Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 18

Nuclear Reactor Theory III

Neutron Transport

Spiritual Thought

"The healing power of the Lord Jesus Christ—whether it removes our burdens or strengthens us to endure and live with them like the Apostle Paul—is available for every affliction in mortality"

- Elder Dallin H. Oaks

Homework 17 (due 3/26)

Homework 17 Text

Homework #18

Web Problem #8

Back to the Future

GROUP WORK OKAY, Due 3/26/24 at beginning of class

(Don't be afraid to "Google" good assumptions!)

Back to the Future

The flux capacitor is the single greatest invention of our time, but the energy requirements are incredible! 1.21 GW? Seriously?!? Luckily Doc brown had a few jars of plutonium (water shielded of course). Assuming that the water/plutonium ratio in the jar is the same ratio as in the reactor, determine whether there is sufficient plutonium (assume it's pure ^{239}Pu) for the reactor to be critical. Also, determine the power of such a critical reactor, assuming a max thermal flux of 2^{16} neut/cm^2/s.

Reactor Power

Me: Epsilon and p are just one, right?

Heterogeneous Reactor:

General Reactor Equation

Mono-energetic neutrons

$$D\nabla^2\phi - \Sigma_a\phi + S = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

For a reactor, $S = \nu\Sigma_f\phi$

$$D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

$$\nabla^2\phi + B^2\phi = -\frac{1}{Dv}\frac{\partial\phi}{\partial t}$$

Bare Slab Reactor Solution

$$\frac{d^2\phi}{dx^2} = -B^2\phi$$ reactor equation

$$\phi\left(\frac{\tilde{a}}{2}\right) = \phi\left(-\frac{\tilde{a}}{2}\right) = \phi'(0) = 0$$ boundary conditions

$$\phi(x) = A\cos(Bx) + C\sin(Bx)$$ general solution

$$C = 0$$ from symmetry or by substitution

$$\phi(x) = A\cos(Bx)$$

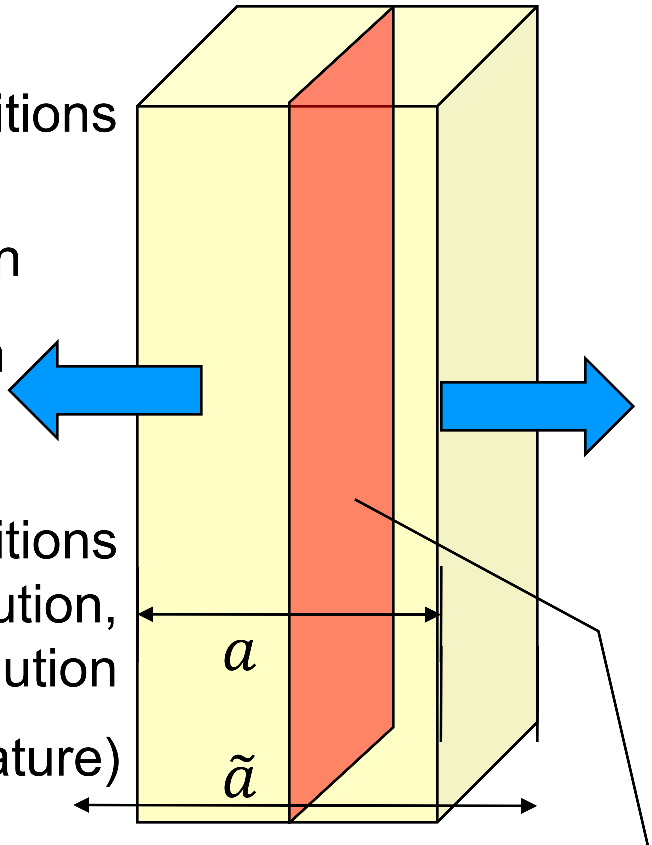
$$\phi(\tilde{a}/2) = 0$$

$$\Rightarrow B_n = \frac{n\pi}{\tilde{a}}$$

Eigenvalues from boundary conditions – all n important in transient solution, only n=1 important for steady solution

$$B_1^2 = -\frac{1}{\phi}\frac{d^2\phi}{dx^2}$$

B_1^2 is buckling (prop. to flux curvature)



The constant A is as yet undetermined and relates to the power. There are different solutions to this problem for every power level.

