

# Chemical Engineering 412

## *Introductory Nuclear Engineering*

### Lecture 19

### Nuclear Reactor Theory IV

### Nuclear Kinetics



# Spiritual Thought

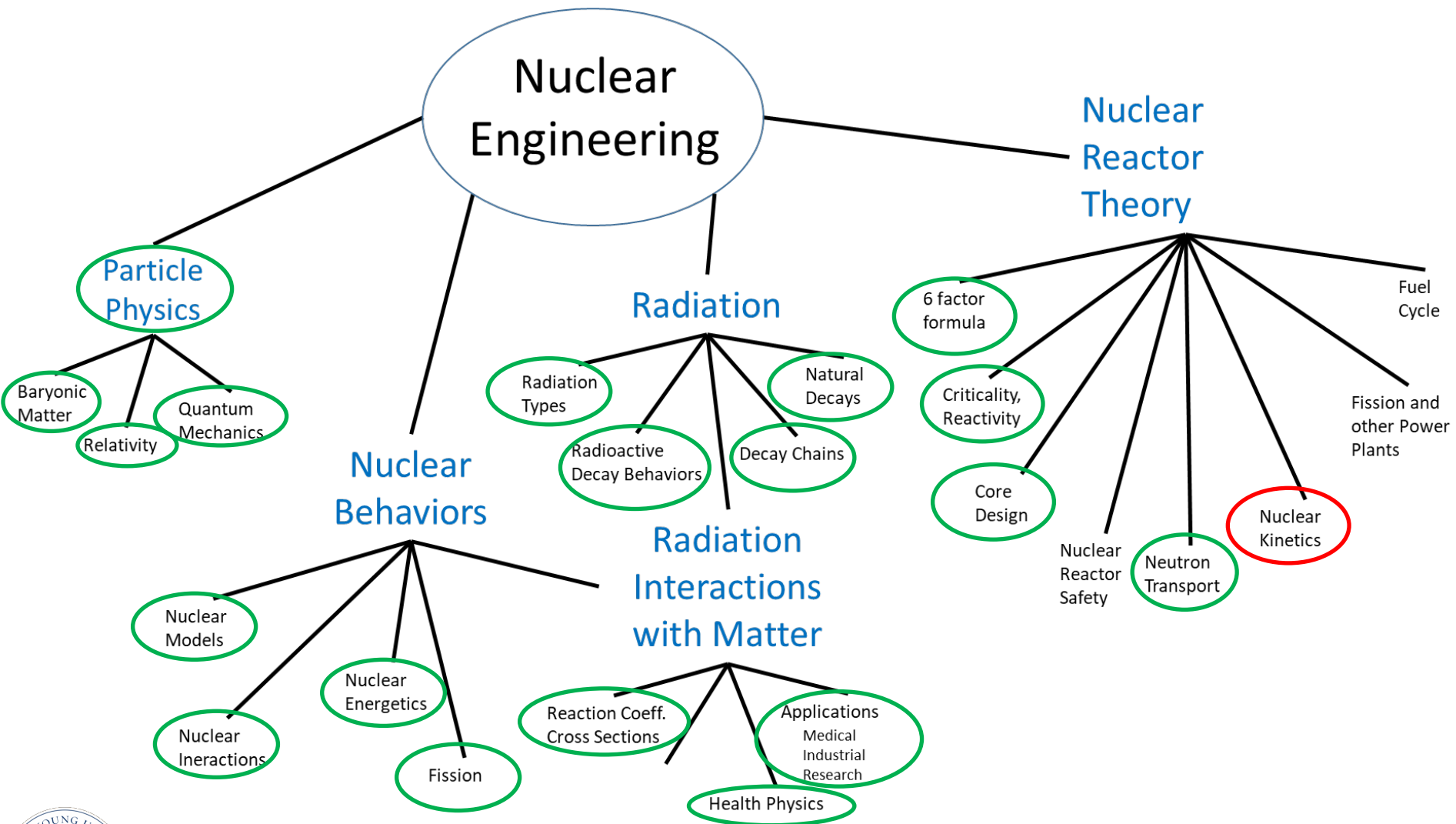
“When we put God first, all other things fall into their proper place or drop out of our lives. Our love of the Lord will govern the claims for our affection, the demands on our time, the interests we pursue, and the order of our priorities.

