Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 4

Nuclear Energetics

Radiation I

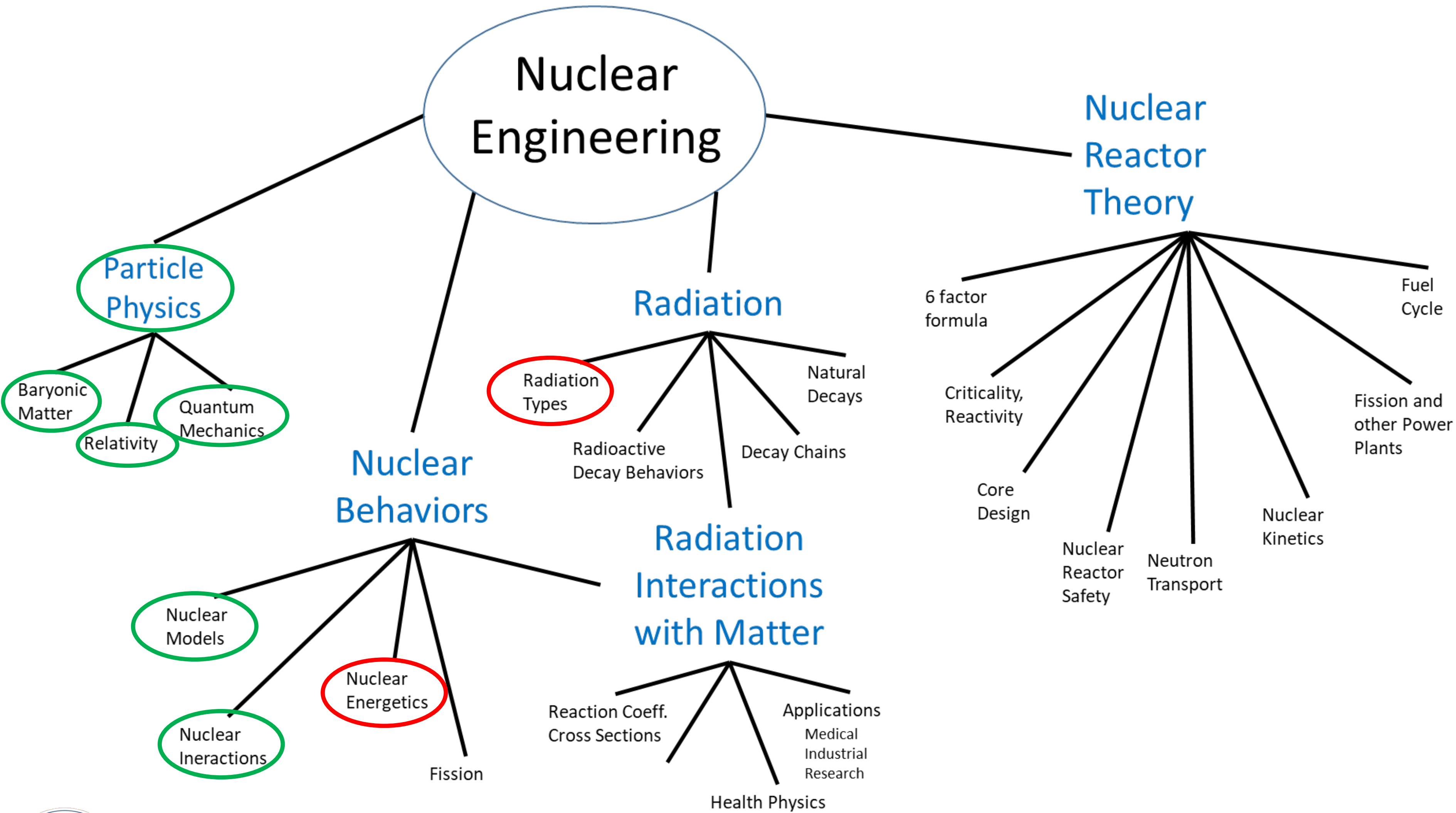
Spiritual Thought

"I add my voice to these wise and inspired brethren and say to you that one of the most important things you can do as priesthood leaders is to immerse yourselves in the scriptures. Search them diligently. Feast upon the words of Christ. Learn the doctrine. Master the principles that are found therein. There are few other efforts that will bring greater dividends to your calling. There are few other ways to gain greater inspiration as you serve.

But that alone, as valuable as it is, is not enough. You must also bend your efforts and your activities to stimulating meaningful scripture study among the members of the Church."

President Ezra Taft Benson

Roadmap


Nuclear Engineering
Particle Physics
Baryonic Matter
Relativity
Quantum Mechanics
Nuclear Behaviors
Nuclear Models
Nuclear Ineractions
Nuclear Energetics
Fission
Radiation
Radiation Types
Radioactive Decay Behaviors
Decay Chains
Natural Decays
Radiation Interactions with Matter
Reaction Coeff. Cross Sections
Health Physics
Applications
Medical
Industrial
Research
Nuclear Reactor Theory
6 factor formula
Criticality, Reactivity
Core Design
Nuclear Reactor Safety
Neutron Transport
Nuclear Kinetics
Fission and other Power Plants
Fuel Cycle

Objectives

- Understand Q and be able to calculate it
- Set up, complete and evaluate nuclear reaction equations
- Understand nuclear decay types:
  - Name
  - Properties
  - Mathematical Descriptions
  - Equations
  - Caveats
  - Q Values
- Decay Charts (KNOW HOW TO USE!)

Parallel reactions

- Chemistry – multiple possible reactions
  - Large quantities of reactants
  - Branching ratios, equilibrium, etc.
- Nuclear – single nuclei at a time
  - Statistical probabilities for various outcomes
- Example, neutron interacting with sulfur
  - $^{32}_{16}S(n,n)^{32}_{16}S$ α%
  - $^{32}_{16}S(n,n')^{32}_{16}S^{*}$ β% where β=0% if E_n<XX
  - $^{32}_{16}S(n,p)^{32}_{15}P$ δ% where δ =0% if E_n<YY
  - $^{32}_{16}S(n,\gamma)^{33}_{16}S$ (1-α-β-δ)%

Q-value

$$Q = KE_{reactants} - KE_{products}$$

$$\frac{Q}{c^2} = reactant\ rest\ mass - product\ rest\ mass$$

note that this is reactants – products, the opposite convention as is used in chemistry

- 3 special challenges:
  - Charge balancing
  - Reactions with changes in the number of protons
  - Excited-state rather than ground-state isotopes
- The next several slides cover these

Simple Q-value Calculations

- No:
  - proton-neutron transformations (β emissions),
  - excited states,
  - other special cases
- For example, the reaction of lithium with deuterium

$$ {}^{6}_{3}Li\left({}^{2}_{1}H, {}^{4}_{2}He\right){}^{4}_{2}He $$

$$ Q = M_{{}^{6}_{3}Li} + M_{{}^{2}_{1}H} - M_{{}^{4}_{2}He} - M_{{}^{4}_{2}He} $$
$$ = 6.015122 + 2.014101 - 2(4.002603) = $$
$$ = 0.02402\,u\left(931.5\,\frac{MeV}{u}\right) = 22.37\,MeV $$