Infinite plane indicates no net flux from sides

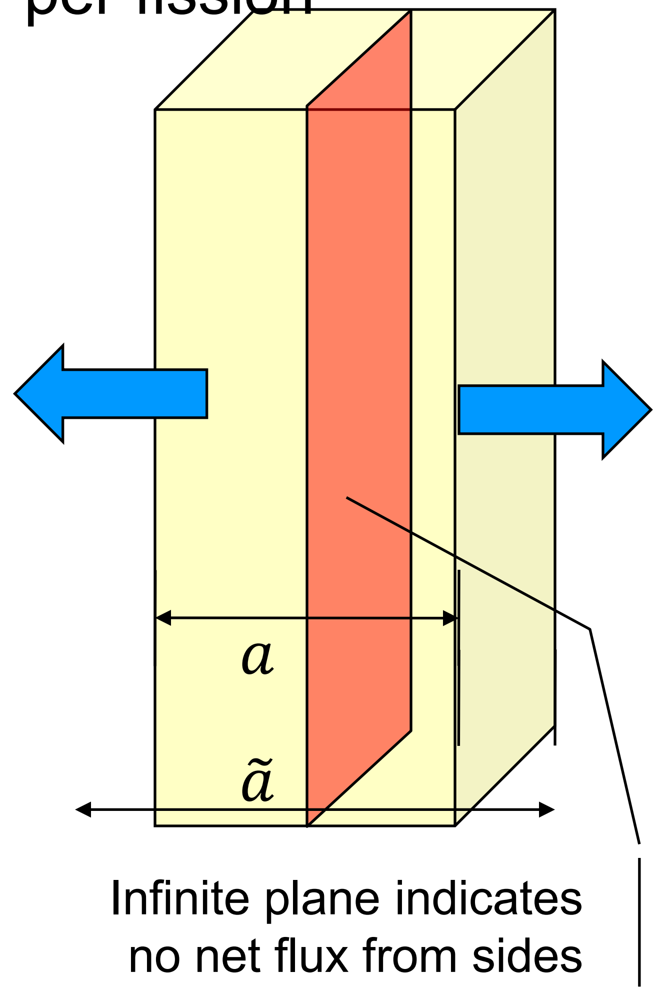
Bare Slab Reactor Power

$$P = E_R \Sigma_f \int_{-a/2}^{a/2} \phi(x) dx$$

E_R is the recoverable energy per fission

$$P = \frac{2A\tilde{a}E_R\Sigma_f}{\pi} \sin\left(\frac{\pi a}{2\tilde{a}}\right)$$

$$\phi(x) = \frac{\pi P}{2\tilde{a}E_R\Sigma_f \sin\left(\frac{\pi a}{2\tilde{a}}\right)} \cos\left(\frac{\pi x}{\tilde{a}}\right)$$



Power Scales with flux !

Spherical Reactor

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = -B^2\phi$$ reactor transport equation

$$\phi(\tilde{R}) = \phi'(0) = 0$$ boundary conditions

$$\phi(r) = A\frac{\sin(Br)}{r} + C\frac{\cos(Br)}{r}$$ general solution

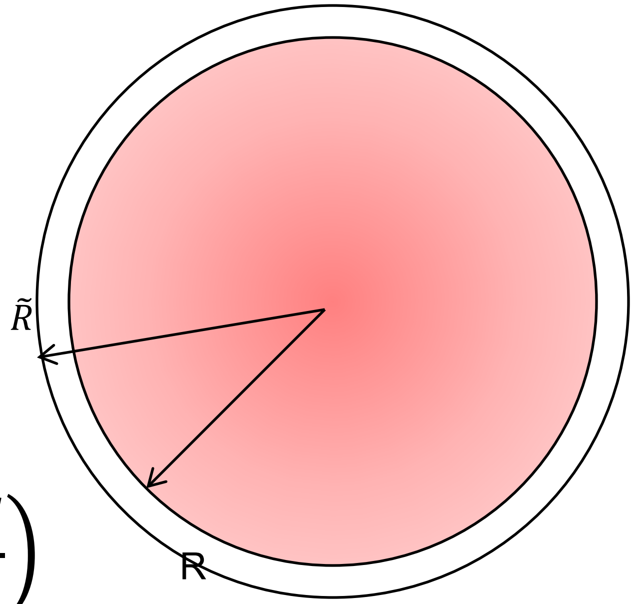
$$C = 0$$ from symmetry or by substitution

