

Chemical Engineering 412

Introductory Nuclear Engineering

Lecture 19

Nuclear Reactor Theory IV

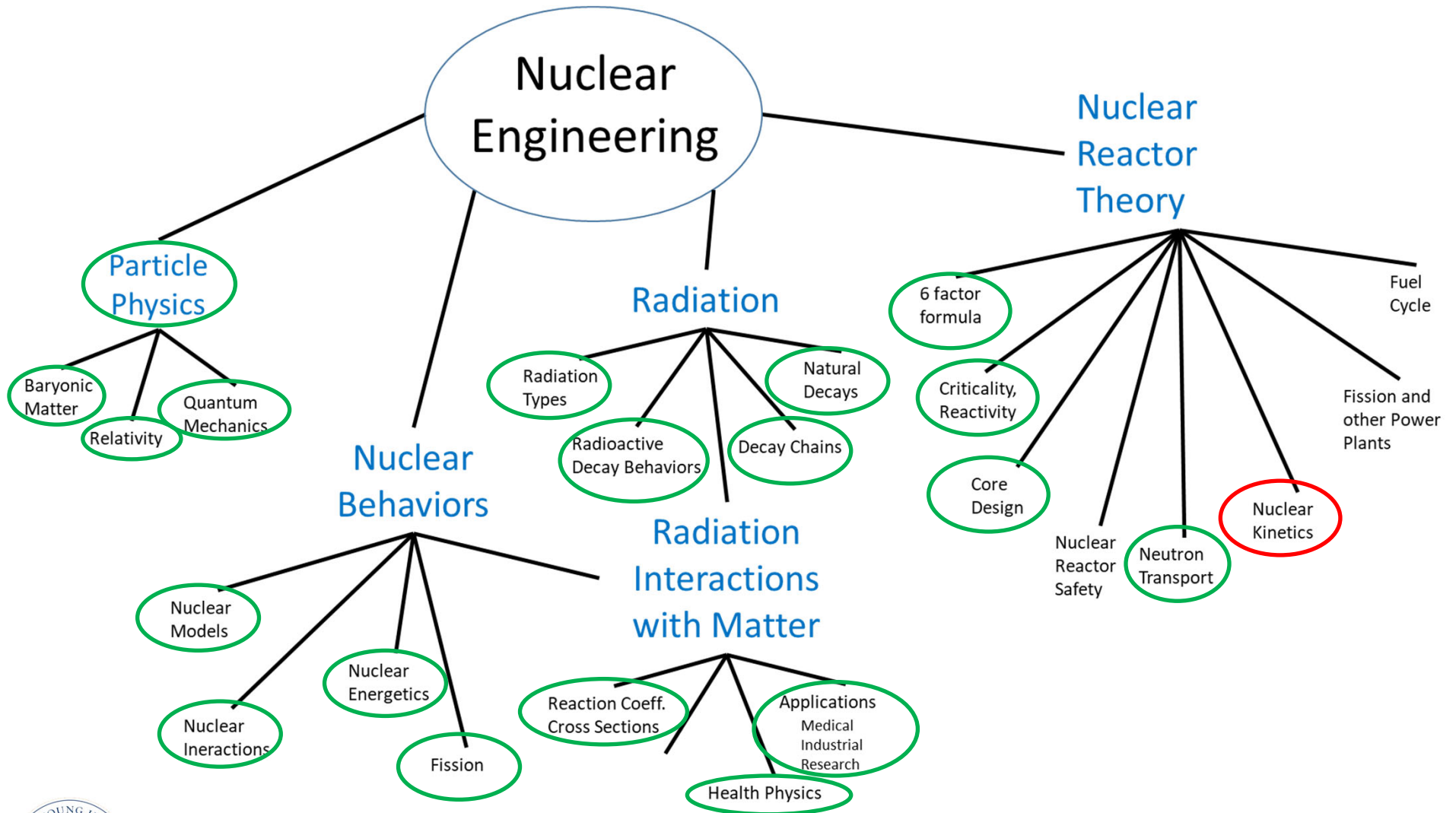
Nuclear Kinetics



Spiritual Thought



Roadmap



Objectives

- Understand the simple, “big picture” of kinetics
- Understand how delayed neutrons/precursors work
- Understand, recognize and do calculations relating to feedback coefficients
- Understand reactor “control strategies”
- Understand xenon peak phenomena