Be sure to conserve charge

- Often, reactions notations don't include electrons
  - $^{16}_{8}O(n,p)^{16}_{7}N$ or, more explicitly, $^{16}_{8}O(n,p)^{16}_{7}N + {}^{0}_{-1}e$

can be represented as

$$^{16}_{8}O(n,{}^{1}_{1}H)^{16}_{7}N$$

- So, to conserve charge, non-ionized particles used.

$$\frac{Q}{c^2} = M({}^{16}_{8}O) + m_n - M({}^{16}_{7}N) - M({}^{1}_{1}H)$$
$$= 15.994915 + 1.0086649156 - 16.006101$$
$$- 1.007825 = -0.01035/c^2\ \mathrm{u} = -9.637\ \mathrm{MeV}$$

Note: Q-values commonly involve mass differences in the 3rd to 4th decimal point.

When Z changes

- Changes in the proton number (Z) require special step
  - Include neutrino or anti-neutrino
  - sometimes a positron (antimatter electron with positive charge).

$$p(p, {}^{0}_{+1}e\,\nu)d \quad \text{or} \quad {}^{1}_{1}p + {}^{1}_{1}p \rightarrow {}^{2}_{1}d + {}^{0}_{+1}e + \nu$$

- drives deuterium formation in the sun
  - Q-value is computed by adding two electrons to each side
  - Form two hydrogens on the left
  - Form deuterium and an extra electron on the right

$${}^{1}_{1}H({}^{1}_{1}H, \nu\, {}^{0}_{+1}e\, {}^{0}_{-1}e){}^{2}_{1}H$$

$$\frac{Q}{c^2} = 2M({}^{1}_{1}H) - M({}^{2}_{1}H) - 2m_e - m_\nu$$

=0.420 MeV

m_ν is unknown but known to be very small (negligible) compared to m_e

Excited Nuclei

- Some reactions produce stable (long decay time) excited-state nuclei
- Excited states are designated by an asterisk
- For these, the energy difference between the ground and excited states must be known, (E^*)

$$ {}^{10}_{5}B(n,\alpha){}^{7}_{3}Li^* $$

$$ \frac{Q}{c^2} = m_n + M\left({}^{10}_{5}B\right) - M\left({}^{7}_{3}Li^*\right) - M\left({}^{4}_{2}He\right) $$
$$ = m_n + M\left({}^{10}_{5}B\right) - M\left({}^{7}_{3}Li\right) - M\left({}^{4}_{2}He\right) - E^* $$
$$ m_n + M\left({}^{10}_{5}B\right) - M\left({}^{7}_{3}Li\right) - M\left({}^{4}_{2}He\right) - Q_{{}^{7}_{3}Li^* \rightarrow {}^{7}_{3}Li} $$
$$ = m_n + M\left({}^{10}_{5}B\right) - M\left({}^{7}_{3}Li\right) - M\left({}^{4}_{2}He\right) - 0.48MeV $$

Some notes

- Text provides isotopic masses for most isotopes.
- The official masses (in the US) are updated by NIST at http://www.nist.gov/pml/data/comp.cfm.

Decay Conservations

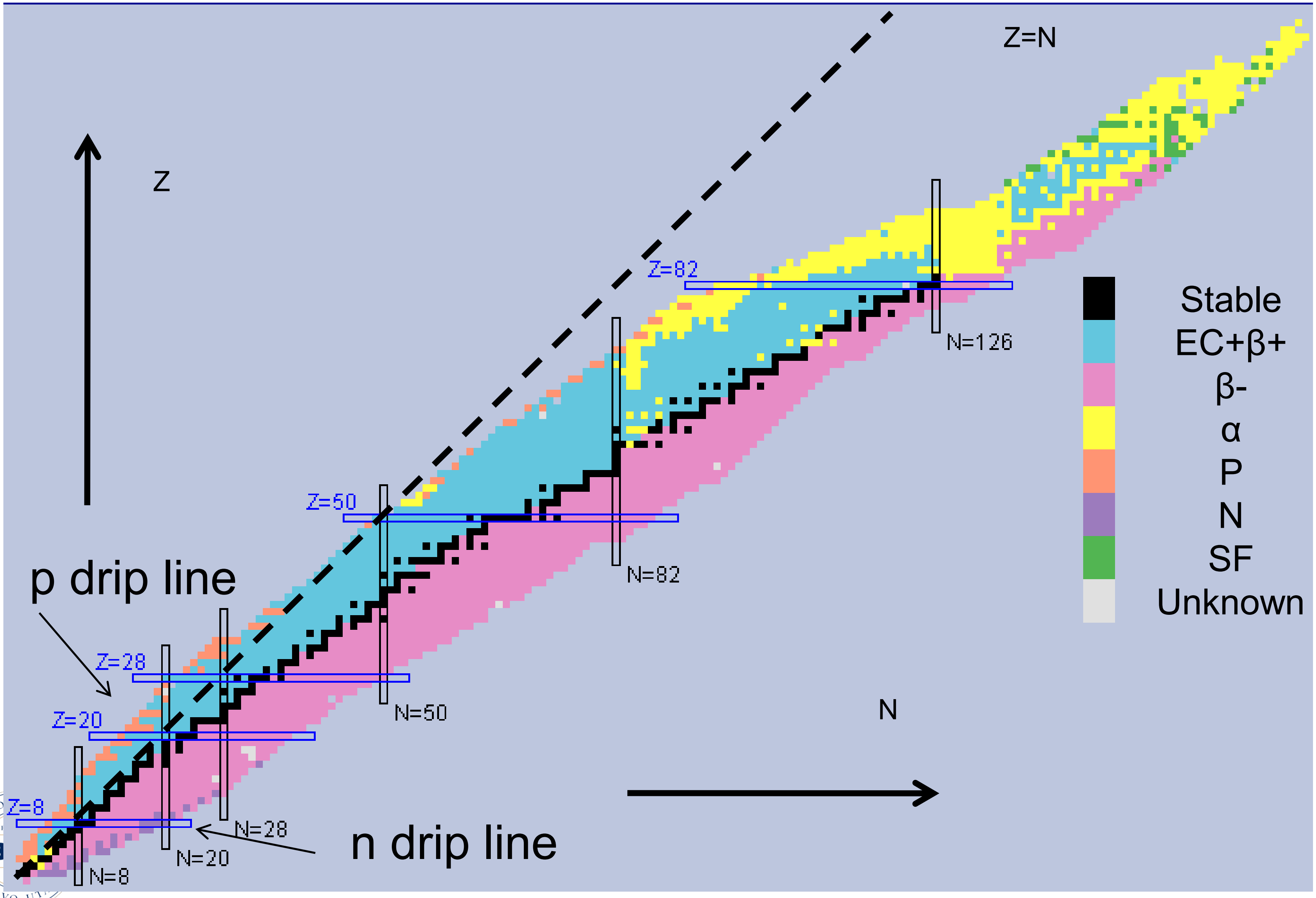
- Charge
- Nucleon Number
- Mass/Energy (Total Energy)
- Linear Momentum
- Angular Momentum