$$\phi(r) = A\frac{\sin(Br)}{r}$$ specific solution

$$B_n = \frac{n\pi}{\tilde{R}}$$ Eigen values

$$B_1^2 = \left(\frac{\pi}{\tilde{R}}\right)^2$$ buckling

$$\phi(r) = A\frac{\sin\left(\frac{\pi r}{\tilde{R}}\right)}{r}$$

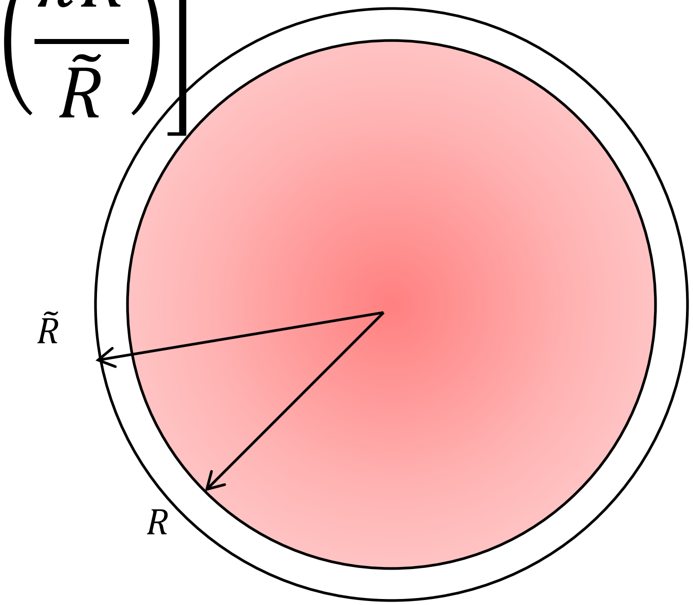


Spherical Reactor Power

Integrate over 2 symmetric dimensions – transform volume integral to radial integral

$$P = E_R \Sigma_f \iiint \phi(r)\,dV = 4\pi E_R \Sigma_f \int_0^R r^2 \phi(r)\,dr$$

$$P = 4\pi E_R \Sigma_f A \frac{\tilde{R}}{\pi}\left[\frac{\tilde{R}}{\pi}\sin\left(\frac{\pi R}{\tilde{R}}\right) - R\cos\left(\frac{\pi R}{\tilde{R}}\right)\right]$$



again, power is proportional to flux and highest at center

$$\phi(r) = \frac{P\sin\left(\frac{\pi r}{\tilde{R}}\right)}{4\,E_R \Sigma_f R^2 r}$$

Infinite Cylindrical Reactor

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right) = -B^2\phi = \frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr}$$

reactor transport equation

$$\phi(\tilde{R}) = \phi'(0) = 0; |\phi(r)| < \infty$$

boundary conditions

$$\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} + \left(B^2 - \frac{m^2}{r^2}\right)\phi = 0$$

zero-order (m=0) Bessel equation

$$\phi(r) = AJ_0(Br) + CY_0(Br)$$

general solution involves Bessel functions of first and second kind

$$\phi(r) = AJ_0(Br)$$

flux is finite

$$B_n = \frac{x_n}{\tilde{R}}$$

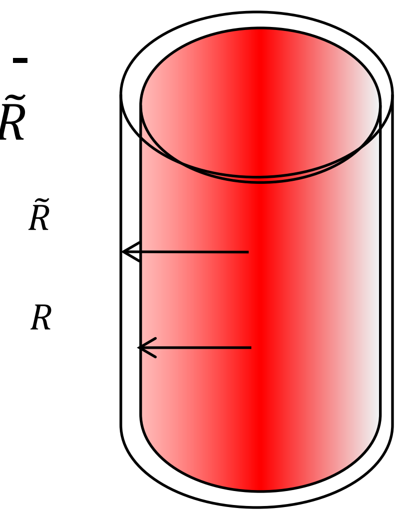
roots of Bessel functions - ϕ is zero at boundary $\tilde{R}$

$$B_1^2 = \left(\frac{x_1}{\tilde{R}}\right)^2 = \left(\frac{2.405}{\tilde{R}}\right)^2$$