Power Plant Operation

- Try to achieve “steady state” operation at all times
- Set up core and fuel to provide minimal changes
- Despite this, still have three scales of change
 1. Short Term (Grid load transients)
 2. Intermediate Term (Fuel composition changes)
 3. Long Term (Burnup & Depletion)



Three Time Scales (short)

- Short Time Constant (load change)
 - An abrupt change in steam demand/load.
 - A load change is seen as:
 - Reactor pressure change in the BWR
 - Reactor temperature change in a PWR.
 - Higher loads lead to higher pressures/temperatures.
- Assumptions
 - Assumes a uniform multiplicative change everywhere
 - Spatial variations in time in the reactor are not considered
 - (This method is called point kinetics.)



Three Time Scales (intermediate)

- Intermediate Time Constant (core composition change)
 - Changing fission product concentrations
 - Generation rates and destruction/decay rates.
 - Many fission products have measurable thermal neutron cross sections $f = \frac{\Sigma_a^F}{\Sigma_a^F + \Sigma_a^{NF}(V^{NF}/V^F)(\phi^{NF}/\phi^F)}$
 - **Change** the value of k (and k_∞).
 - Point kinetics can be used if spatial variations in concentrations are negligible.
 - Otherwise, detailed spatial and temporal equations must be used!



Three Time Scales (long)

- Long Time Constant (Fuel Depletion)

- Treated as a series of steady-state problems

$$D\nabla^2\phi - \Sigma_a\phi = -\lambda\nu\Sigma_f\phi$$

- Two things are adjusted to maintain $\lambda = 1$ (i.e. $\lambda = 1/k$)

- Material buckling
- Reactor dimensions
- If $\lambda \neq 1$, the equation is not valid
 - Why?