- Classical behavior if liberated energy is much less than rest mass energy

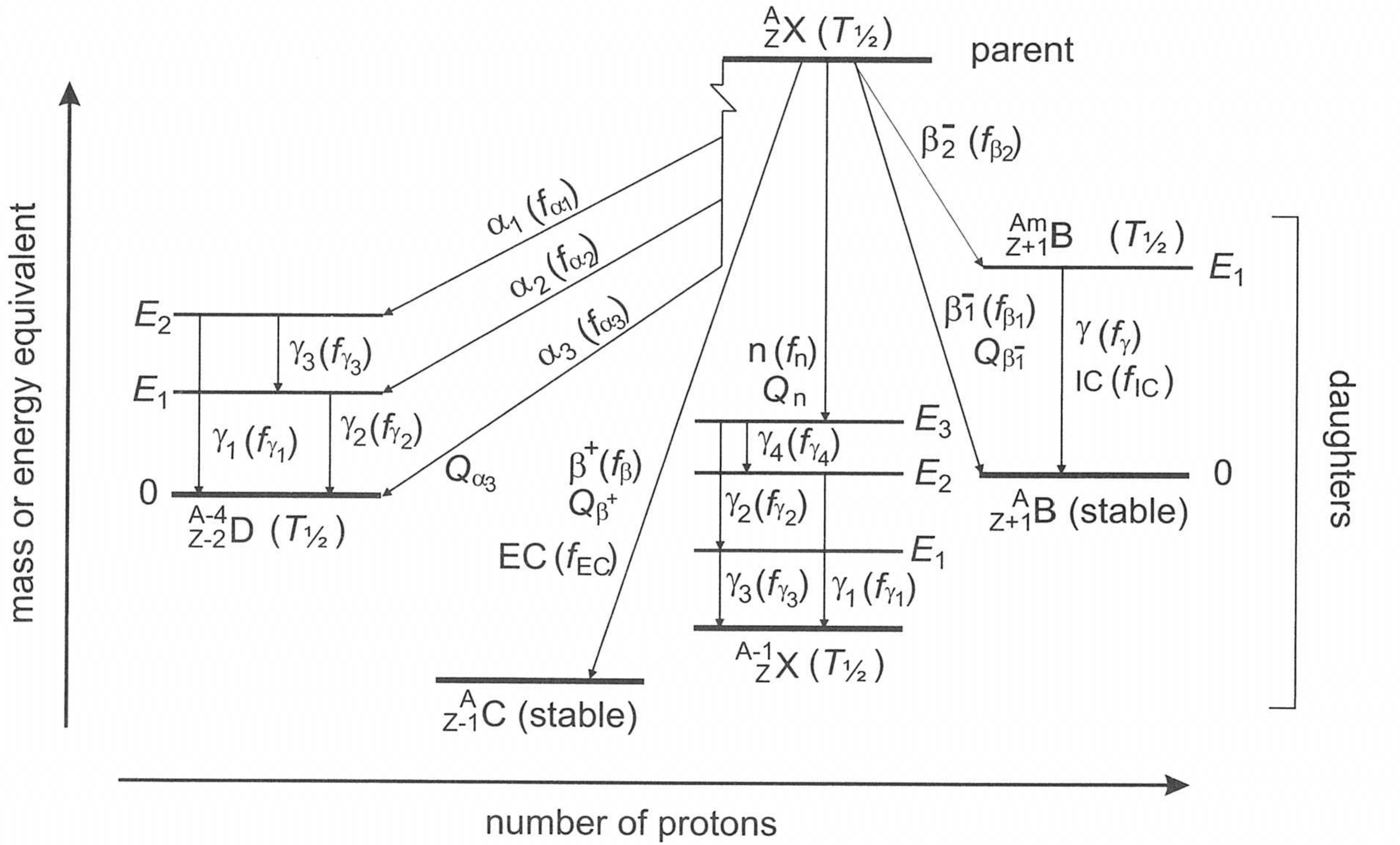
Decay Mechanisms

- Alpha ($\boldsymbol{\alpha}$)
- Beta (+/-) ($\boldsymbol{\beta}^+, \boldsymbol{\beta}^-$)
- Gamma ($\boldsymbol{\gamma}$)
- Electron capture (EC)
- Proton (P), Deuteron (D) and Triton (T)
- Neutron (N)
- Internal conversion (IC)
- Spontaneous fission (SF)

Chart of the Nuclides



Energy Diagram



Alpha Decay

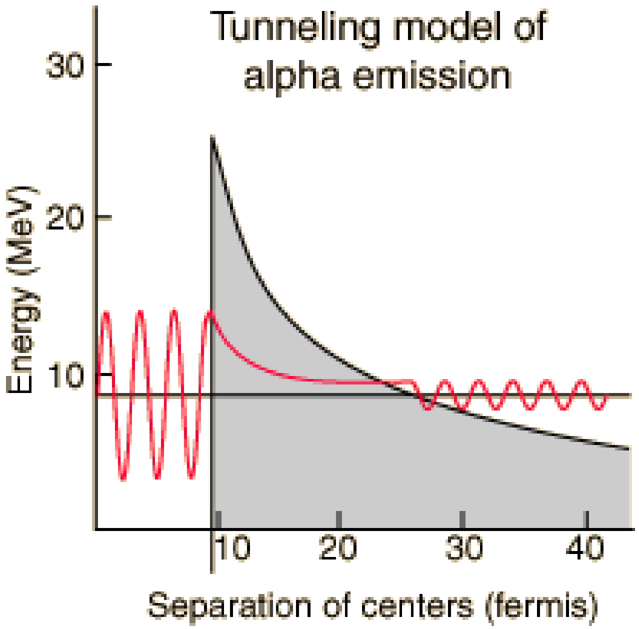
- Emission of a ^4He nucleus – a (2+) charged particle.
- Reduces Z by 2, N by 2, and A by 4.
- Common in heavy (> Pb, 82) nuclides, otherwise rare.
- Alpha particles have discrete energies (quantized)
- Very highly energetic (LARGE)
  - rapidly absorbed by other material – WHY?
  - **generally stopped by piece of paper or outer layers of skin.**
- Minor source of ionizing external radiation exposure.
- Significant exposure if ingested/inhaled.
  - Direct damage to lungs and alveoli