We should put God ahead of *everyone else* in our lives.”

President Ezra Taft Benson



# Roadmap



# Objectives

- Understand the simple, “big picture” of kinetics
- Understand how delayed neutrons/precursors work
- Understand, recognize and do calculations relating to feedback coefficients
- Understand reactor “control strategies”
- Understand xenon peak phenomena



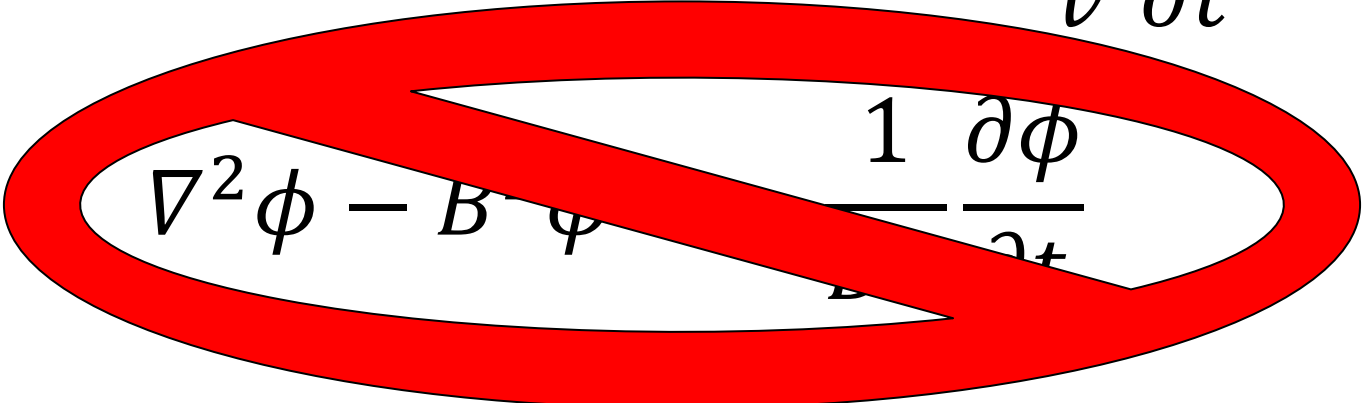
# General Transient Problem

## Mono-energetic neutrons

$$D\nabla^2\phi - \Sigma_a\phi + S = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

For a reactor,  $S = \nu\Sigma_f\phi$

$$D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$


$$\nabla^2\phi - B^2\phi = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

# Prompt Neutrons

- Lifetime,  $l_p$ , is time between emission and absorption.
- Neutrons in thermal reactors:
  - Spend more time (most of  $l_p$ ) in the thermal regime
  - Travel further as fast neutrons
- Average lifetime of a thermal neutron in an infinite reactor is the mean diffusion time,  $t_d$ , and is approximately the same as  $l_n$  in an infinite reactor.  $l_p \approx t_d$

$$t_d = t_{dM}(1 - f)$$

TABLE 7.1 APPROXIMATE DIFFUSION TIMES FOR SEVERAL MODERATORS

| Moderator         | $t_d$ , sec          |
|-------------------|----------------------|
| H <sub>2</sub> O  | $2.1 \times 10^{-4}$ |
| D <sub>2</sub> O* | $4.3 \times 10^{-2}$ |
| Be                | $3.9 \times 10^{-3}$ |
| Graphite          | 0.017                |

\*With 0.25% H<sub>2</sub>O impurity.

In fast reactors, prompt neutron lifetimes are much shorter, on the order of  $10^{-7}$  seconds



# Simple Kinetics Model

$$\Delta n(t) \equiv \ell_p \frac{dn(t)}{dt} = (k_{eff} - 1)n(t)$$

$$\frac{dn(t)}{dt} = \frac{k_{eff} - 1}{\ell_p} n(t)$$

$$\Rightarrow n(t) = n(0) \exp\left(\frac{k_{eff} - 1}{\ell_p} t\right)$$

- For  $^{235}\text{U}$ 
  - $\ell_p = 2.1 \times 10^{-4} \text{ s}$
  - $k_{eff} - 1 = 0.001$
  - and  $t = 1 \text{ s}$ ,
- $n/n^0 = 117$  (22,027 if  $\ell' = 10^{-4}$  as in text)
- Far too rapid to control!!!