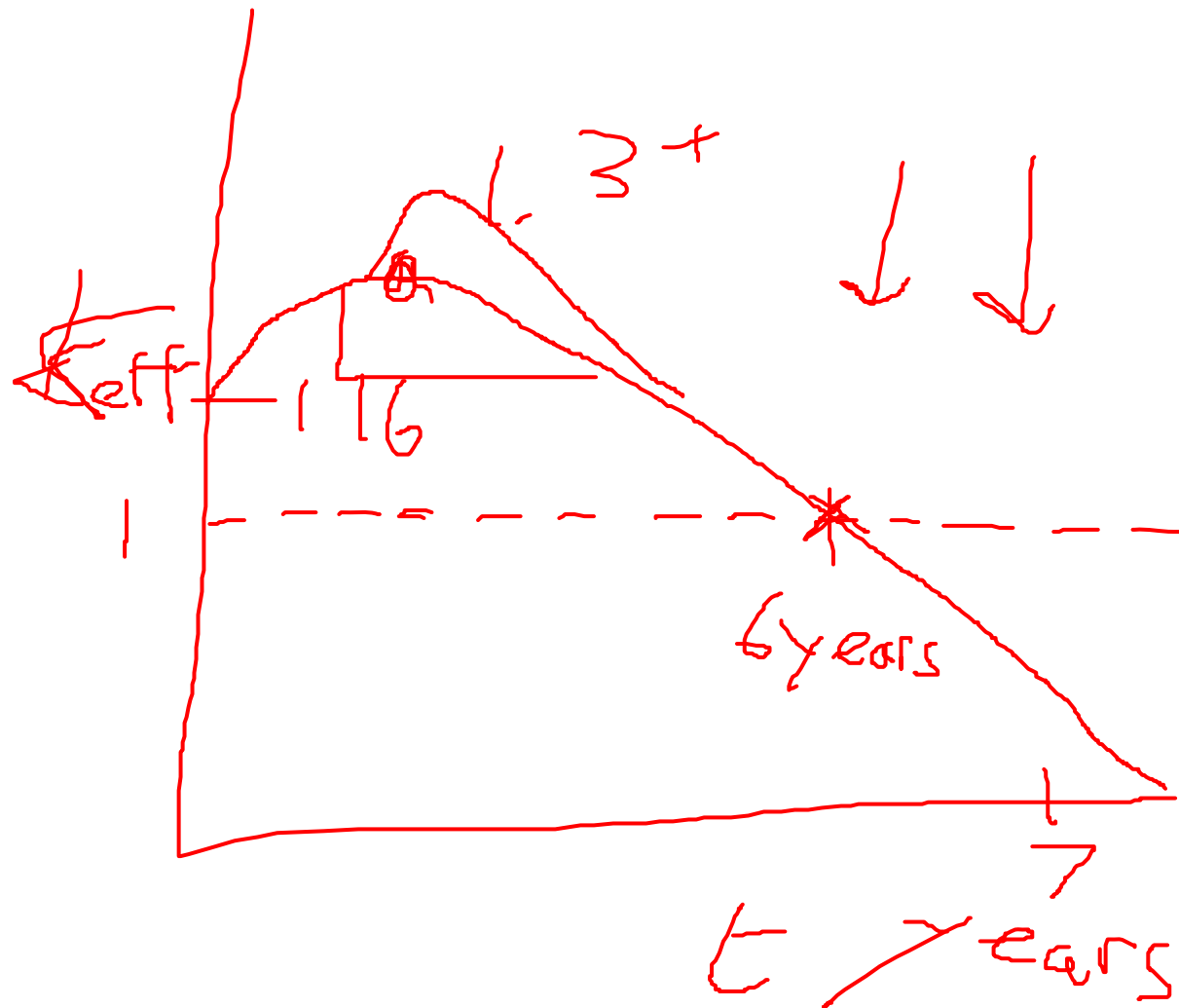
- In operating reactor, cannot change dimensions (much)

- k is adjusted slowly in time by changing chemical shim conc.
 - Chemical shim is an isotope that absorbs neutrons in the reactor
- k is also adjusted via the control rods.

- Managing fuel consumption is a classical example of this type of transient



K_{eff} vs. time curve for Power plant?



Reactivity and Worth

$$\rho \equiv \frac{k_{eff} - 1}{k_{eff}} = \frac{\delta k}{k_{eff}} \quad \text{reactivity } \rho \text{ and } \delta k$$

$$k(\$) \equiv \frac{\rho}{\beta} \quad \begin{array}{l} \beta \text{ is delayed neutron fraction} \\ \text{worth can be measured in units of } k(\$) \text{ or } k \end{array}$$

- cents?
- Percent Mil?