Alpha Particles Tunnel

$$\psi_I(x) = A_i \mathrm{e}^{\mathrm{i}kx} + A_r \mathrm{e}^{-\mathrm{i}kx}$$

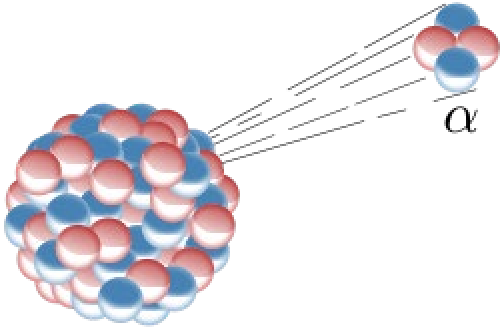
$$\psi_{\mathrm{II}}(x) = B\mathrm{e}^{-x\chi} - B'\mathrm{e}^{x\chi}$$

$$\psi_{\mathrm{III}}(x) = A_t \mathrm{e}^{\mathrm{i}kx}$$



Alpha Decay

$$ {}^{A}_{Z}P \rightarrow \left[{}^{A-2}_{Z-2}D\right]^{2-} + {}^{4}_{2}\alpha \rightarrow {}^{A-2}_{Z-2}D + {}^{4}_{2}He $$



First look at Q for mass (mass difference)

$$ \frac{Q_\alpha}{c^2} = M\left({}^{A}_{Z}P\right) - \left[M\left({}^{A-4}_{Z-2}D^{2-}\right) + m\left({}^{4}_{2}\alpha\right)\right] $$

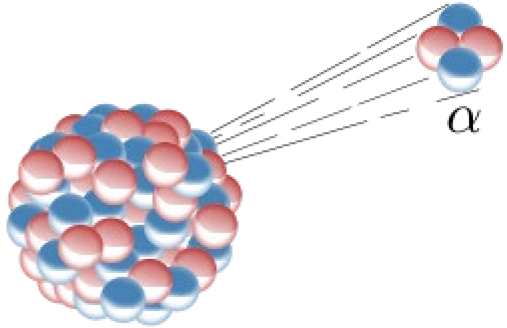
$$ \cong M\left({}^{A}_{Z}P\right) - \left[M\left({}^{A-4}_{Z-2}D\right) + 2m_e + m\left({}^{4}_{2}\alpha\right)\right] $$

$$ \cong M\left({}^{A}_{Z}P\right) - \left[M\left({}^{A-4}_{Z-2}D\right) + m\left({}^{4}_{2}He\right)\right] $$

- Two approximations;
  - Neglect daughter electron Binding Energy (1^{st} Eq.)
  - Neglect He electron Binding energy (2^{nd} Eq.)
- ^{4}He atom differs from an α particle?
  - two electrons – the α particle has a net 2+ charge.

Alpha Decay

$$ {}^{A}_{Z}P \rightarrow \left[{}^{A-2}_{Z-2}D\right]^{2-} + {}^{4}_{2}\alpha \rightarrow {}^{A-2}_{Z-2}D + {}^{4}_{2}He $$



Now look at Q for energy (kinetic energy)

$$ Q_{\alpha} = E_D + E_{\alpha} = \frac{1}{2}M_{\alpha}v_{\alpha}^2 + \frac{1}{2}M_D v_D^2 $$

$$ M_D v_D = M_{\alpha} v_{\alpha} $$

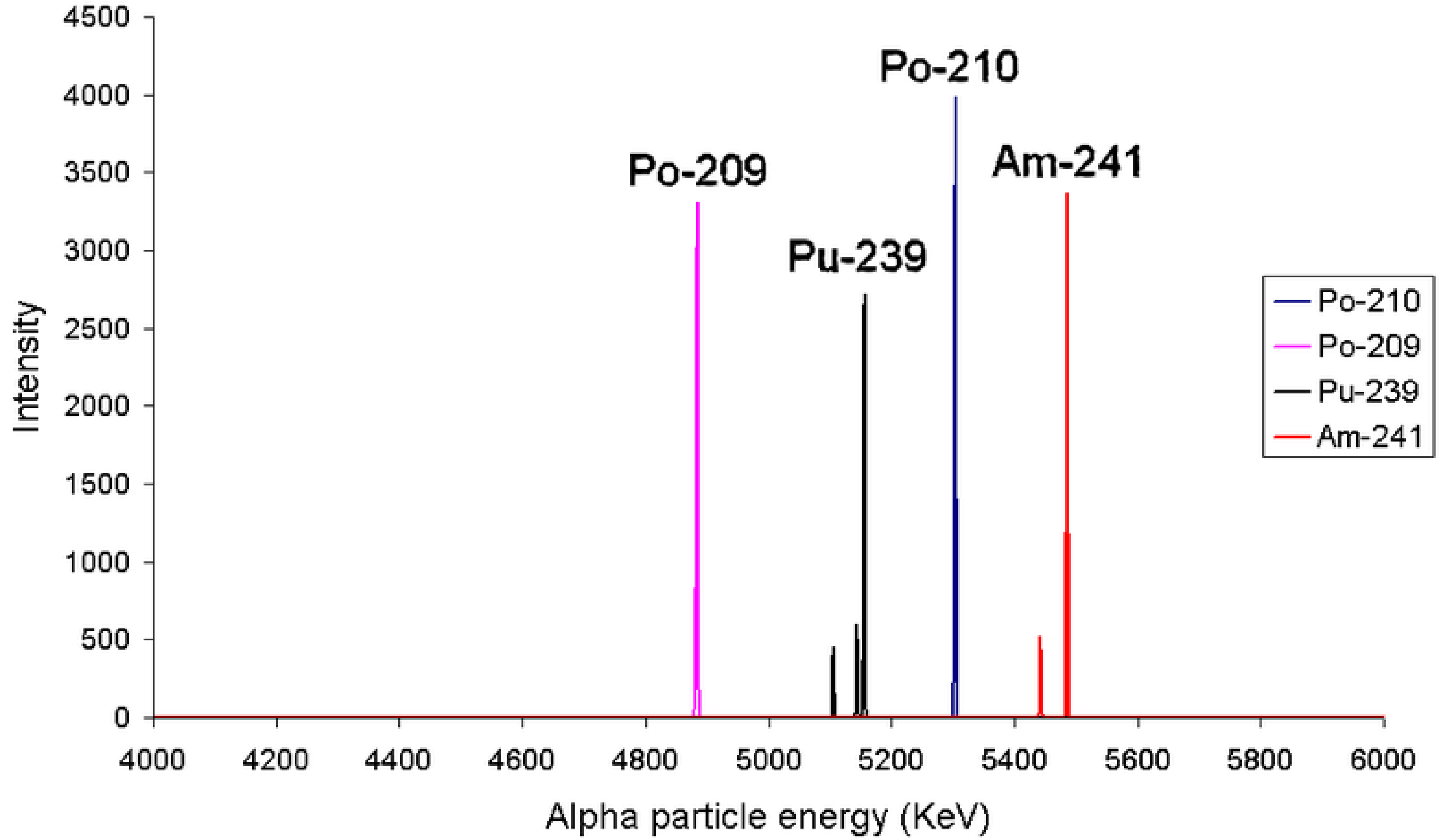
$$ \Rightarrow Q_{\alpha} = \frac{1}{2}M_{\alpha}v_{\alpha}^2\left(\frac{M_{\alpha}}{M_D} + 1\right) $$

$$ E_{\alpha} = Q_{\alpha}\left[\frac{M_D}{M_D + M_{\alpha}}\right] \cong Q_{\alpha}\left[\frac{A_D}{A_D + A_{\alpha}}\right] $$

$$ E_D = Q_{\alpha}\left[\frac{M_{\alpha}}{M_D + M_{\alpha}}\right] \cong Q_{\alpha}\left[\frac{A_{\alpha}}{A_D + A_{\alpha}}\right] $$

Energies of alpha and daughter particles are fixed, and alpha particle energy is uniquely associated with isotope.