# Reactors with delayed neutrons

Delayed neutrons:  $\beta$  is fraction of total neutrons

$$\bar{\ell}_p = (1 - \beta)\ell_p + \beta(\ell_p + \tau) \approx \ell_p + \beta\tau$$

$\tau$  is lifetime of delayed neutrons  $= \frac{T_{1/2}}{\ln 2} \approx 12.8 \text{ s}$

For  $\delta k \ll \beta$

$$\frac{n(t)}{n_0} = \exp\left(\frac{k_{eff} - 1}{\bar{\ell}_p}\right) = \exp\left(\frac{t}{T}\right)$$
$$T = \frac{\bar{\ell}_p}{k_{eff} - 1} = \frac{\beta\tau}{\delta k}$$

- For  $^{235}\text{U}$ ,  $T = 83 \text{ s}$ ,  $k_{eff} - 1 = 0.001$ ,
- $n/n^0 = 1.012$
- This can be controlled!



# Delayed Neutrons

**TABLE 3.5** DELAYED NEUTRON DATA FOR THERMAL FISSION IN  $^{235}\text{U}^*$

| Group | Half-Life<br>(sec) | Decay Constant<br>( $\lambda_i$ , $\text{sec}^{-1}$ ) | Energy<br>(ke V) | Yield, Neutrons<br>per Fission | Fraction<br>( $\beta_i$ ) |
|-------|--------------------|-------------------------------------------------------|------------------|--------------------------------|---------------------------|
| 1     | 55.72              | 0.0124                                                | 250              | 0.00052                        | 0.000215                  |
| 2     | 22.72              | 0.0305                                                | 560              | 0.00346                        | 0.001424                  |
| 3     | 6.22               | 0.111                                                 | 405              | 0.00310                        | 0.001274                  |
| 4     | 2.30               | 0.301                                                 | 450              | 0.00624                        | 0.002568                  |
| 5     | 0.610              | 1.14                                                  | —                | 0.00182                        | 0.000748                  |
| 6     | 0.230              | 3.01                                                  | —                | 0.00066                        | 0.000273                  |

Total yield: 0.0158

Total delayed fraction ( $\beta$ ) 0.0065

\*Based in part on G. R. Keepin, *Physics of Nuclear Kinetics*, Reading, Mass.: Addison-Wesley, 1965.

For 1-group model,  $T_{\frac{1}{2}}$  for  $^{235}\text{U}$  is about 8.87 s and  $\tau$  is about 12.8 s.



# Delayed Neutron Fractions

| Group | $^{235}\text{U}$ |                    | $^{233}\text{U}$ |                    | $^{239}\text{Pu}$ |                    |
|-------|------------------|--------------------|------------------|--------------------|-------------------|--------------------|
|       | Half-life (s)    | fraction $\beta_i$ | Half-life (s)    | fraction $\beta_i$ | Half-life (s)     | fraction $\beta_i$ |
| 1     | 55.7             | 0.00021            | 55.0             | 0.00022            | 54.3              | 0.00007            |
| 2     | 22.7             | 0.00142            | 20.6             | 0.00078            | 23.0              | 0.00063            |
| 3     | 6.22             | 0.00127            | 5.00             | 0.00066            | 5.60              | 0.00044            |
| 4     | 2.30             | 0.00257            | 2.13             | 0.00072            | 2.13              | 0.00068            |
| 5     | 0.610            | 0.00075            | 0.615            | 0.00013            | 0.618             | 0.00018            |
| 6     | 0.230            | 0.00027            | 0.277            | 0.00009            | 0.257             | 0.00009            |
| total | -                | 0.0065             | -                | 0.0026             | -                 | 0.0021             |