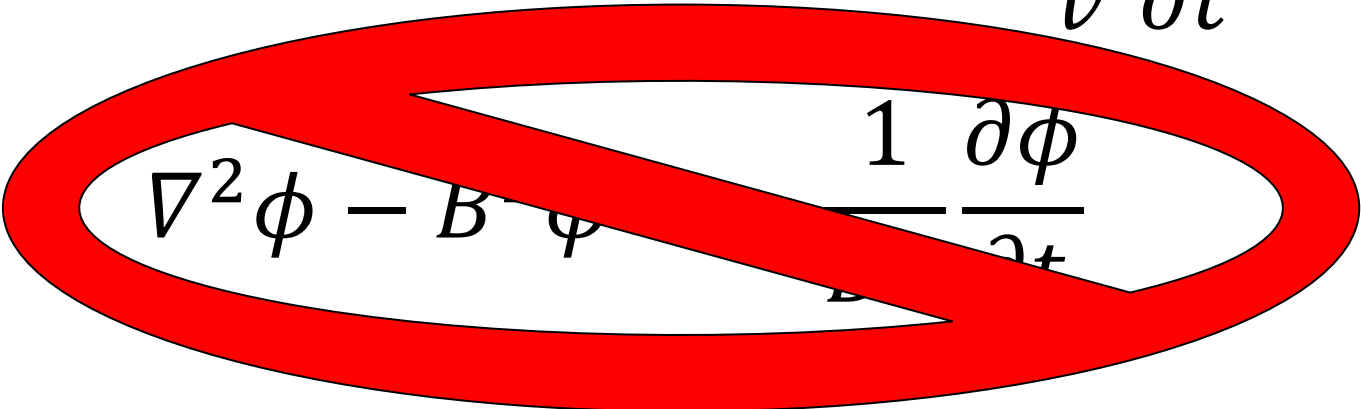
General Transient Problem

Mono-energetic neutrons

$$D\nabla^2\phi - \Sigma_a\phi + S = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

For a reactor, $S = \nu\Sigma_f\phi$

$$D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$


$$\nabla^2\phi - B^2\phi = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$



Prompt Neutrons

- Lifetime, l_p , is time between emission and absorption.
- Neutrons in thermal reactors:
 - Spend more time (most of l_p) in the thermal regime
 - Travel further as fast neutrons
- Average lifetime of a thermal neutron in an infinite reactor is the mean diffusion time, t_d , and is approximately the same as l_n in an infinite reactor. $l_p \approx t_d$

$$t_d = t_{dM}(1 - f)$$

TABLE 7.1 APPROXIMATE DIFFUSION TIMES FOR SEVERAL MODERATORS

Moderator	t_d , sec
H ₂ O	2.1×10^{-4}
D ₂ O*	4.3×10^{-2}
Be	3.9×10^{-3}
Graphite	0.017

*With 0.25% H₂O impurity.



In fast reactors, prompt neutron lifetimes are much shorter, on the order of 10^{-7} seconds

Simple Kinetics Model

$$\Delta n(t) \equiv \ell_p \frac{dn(t)}{dt} = (k_{eff} - 1)n(t)$$

$$\frac{dn(t)}{dt} = \frac{k_{eff} - 1}{\ell_p} n(t)$$

$$\Rightarrow n(t) = n(0) \exp\left(\frac{k_{eff} - 1}{\ell_p} t\right)$$

- For ^{235}U
 - $\ell_p = 2.1 \times 10^{-4} \text{ s}$
 - $k_{eff} - 1 = 0.001$
 - and $t = 1 \text{ s}$,
- $n/n^0 = 117$ (22,027 if $\ell' = 10^{-4}$ as in text)
- Far too rapid to control!!!



Reactors with delayed neutrons

Delayed neutrons: β is fraction of total neutrons

$$\bar{\ell}_p = (1 - \beta)\ell_p + \beta(\ell_p + \tau) \approx \ell_p + \beta\tau$$

τ is lifetime of delayed neutrons $= \frac{T_{1/2}}{\ln 2} \approx 12.8 \text{ s}$

For $\delta k \ll \beta$

$$\frac{n(t)}{n_0} = \exp\left(\frac{k_{eff} - 1}{\bar{\ell}_p} t\right) = \exp\left(\frac{t}{T}\right)$$
$$T = \frac{\bar{\ell}_p}{k_{eff} - 1} = \frac{\beta\tau}{\delta k}$$

- For ^{235}U , $T = 83 \text{ s}$, $k_{eff} - 1 = 0.001$,
- $n/n^0 = 1.012$
- This can be controlled!



Delayed Neutrons

TABLE 3.5 DELAYED NEUTRON DATA FOR THERMAL FISSION IN ^{235}U *

Group	Half-Life (sec)	Decay Constant (l_i, sec^{-1})	Energy (ke V)	Yield, Neutrons per Fission	Fraction (β_i)
1	55.72	0.0124	250	0.00052	0.000215
2	22.72	0.0305	560	0.00346	0.001424
3	6.22	0.111	405	0.00310	0.001274
4	2.30	0.301	450	0.00624	0.002568
5	0.610	1.14	—	0.00182	0.000748
6	0.230	3.01	—	0.00066	0.000273

Total yield: 0.0158

Total delayed fraction (β) 0.0065

*Based in part on G. R. Keepin, *Physics of Nuclear Kinetics*, Reading, Mass.: Addison-Wesley, 1965.