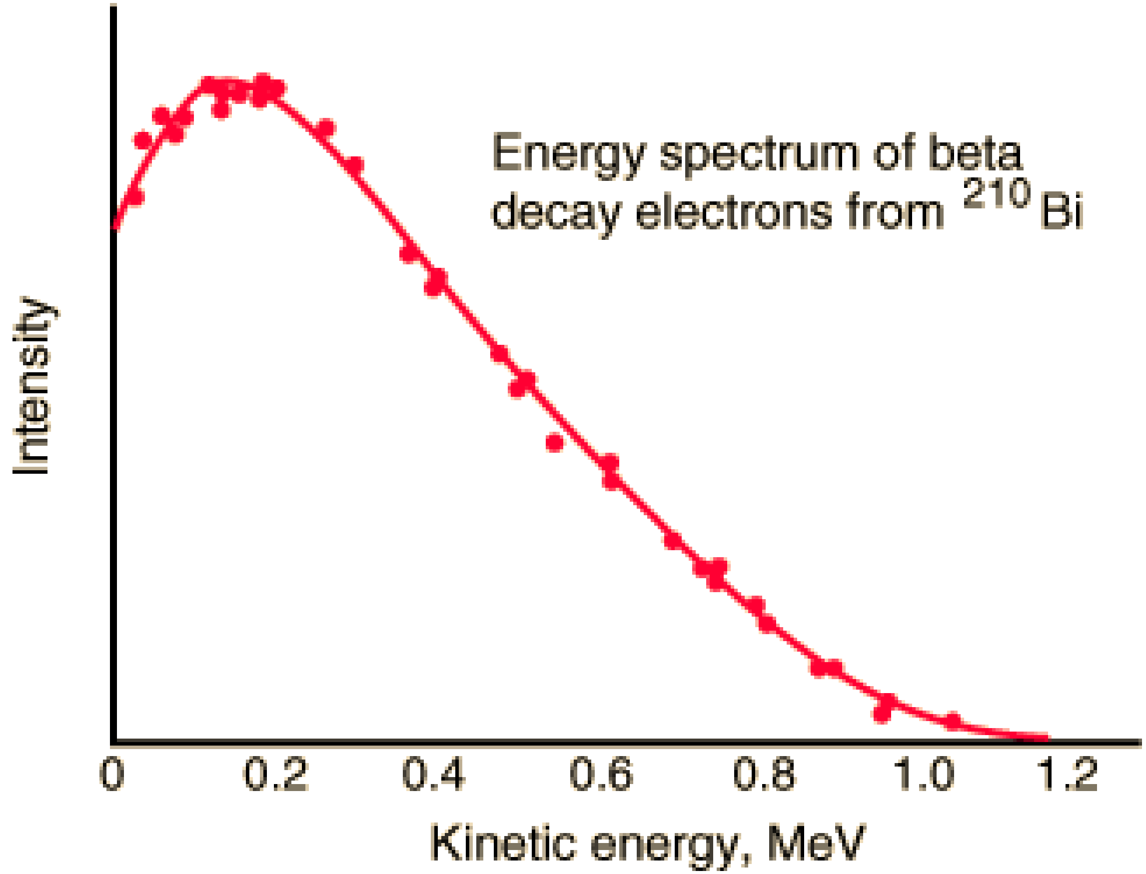
Alpha Decay Example



Beta Decay (I)

- β^+ (positron) decay
  - occurs in proton-rich nuclei.
  - proton transforms to a neutron
    - positron and neutrino emitted
    - reduces Z and increases N by 1 without changing A.
  - requires parent particle to be at least 2 M_e heavier than daughter.
- β^- (electron) decay
  - occurs in neutron-rich nuclei
  - neutron transforms to a proton
    - electron and anti-neutrino emitted
    - increases Z and decreases N by 1 without changing A.
  - requires parent particle to be at least n-p^+-e^- heavier than daughter.

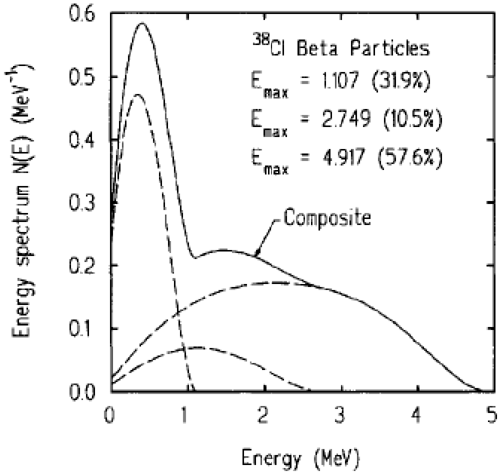
Experimental β Decay Spectrum



Source: G. J. Neary, [Roy. Phys. Soc. (London)](), A175, 71 (1940).

Beta Decay (II)

- Continuous energy spectrum (not discrete)
  - shares its energy with the neutrino.
- Positron/electrons have a well-defined upper energy limit
  - with an average energy of about 0.3 E_{max}.
- High-energy ionizing radiation
  - penetrates outer skin
  - Doesn't penetrate plywood or most construction materials.
- Responsible for both external and internal exposures.