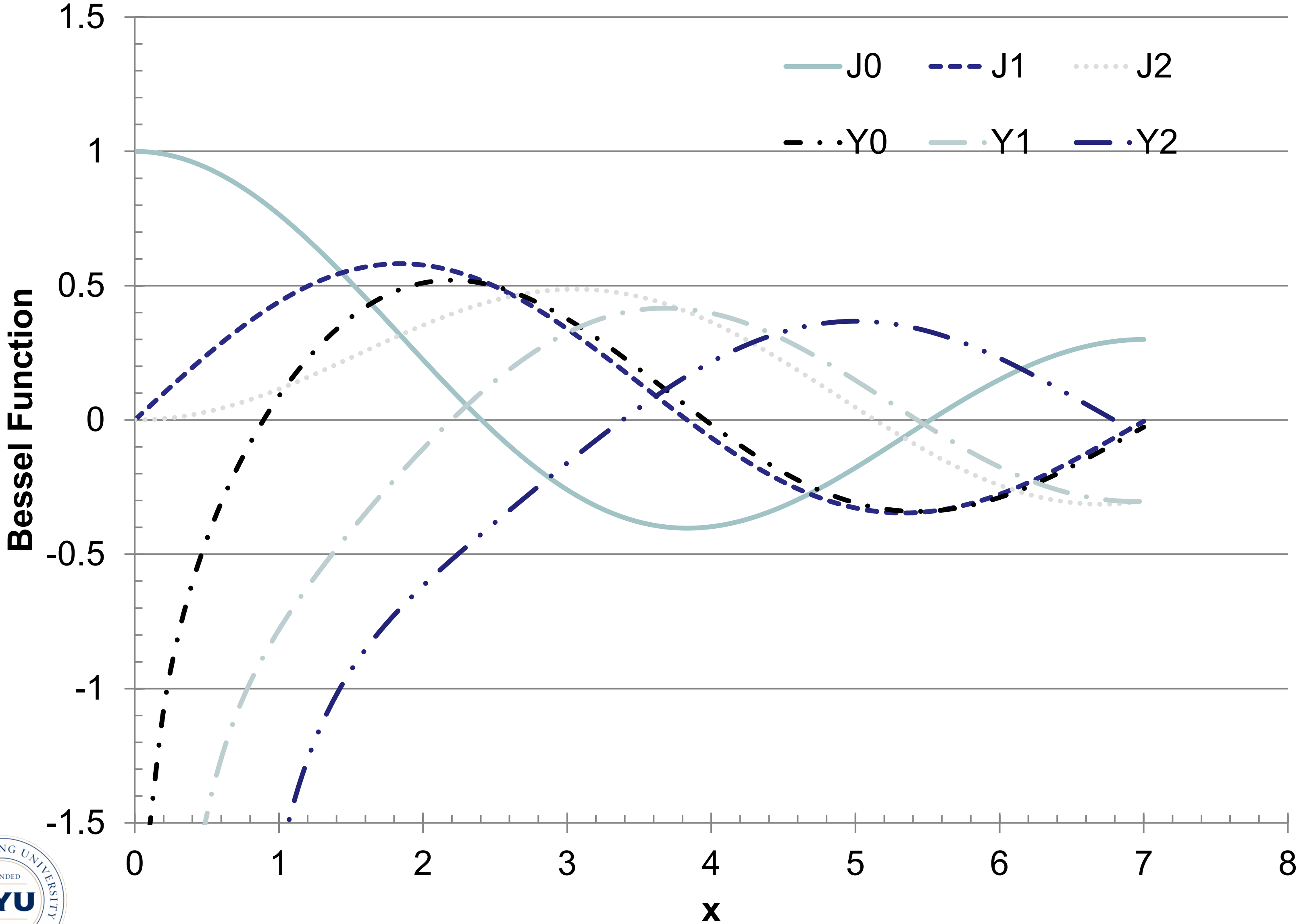
first root

$$\phi(r) = AJ_0\left(\frac{2.405\,r}{\tilde{R}}\right)$$

solution (power production determines A)