For 1-group model, $T_{\frac{1}{2}}$ for ^{235}U is about 8.87 s and τ is about 12.8 s.

Delayed Neutron Fractions

Group	^{235}U		^{233}U		^{239}Pu	
	Half-life (s)	fraction β_i	Half-life (s)	fraction β_i	Half-life (s)	fraction β_i
1	55.7	0.00021	55.0	0.00022	54.3	0.00007
2	22.7	0.00142	20.6	0.00078	23.0	0.00063
3	6.22	0.00127	5.00	0.00066	5.60	0.00044
4	2.30	0.00257	2.13	0.00072	2.13	0.00068
5	0.610	0.00075	0.615	0.00013	0.618	0.00018
6	0.230	0.00027	0.277	0.00009	0.257	0.00009
total	-	0.0065	-	0.0026	-	0.0021



Point Kinetic Equations

$$\frac{dn(t)}{dt} = \frac{\rho - \beta}{l} n(t) + \sum_{i=1}^G \lambda_i C_i(t) + S(t)$$
$$\frac{dC_i(t)}{dt} = \frac{\beta}{l} n(t) - \lambda_i C_i(t), i = 1, \dots, G$$



Reactivity Equation Solutions

$$\phi_T = \underbrace{A_1 \exp(\omega_1 t)}_{\substack{\text{dominant term} \\ \text{as } t \rightarrow \infty}} + \underbrace{A_2 \exp(\omega_2 t)}_{\text{approaches 0 rapidly}}$$

General solution for single group of delayed neutrons

$$T = \frac{1}{\omega_1}$$

Definition of reactor or stable period

$$\phi_T = \exp\left(\frac{t}{T}\right)$$

General solution for single group of delayed neutrons

$$\rho = \frac{\omega l_p}{1 + \omega l_p} + \frac{\omega}{1 + \omega l_p} \sum_{i=1}^6 \frac{\beta_i}{\omega + \lambda_i}$$

Reactivity equation for six group model



Power Changes

$$T = \frac{\beta\tau}{\delta k} = \frac{\beta\tau}{keff \cdot \rho} = \frac{\tau}{keff \cdot k(\$)} \sim \frac{\tau}{k(\$)}$$

- T = Reactor Period (units of time)
 - Time required to increase reactor power (or neutron flux) by 2.72
- Integrate solution with specific values (initial time power to final time/power)

$$\frac{t}{T} = \ln \left[\frac{P(t)}{P(0)} \right]$$



Reactivity Equation Solutions

$$T = \frac{\bar{l}}{k_{eff} - 1} = \frac{\bar{l}}{\delta k} = \frac{\beta\tau}{\delta k} =$$

Reactor period - The time required for a neutron population to change by a factor of e

$$k_{eff} = 1 + \delta k = 1 + \frac{\beta\tau}{T}$$

τ = Lifetime of delayed neutrons
~12.8s (U235)

$$T = \frac{\beta\tau}{\delta k} = \frac{\beta\tau}{k_{eff} - 1} = \frac{\beta\tau}{k_{eff}\rho} = \frac{\tau}{k_{eff}\rho(\$)} \approx \frac{\tau}{k(\$)}$$

$$\phi(t) = \exp\left(\frac{t}{T}\right)$$

Remember, Flux is proportional to power....

$$C \cdot P(t) = \exp\left(\frac{t}{T}\right) \quad \frac{t}{T} = \ln\left(\frac{P(t) \cdot C}{P(0) \cdot C}\right) \quad \frac{P(t)}{P(0)} = \exp\left(\frac{t}{T}\right)$$