# Point Kinetic Equations

$$\frac{dn(t)}{dt} = \frac{\rho - \beta}{l} n(t) + \sum_{i=1}^G \lambda_i C_i(t) + S(t)$$
$$\frac{dC_i(t)}{dt} = \frac{\beta}{l} n(t) - \lambda_i C_i(t), i = 1, \dots, G$$



# Reactivity and Worth

$$\rho \equiv \frac{k_{eff} - 1}{k_{eff}} = \frac{\delta k}{k_{eff}} \quad \text{reactivity } \rho \text{ and } \delta k$$

$$k(\$) \equiv \frac{\rho}{\beta} \quad \begin{array}{l} \beta \text{ is delayed neutron fraction} \\ \text{worth can be measured in units of } k(\$) \text{ or } k \end{array}$$

- cents?
- Percent Mil?



# Reactivity Equation Solutions

$$\phi_T = \underbrace{A_1 \exp(\omega_1 t)}_{\substack{\text{dominant term} \\ \text{as } t \rightarrow \infty}} + \underbrace{A_2 \exp(\omega_2 t)}_{\text{approaches 0 rapidly}}$$

General solution for single group of delayed neutrons

$$T = \frac{1}{\omega_1}$$

Definition of reactor or stable period

$$\phi_T = \exp\left(\frac{t}{T}\right)$$

General solution for single group of delayed neutrons

$$\rho = \frac{\omega l_p}{1 + \omega l_p} + \frac{\omega}{1 + \omega l_p} \sum_{i=1}^6 \frac{\beta_i}{\omega + \lambda_i}$$

Reactivity equation for six group model



# Power Changes

$$T = \frac{\beta\tau}{\delta k} = \frac{\beta\tau}{keff \cdot \rho} = \frac{\tau}{keff \cdot k(\$)} \sim \frac{\tau}{k(\$)}$$

- T = Reactor Period (units of time)
  - Time required to increase reactor power (or neutron flux) by 2.72
- Integrate solution with specific values (initial time power to final time/power)

$$\frac{t}{T} = \ln \left[ \frac{P(t)}{P(0)} \right]$$



# Reactivity Equation Solutions

$$T = \frac{\bar{l}}{k_{eff} - 1} = \frac{\bar{l}}{\delta k} = \frac{\beta\tau}{\delta k} =$$

**Reactor period** - The time required for a neutron population to change by a factor of e

$$k_{eff} = 1 + \delta k = 1 + \frac{\beta\tau}{T}$$

$\tau$  = Lifetime of delayed neutrons  
~12.8s (U235)

$$T = \frac{\beta\tau}{\delta k} = \frac{\beta\tau}{k_{eff} - 1} = \frac{\beta\tau}{k_{eff}\rho} = \frac{\tau}{k_{eff}\rho(\$)} \approx \frac{\tau}{\rho(\$)}$$

$$\phi(t) = \exp\left(\frac{t}{T}\right)$$

Remember, Flux is proportional to power....

$$C \cdot P(t) = \exp\left(\frac{t}{T}\right) \quad \frac{t}{T} = \ln\left(\frac{P(t) \cdot C}{P(0) \cdot C}\right) \quad \frac{P(t)}{P(0)} = \exp\left(\frac{t}{T}\right)$$



# Example

- Following a reactor scram in which all the control rods are inserted into a power reactor, how long is it before the reactor power decreases to 0.0001 of the steady-state power prior to shutdown? (Assume a reactor period of -80 s)