Bessel Functions



Infinite Cylindrical Reactor Power

$$P = E_R \Sigma_f \iiint \phi(r) dV = 2\pi E_R \Sigma_f \int_0^R r\phi(r) dr$$

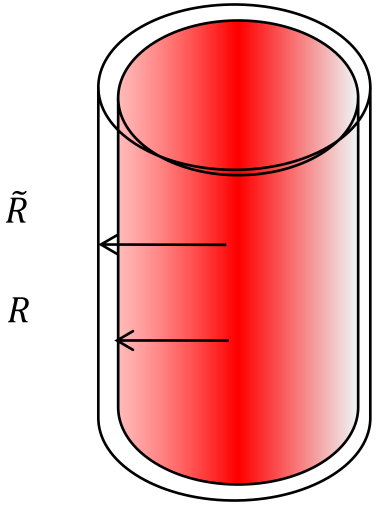
transform volume integral to radial integral – becomes power per unit length

$$P = 2\pi E_R \Sigma_f \int_0^R r J_0\left(\frac{2.405r}{R}\right) dr$$

$$\int_0^R x' J_0(x') dx' = x J_1(x)$$

$$P = \frac{2\pi E_R \Sigma_f R^2 A J_1(2.405)}{2.405}$$

$$\phi(r) = \frac{0.738P}{E_R \Sigma_f R^2} J_0\left(\frac{2.405r}{R}\right)$$



again, power is proportional to power and highest at center

Finite Cylindrical Reactor

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial \phi}{\partial r}\right)+\frac{\partial^2 \phi}{\partial z^2}=-B^2\phi$$

reactor transport equation

$$\phi\left(\tilde{R},z\right)=\phi'(0,z)=\phi\left(r,\frac{\widetilde{H}}{2}\right)=\phi\left(r,-\frac{\widetilde{H}}{2}\right)=0$$

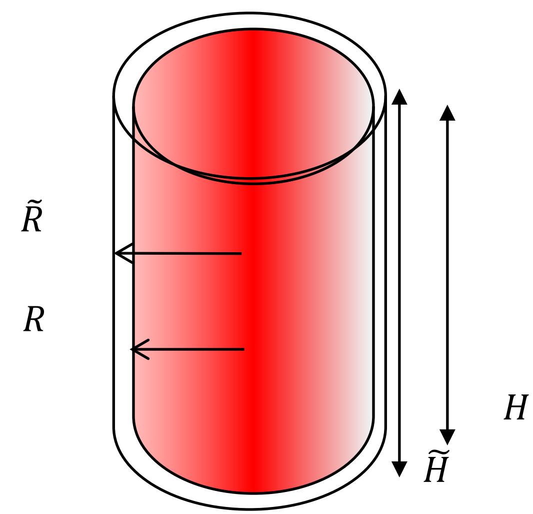
boundary conditions

$$\phi\left(r,z\right)=R(r)Z(z)$$

separation of variables

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial \phi}{\partial r}\right)+\frac{\partial^2 \phi}{\partial z^2}=\frac{Z}{r}\frac{\partial}{\partial r}\left(\frac{\partial R}{\partial r}\right)+R\,\frac{\partial^2 Z}{\partial z^2}R=-B^2R(r)Z(z)$$

$$\frac{1}{R}\frac{\partial}{\partial r}\left(\frac{\partial R}{\partial r}\right)+\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}R=-B^2$$



since R and Z vary independently, both portions of the equation must equal (generally different) constants, designated as B_R and B_Z, respectively

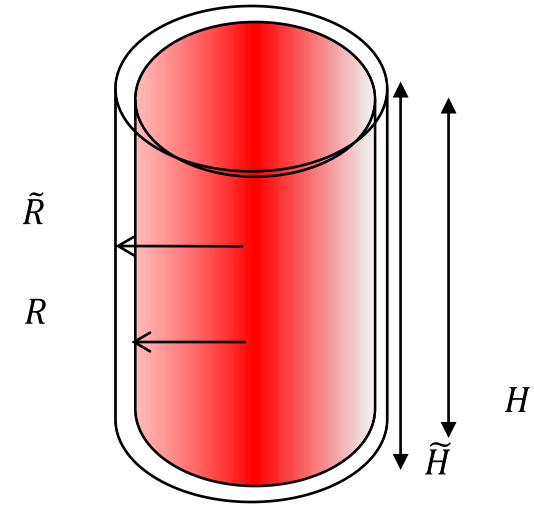
Finite Cylinder Solution

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) = -B_R^2 R$$ a problem we already solved, w/ same bcs