β^- Decay

$${}^{A}_{Z}P \rightarrow \left[{}^{A}_{Z+1}D\right]^{+} + {}^{0}_{-1}e + \overline{\nu}_e$$

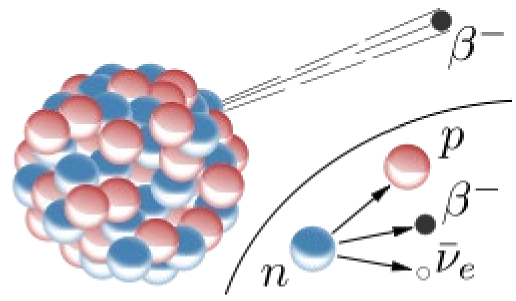
$$\frac{Q_{\beta^-}}{c^2} = M\left({}^{A}_{Z}P\right) - \left[M\left({}^{A}_{Z+1}D^{+}\right) + m_{\beta^-} + m_{\overline{\nu}_e}\right]$$

$$\cong M\left({}^{A}_{Z}P\right) - \left\{\left[M\left({}^{A}_{Z+1}D\right) - m_e\right] + m_{\beta^-} + m_{\overline{\nu}_e}\right\}$$

$$= M\left({}^{A}_{Z}P\right) - M\left({}^{A}_{Z+1}D\right)$$ ground-state daughter

$$\frac{Q_{\beta^-}}{c^2} = M\left({}^{A}_{Z}P\right) - M\left({}^{A}_{Z+1}D\right) - \frac{E^*}{c^2}$$ excited-state daughter

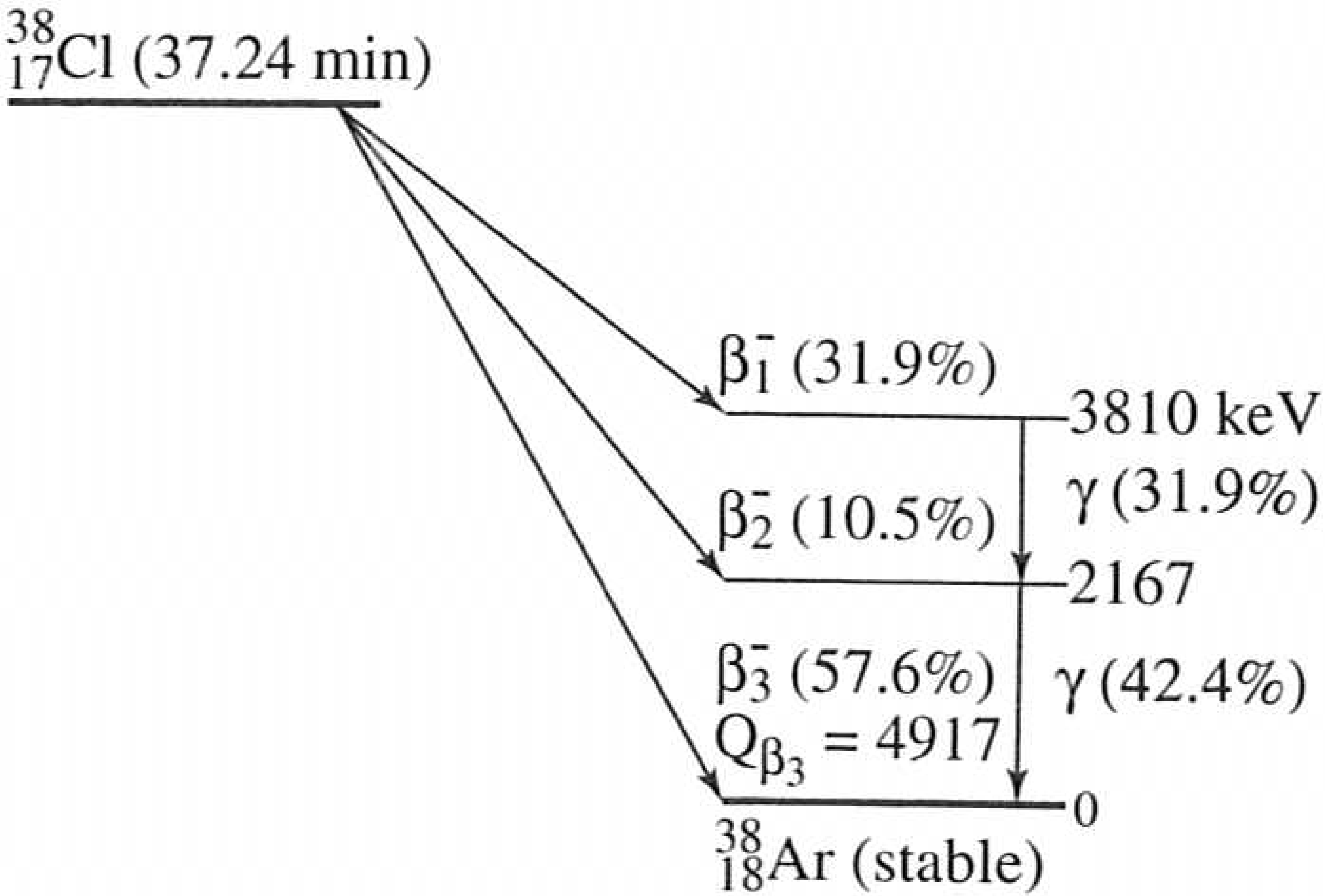
$$E_{\beta^-,\max} = \left(\frac{M_D}{M_D + M_e}\right) Q_{\beta^-} \cong Q_{\beta^-}$$



β^- decay

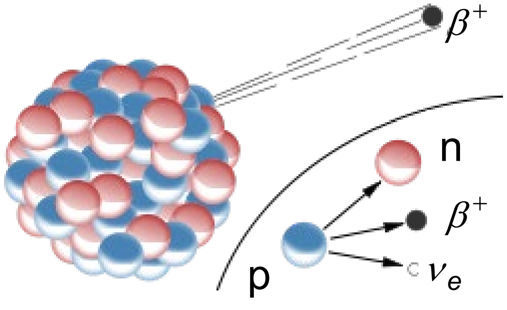
- specific particle energies cannot be determined
- continuous range of energies with a defined maximum value.

β^- Decay Example: Chlorine-38



β^+ or Positron Decay

$$ {}^{A}_{Z}P \rightarrow \left[{}^{A}_{Z-1}D\right]^{-} + {}^{0}_{+1}e + \nu_e $$



β^+ decay

$$ \frac{Q_{\beta^-}}{c^2} = M\left({}^{A}_{Z}P\right) - \left[M\left({}^{A}_{Z-1}D^-\right) + m\left({}^{0}_{+1}e\right) + m_{\nu_e}\right] $$

$$ \cong M\left({}^{A}_{Z}P\right) - \left\{\left[M\left({}^{A}_{Z-1}D\right) + m_e\right] + m_{\beta^+} + m_{\nu_e}\right\} $$