# 1-level Model Parameters

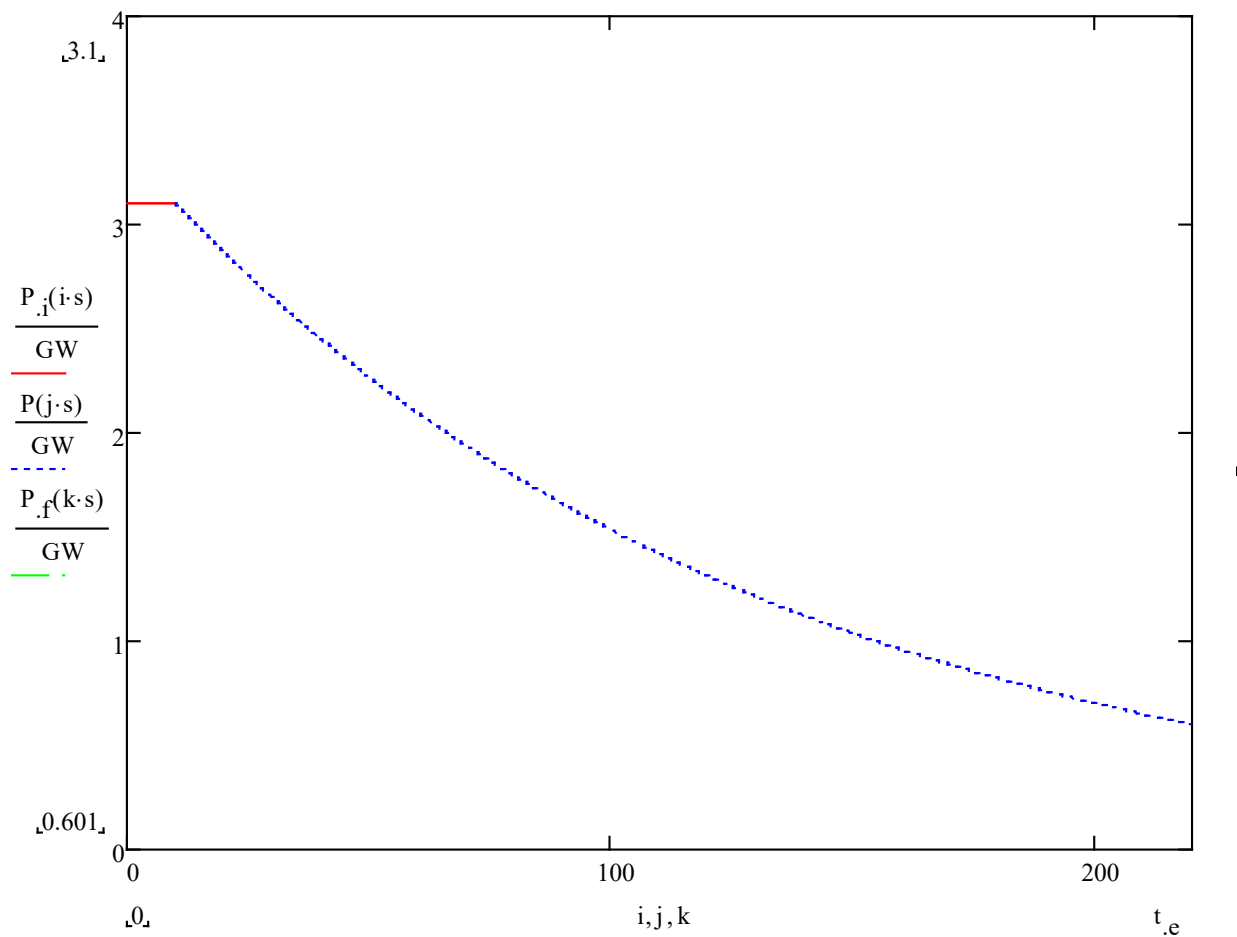
|                   | $\beta$ | $T_{\frac{1}{2},d}(s)$ | $\tau_d(s)$ |
|-------------------|---------|------------------------|-------------|
| $^{232}\text{Th}$ | 0.0203  | 6.98                   | 10.07       |
| $^{233}\text{U}$  | 0.0026  | 12.4                   | 17.89       |
| $^{235}\text{U}$  | 0.0064  | 8.82                   | 12.72       |
| $^{238}\text{U}$  | 0.0148  | 5.32                   | 7.68        |
| $^{239}\text{Pu}$ | 0.002   | 7.81                   | 11.27       |
| $^{241}\text{Pu}$ | 0.0054  | 104.1                  | 150.18      |
| $^{241}\text{Am}$ | 0.0013  | 10                     | 14.43       |
| $^{243}\text{Am}$ | 0.0024  | 10                     | 14.43       |
| $^{242}\text{Cm}$ | 0.0004  | 10                     | 14.43       |