Example

- Following a reactor scram in which all the control rods are inserted into a power reactor, how long is it before the reactor power decreases to 0.0001 of the steady-state power prior to shutdown?
(Assume a reactor period of -80 s)

$$T = -80 \text{ s} \quad \frac{P(t)}{P(0)} = .0001$$

$$t = T \ln \left(\frac{P(t)}{P(0)} \right)$$

$$t = 736.8 \text{ sec}$$



1-level Model Parameters

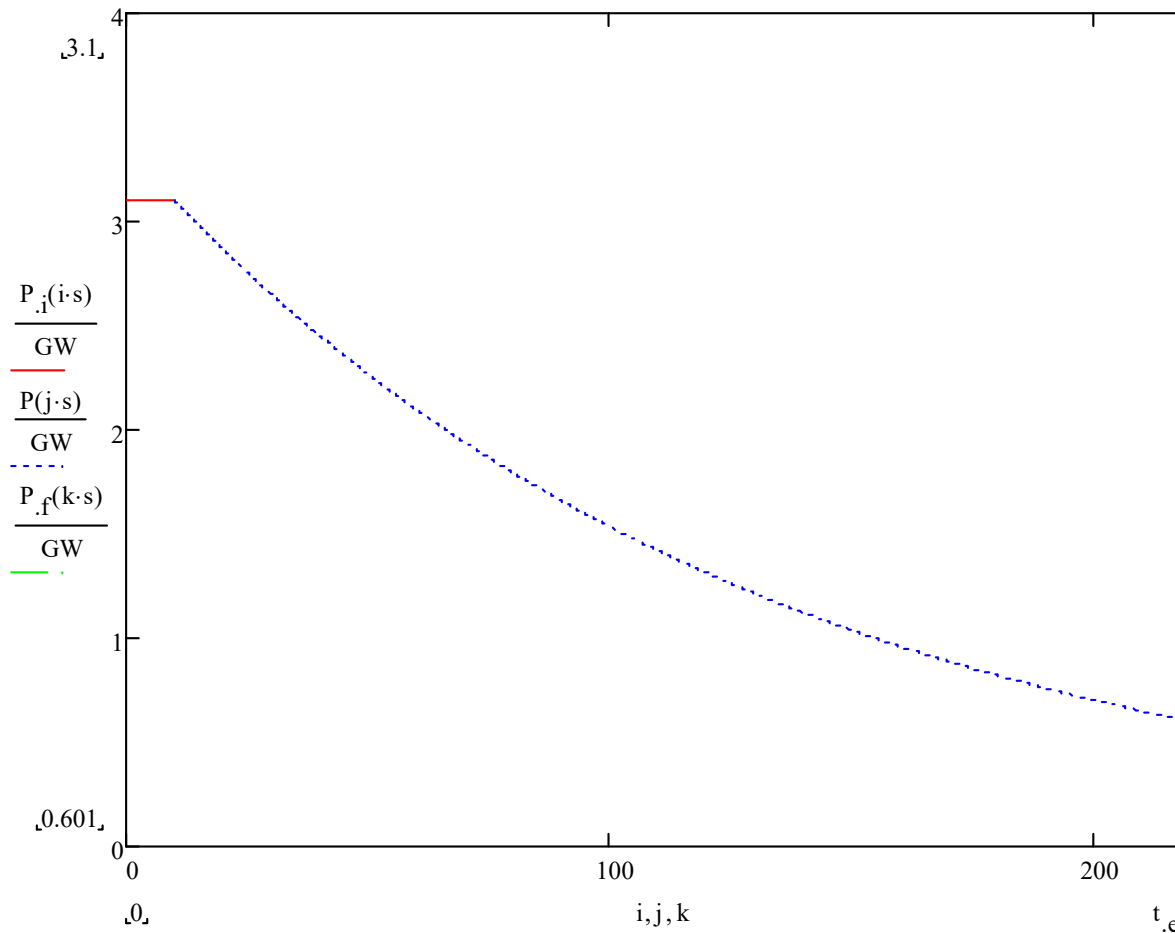
	β	$T_{\frac{1}{2},d}(s)$	$\tau_d(s)$
^{232}Th	0.0203	6.98	10.07
^{233}U	0.0026	12.4	17.89
^{235}U	0.0064	8.82	12.72
^{238}U	0.0148	5.32	7.68
^{239}Pu	0.002	7.81	11.27
^{241}Pu	0.0054	104.1	150.18
^{241}Am	0.0013	10	14.43
^{243}Am	0.0024	10	14.43
^{242}Cm	0.0004	10	14.43