$$\Rightarrow R(r) = A\,J_0\left(\frac{2.405}{\tilde{R}}r\right)$$

$$\frac{\partial^2 Z}{\partial z^2} = -B_Z^2 Z$$ again a problem we already solved, w/ same bcs

$$\Rightarrow Z(z) = A\cos\frac{\pi z}{\tilde{H}}$$



$$\phi(r,z) = A\,J_0\left(\frac{2.405}{\tilde{R}}r\right)\cos\frac{\pi z}{\tilde{H}}$$

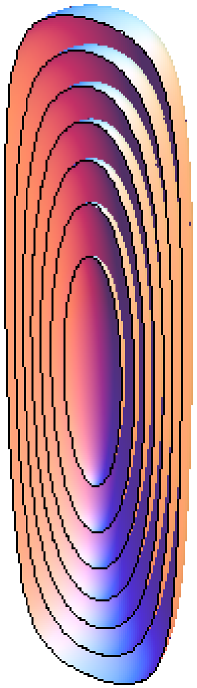
$$B^2 = B_R^2 + B_H^2$$

solution is the product of the infinite cylinder and infinite slab solutions

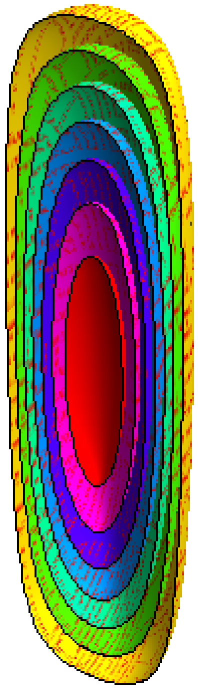
Buckling is higher than for either the infinite plane or the infinite cylinder. Buckling generally increases with increasing leakage, and there are more surfaces to leak here than either of the infinite cases.

Neutron Flux Contours

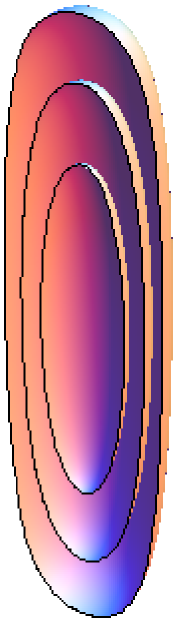
Neutron flux in finite cylindrical reactor