Source: Laboratoire de Physique Subatomique et de Cosmologie

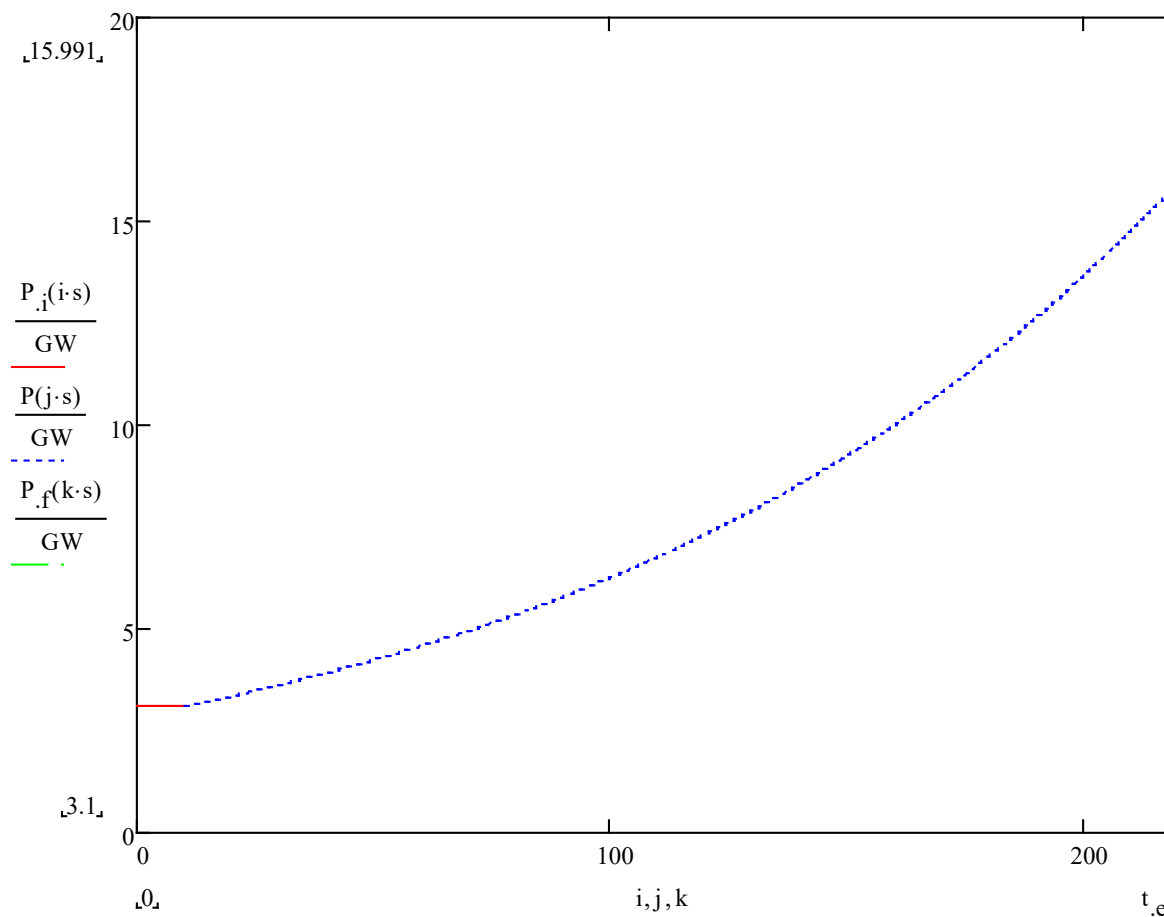
# Exploration 1

- What if we add  $-\$0.1$  to AP1000 core?