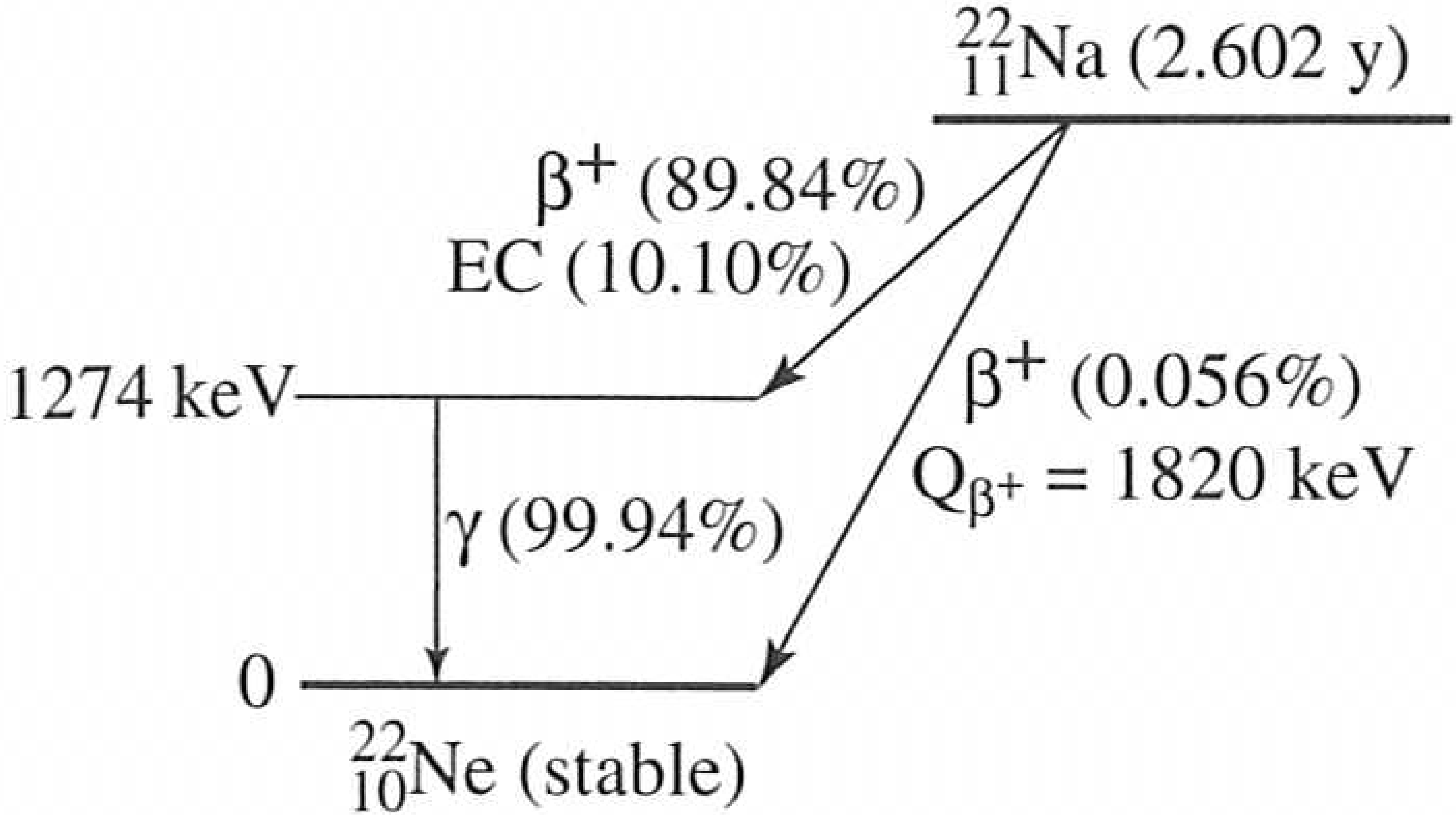
$$ = M\left({}^{A}_{Z}P\right) - M\left({}^{A}_{Z-1}D\right) - 2m_e $$ ground-state daughter

$$ \frac{Q_{\beta^+}}{c^2} = M\left({}^{A}_{Z}P\right) - M\left({}^{A}_{Z-1}D\right) - 2m_e - \frac{E^*}{c^2} $$ excited-state daughter

$$ E_{\beta^+,\max} \cong Q_{\beta^+} $$

The parent mass must exceed the daughter mass by at least 2 m_e for this to spontaneously proceed.

Positron/β^+ Decay: Sodium-22



Electron Capture

- Conversion of proton to neutron & neutrino by capture of inner shell electron.
- Similar to (competes with) β^+:
  - Z decreases and N increases, no change in A
- Differs from β^+:
  - Consumes an electron
  - Also γ-ray emission and/or Auger electrons
- K-capture (innermost, K electron shell)
- No charged particle ***must*** be emitted - unique
- Gamma rays – excited state decay

Electron Capture

$$ {}_{Z}^{A}P \rightarrow {}_{Z-1}^{A}D^{*} + \nu_e $$

$$ \frac{Q_{EC}}{c^2} = M\left({}_{Z}^{A}P\right) - \left[M\left({}_{Z-1}^{A}D^{*}\right) + m_{\nu_e}\right] $$