3D contours of neutron flux at high power



3D contours w color scaled to magnitude – intermediate power



3D contours of neutron flux at low power

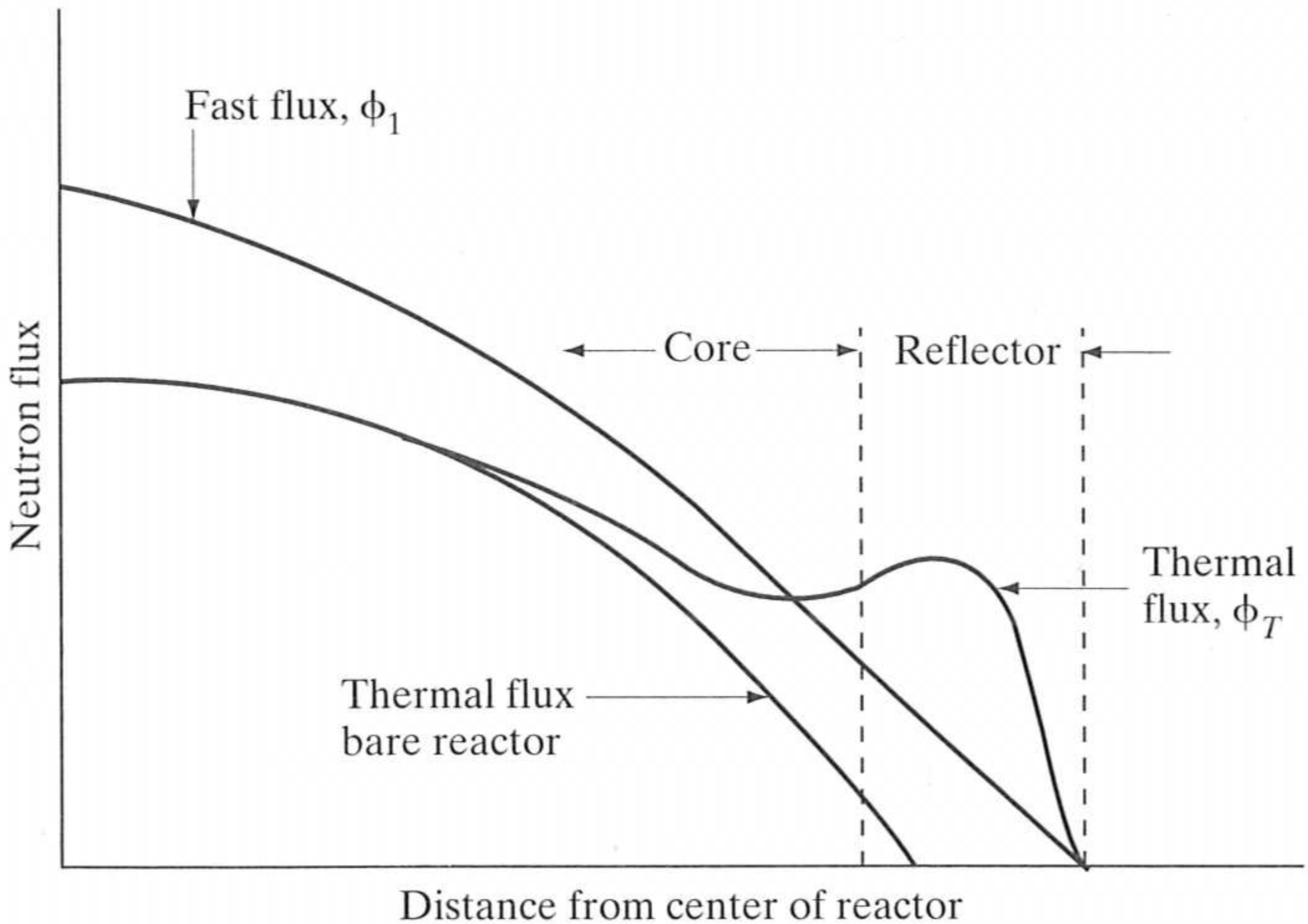
Bare Reactor Summary

geometry	Buckling (B^2)	Flux	A	$\Omega = \dfrac{\phi_{max}}{\phi_{av}}$
$plate - 1D$	$\left(\frac{\pi}{a}\right)^2$	$A\cos\frac{\pi x}{a}$	$1.57P/aE_R\Sigma_f$	1.57
$plate - 3D$	$\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$	$A\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}\cos\frac{\pi z}{c}$	$3.85P/VE_R\Sigma_f$	3.88
$cylinder - 1D$	$\left(\frac{2.405}{R}\right)^2$	$A J_0\left(\frac{2.405}{R}\right)$	$0.738P/R^2E_R\Sigma_f$	2.32
$cylinder - 3D$	$\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$	$A J_0\left(\frac{2.405}{R}\right)\cos\frac{\pi z}{H}$	$3.63P/VE_R\Sigma_f$	3.64
$sphere$	$\left(\frac{\pi}{R}\right)^2$	$\frac{A}{r}\sin\frac{\pi r}{R}$	$P/4R^2E_R\Sigma_f$	3.29