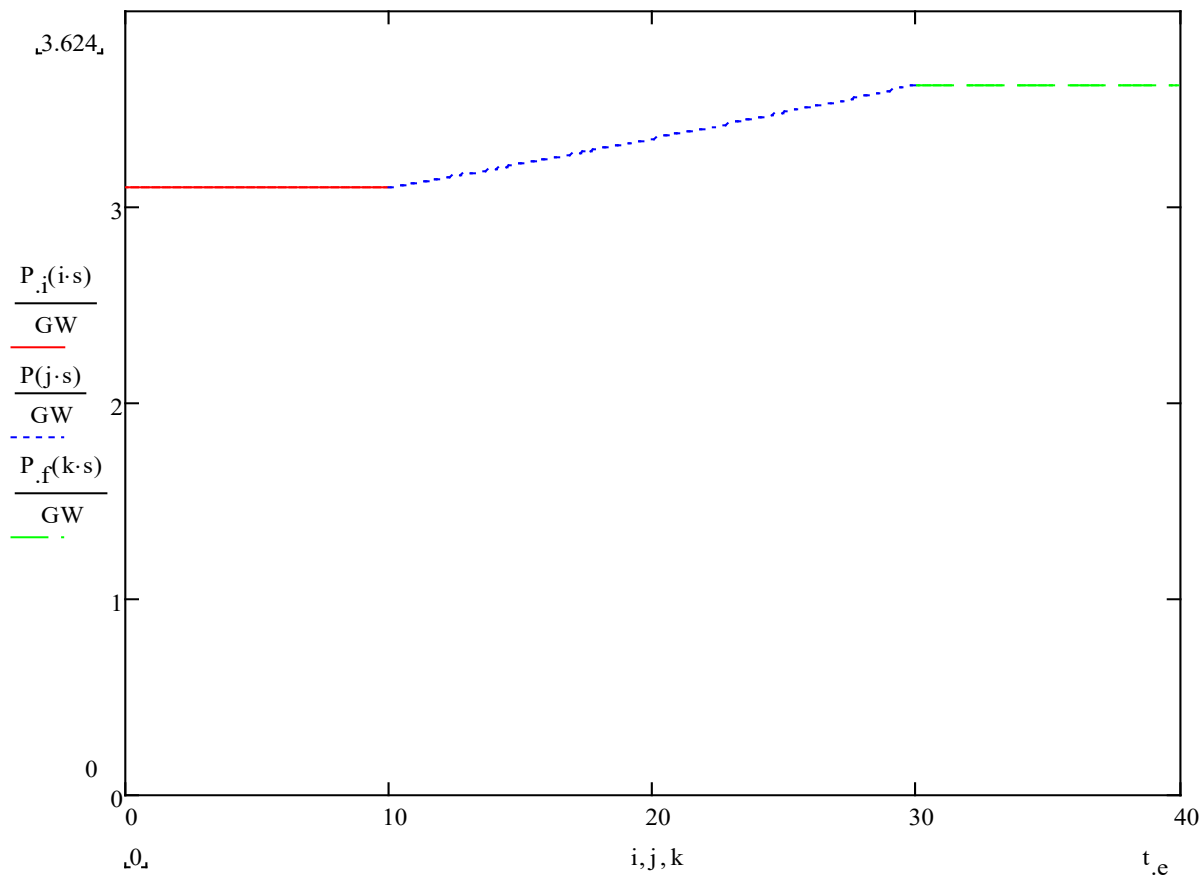
# Exploration 2

- What if we add \$0.1 to AP1000 core?