$$ \cong M\left({}_{Z}^{A}P\right) - M\left({}_{Z-1}^{A}D\right) $$

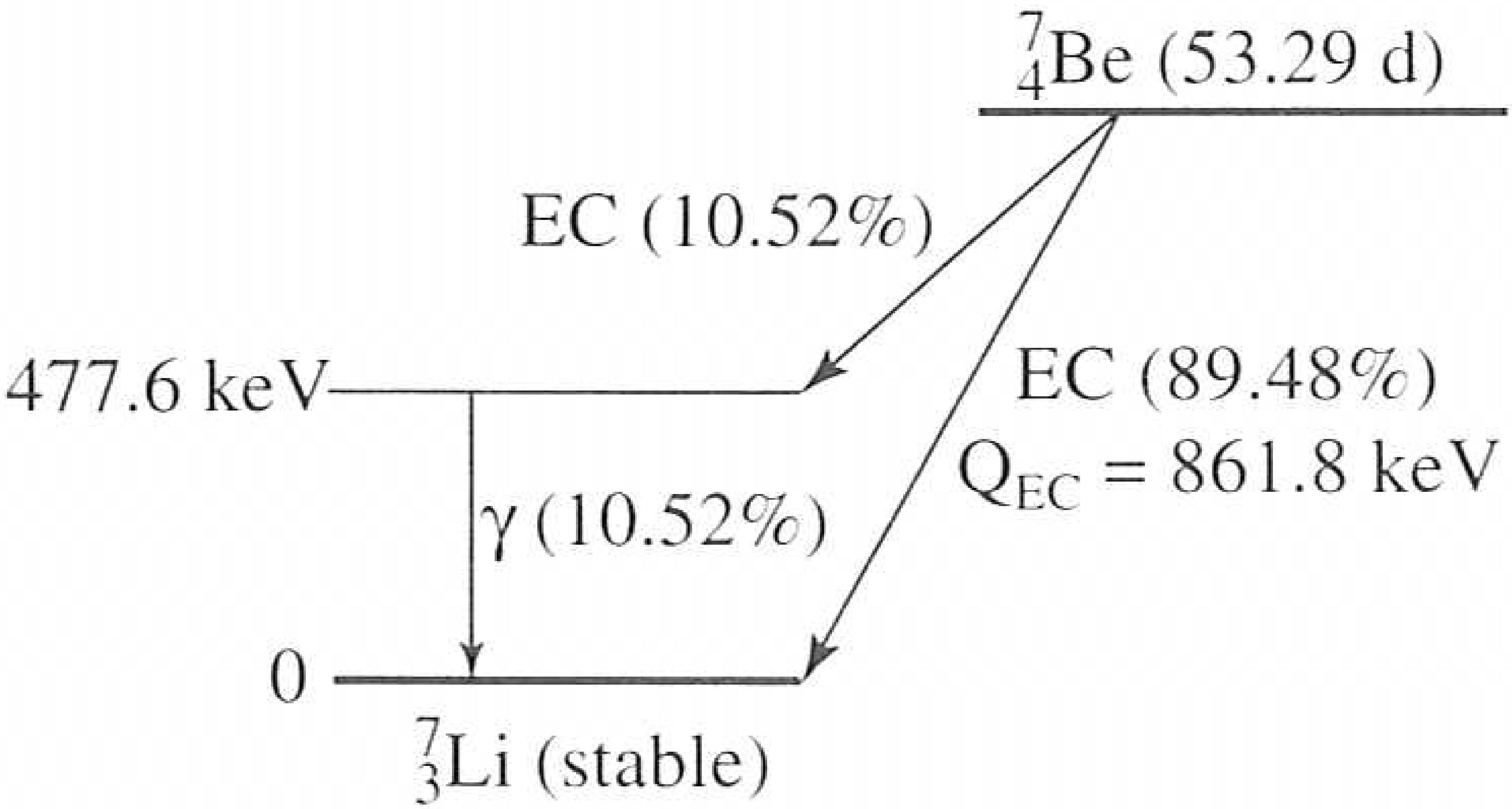
ground-state daughter

$$ \frac{Q_{EC}}{c^2} = M\left({}_{Z}^{A}P\right) - M\left({}_{Z-1}^{A}D\right) - \frac{E^{*}}{c^2} $$

excited-state daughter

Similar to β^{+} decay but less energy constrained since it involves a locally bound electron. Daughter must be just slightly lighter than parent to proceed.

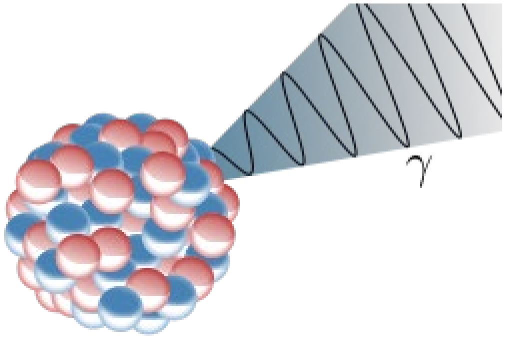
Electron Capture: Beryllium-7



Gamma Emission

- Results from energy level rearrangement of nuclides
- Does not change Z, N, or A
- Exhibits discrete energy levels (quantized)
  - quantized energy states in nucleus.
- Low energy compared to α or β+/-
  - Not an ion, thus not easily absorbed
  - penetrates deeply in skin, concrete, etc.
  - Stopped by thick lead
- Major source of external radiation exposure.

Gamma emission



No change in Z, N, or A. Energy determined by nuclear states.

Neutron Decay

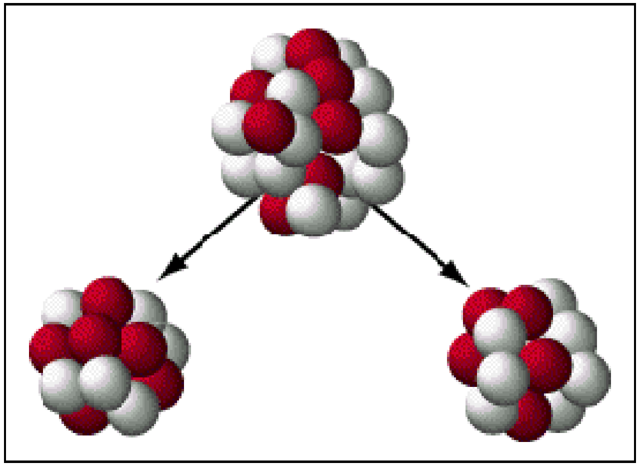
- Some drip line isotopes emit a neutron
  - Daughter has one less nucleon
  - Daughter generally in an excited state.
- Excited state generally decays via gamma emission.
- Critical to reactor control
  - "Delayed neutron" generation
  - Seconds to minutes
  - Fission neutron lifetime: "prompt neutron" = 0.1 ms
  - Delayed neutrons slow the reaction - controllability

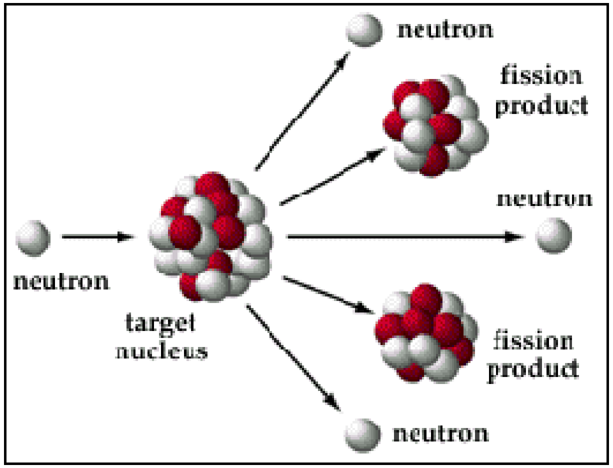
Neutron Decay

$$ {}_{Z}^{A}P \rightarrow {}_{Z}^{A-1}P^{*} + {}_{0}^{1}n \qquad \frac{Q_n}{c^2} = M\left({}_{Z}^{A}P\right) - \left[M\left({}_{Z}^{A-1}P^{*}\right) + m_n\right] $$

$$ = M\left({}_{Z}^{A}P\right) - M\left({}_{Z}^{A-1}P^{*}\right) - m_n - \frac{E^{*}}{c^2} $$

- Relatively rare
- slow (sometimes minutes)
- Makes fission reactions controllable.





Proton Decay

- Proton emission happens at the proton drip line.
- Generally rare and relatively unimportant.

Proton Decay

$$ {}^{A}_{Z}P \rightarrow \left[{}^{A-1}_{Z-1}D^{*} \right]^{-} + {}^{1}_{1}p $$

$$ \frac{Q_n}{c^2} = M\left({}^{A}_{Z}P\right) - \left[M\left({}^{A-1}_{Z-1}D^{*}\right)^{-} + m_p \right] $$

$$ \cong M\left({}^{A}_{Z}P\right) - \left[M\left({}^{A-1}_{Z-1}D^{*}\right) + m_e + m_p \right] $$

$$ = M\left({}^{A}_{Z}P\right) - \left[M\left({}^{A-1}_{Z-1}D\right) + \frac{E^{*}}{c^2} + m_e + m_p \right] $$

$$ \cong M\left({}^{A}_{Z}P\right) - M\left({}^{A-1}_{Z-1}D\right) - M\left({}^{1}_{1}H\right) - \frac{E^{*}}{c^2} $$