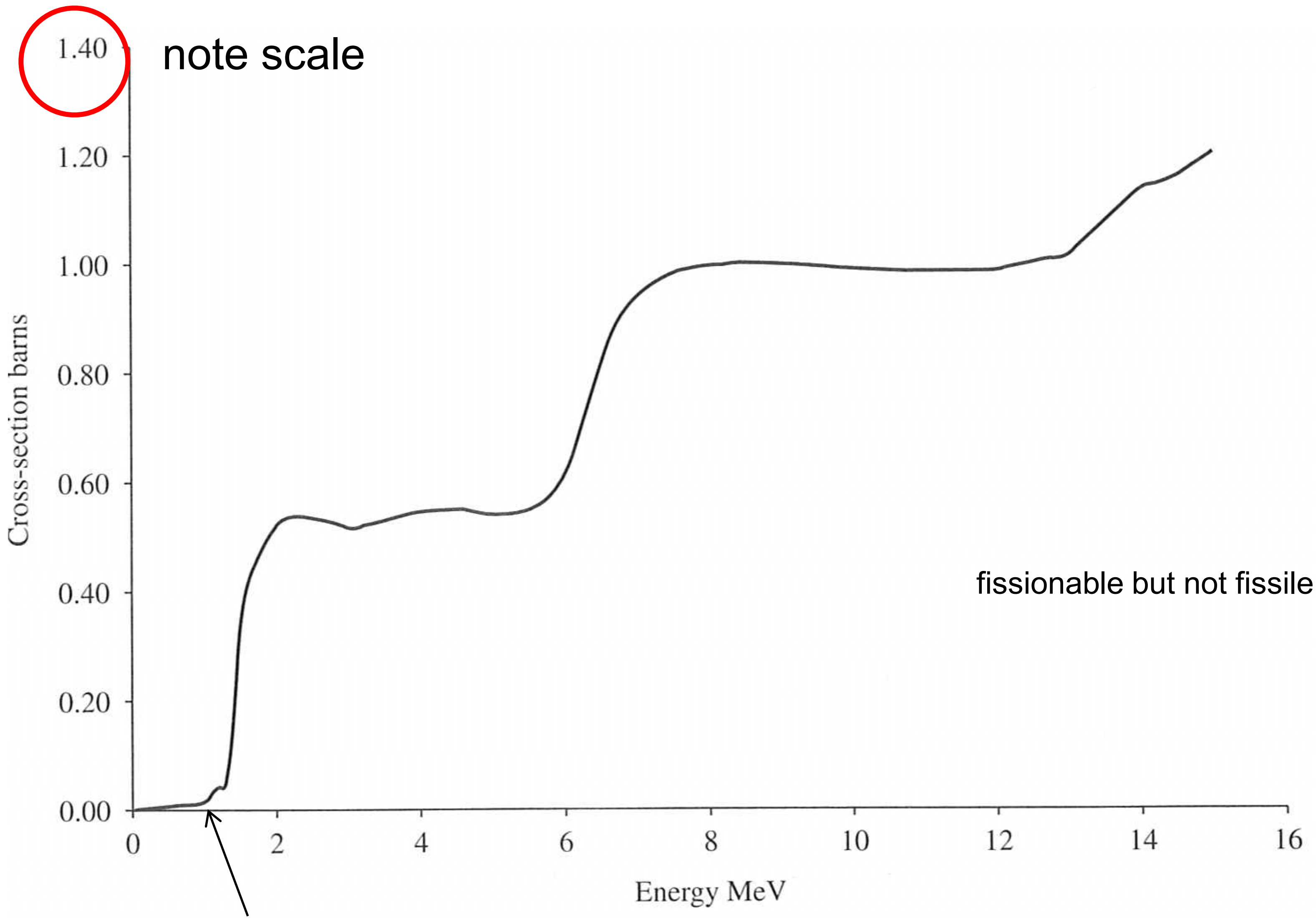
Fission Cross Sections ^{235}U



Cross section over entire range



Fission Cross Section of ^{238}U



threshold energy (> resonance region energy)

Fissionable Cross Sections