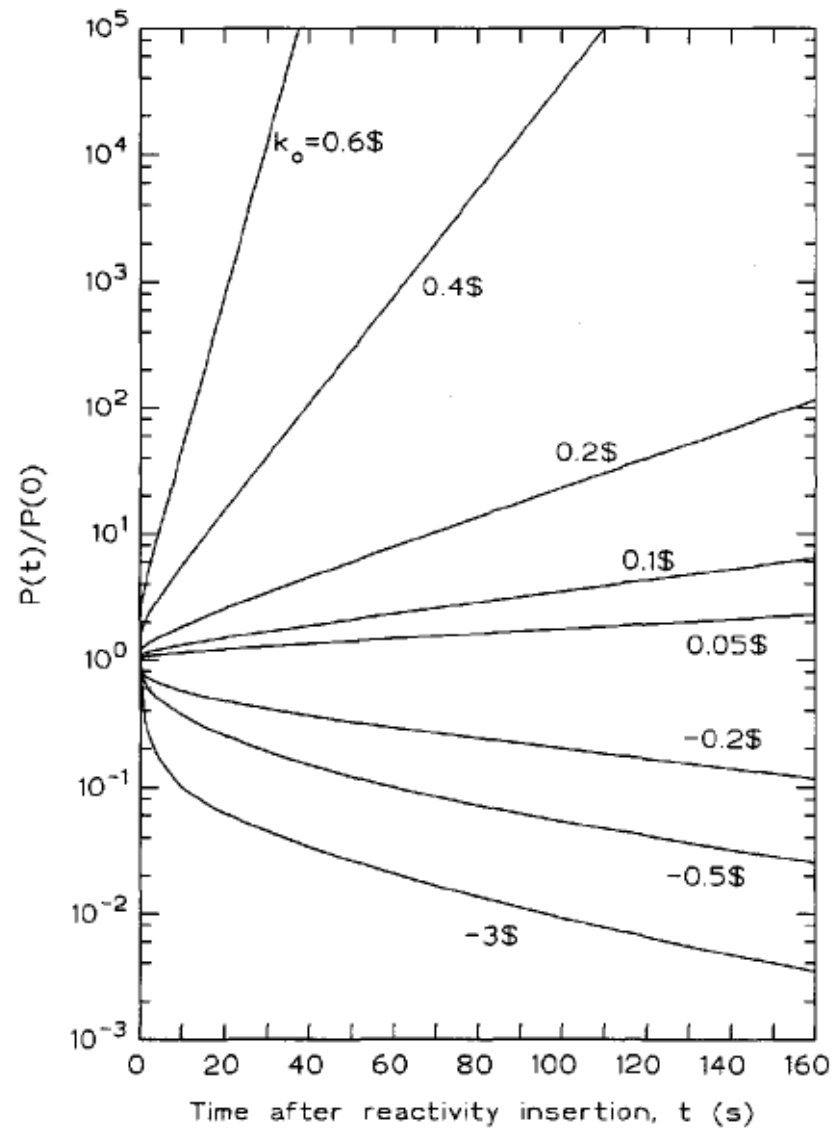


# Exploration 3

- What if we add \$0.1 to AP1000 core, then after 10 seconds we add -\$0.1?



# Reality



# Power Plant Operation

- Try to achieve “steady state” operation at all times
- Set up core and fuel to provide minimal changes
- Despite this, still have three scales of change
  1. Short Term (Grid load transients)
  2. Intermediate Term (Fuel composition changes)
  3. Long Term (Burnup & Depletion)



# $K_{\text{eff}}$ vs. time curve for Power plant?



# Three Time Scales (short)

- Short Time Constant (load change)
  - An abrupt change in steam demand/load.
  - A load change is seen as:
    - Reactor pressure change in the BWR
    - Reactor temperature change in a PWR.
  - Higher loads lead to higher pressures/temperatures.
- Assumptions
  - Assumes a uniform multiplicative change everywhere
    - Spatial variations in time in the reactor are not considered
  - (This method is called point kinetics.)



# Three Time Scales (intermediate)

- Intermediate Time Constant (core composition change)
  - Changing fission product concentrations
    - Generation rates and destruction/decay rates.
  - Many fission products have measurable thermal neutron cross sections  $f = \frac{\Sigma_a^F}{\Sigma_a^F + \Sigma_a^{NF}(V^{NF}/V^F)(\phi^{NF}/\phi^F)}$ 
    - **Change** the value of  $k$  (and  $k_\infty$ ).
  - Point kinetics can be used if spatial variations in concentrations are negligible.
  - Otherwise, detailed spatial and temporal equations must be used!



# Three Time Scales (long)

- Long Time Constant (Fuel Depletion)

- Treated as a series of steady-state problems

$$D\nabla^2\phi - \Sigma_a\phi = -\lambda\nu\Sigma_f\phi$$

- Two things are adjusted to maintain  $\lambda = 1$  (i.e.  $\lambda = 1/k$ )

- Material buckling
- Reactor dimensions
- If  $\lambda \neq 1$ , the equation is not valid
  - Why?

- In operating reactor, cannot change dimensions (much)

- $k$  is adjusted slowly in time by changing chemical shim conc.
  - Chemical shim is an isotope that absorbs neutrons in the reactor
- $k$  is also adjusted via the control rods.

- Managing fuel consumption is a classical example of this type of transient