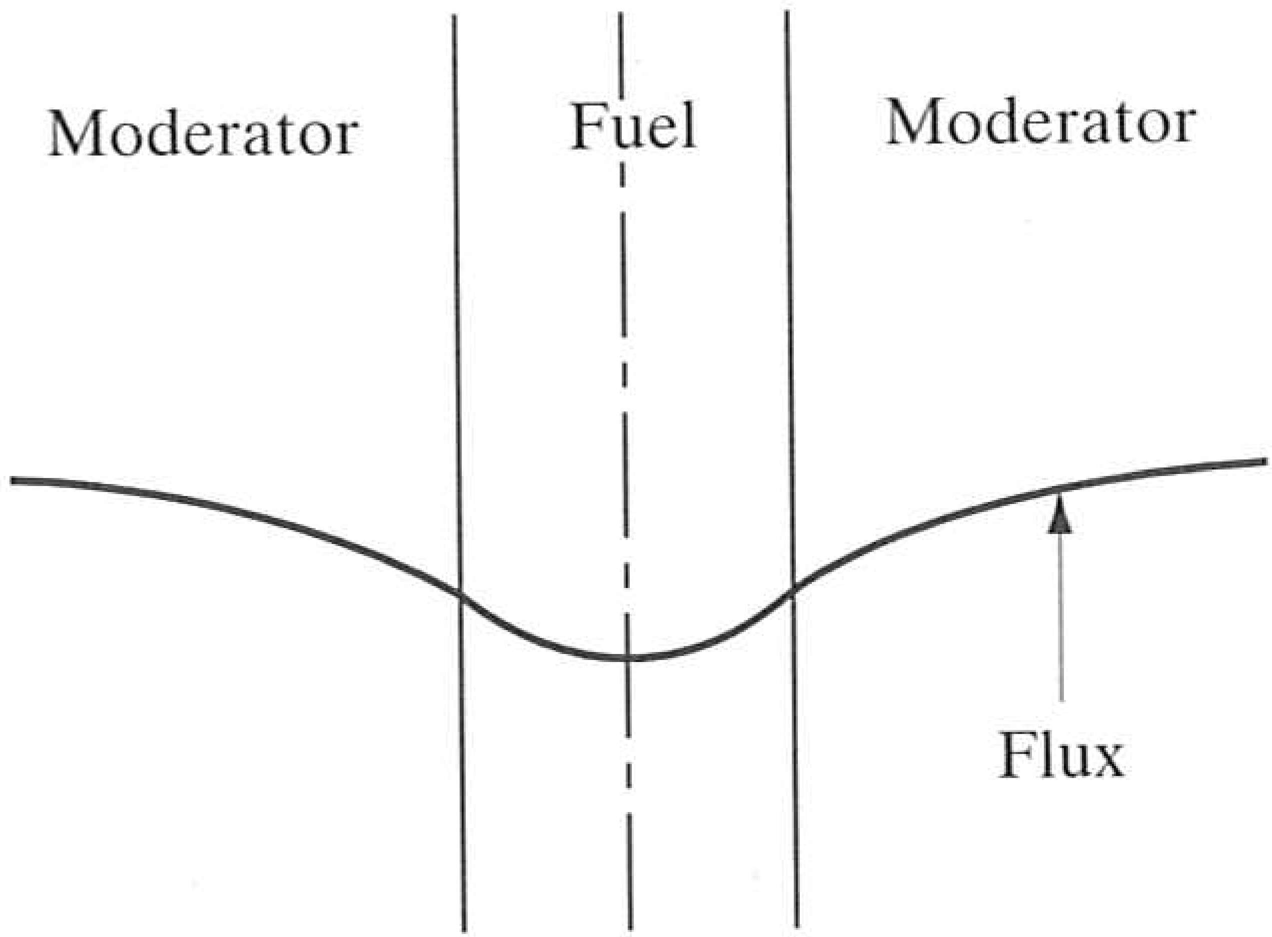
Some details

- Reflected reactors lend themselves less easily to analytical solution – commonly reactors are considered as sphere equivalents rather than trying to solve the equations.
- Reasonable representation for fast neutrons – not for thermal reactors
- Reflector savings in size is typically about the thickness of the extrapolated distance.

Flux Comparisons



Thermal Flux Variations



General Reactor Equation

Mono-energetic neutrons

$$D\nabla^2\phi - \Sigma_a\phi + S = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

For a reactor, $S = \nu\Sigma_f\phi$

$$D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

$$\nabla^2\phi - B^2\phi = -\frac{1}{Dv}\frac{\partial\phi}{\partial t}$$

Multiple Energy Groups

Multi-energetic neutrons?

$$D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

- Source/sink terms change
- Source = fission + scattering from other groups
- Sink = absorption + scattering to other groups
- Fission neutrons only sourcd in top groups
- Fission only in thermal groups