Source: Laboratoire de Physique Subatomique et de Cosmologie

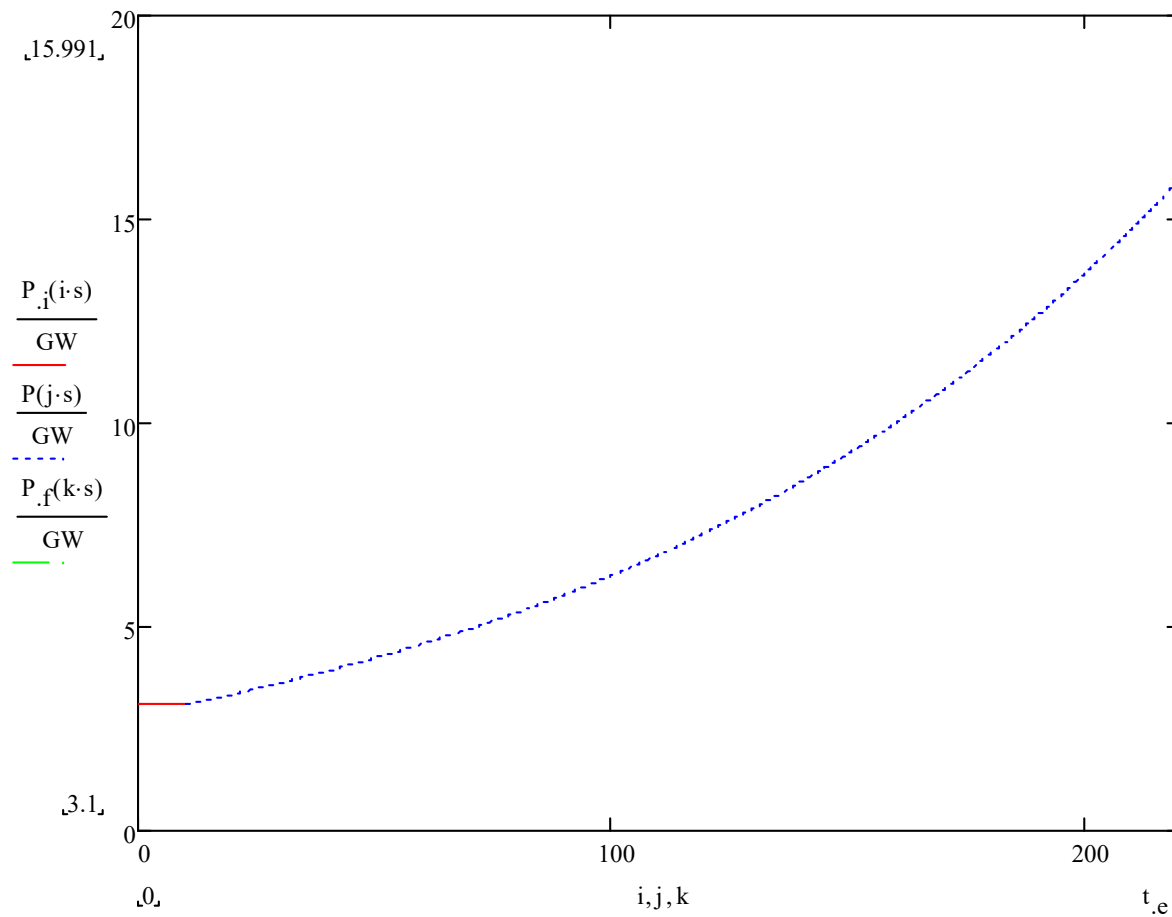
Exploration 1

- What if we add $-\$0.1$ to AP1000 core?



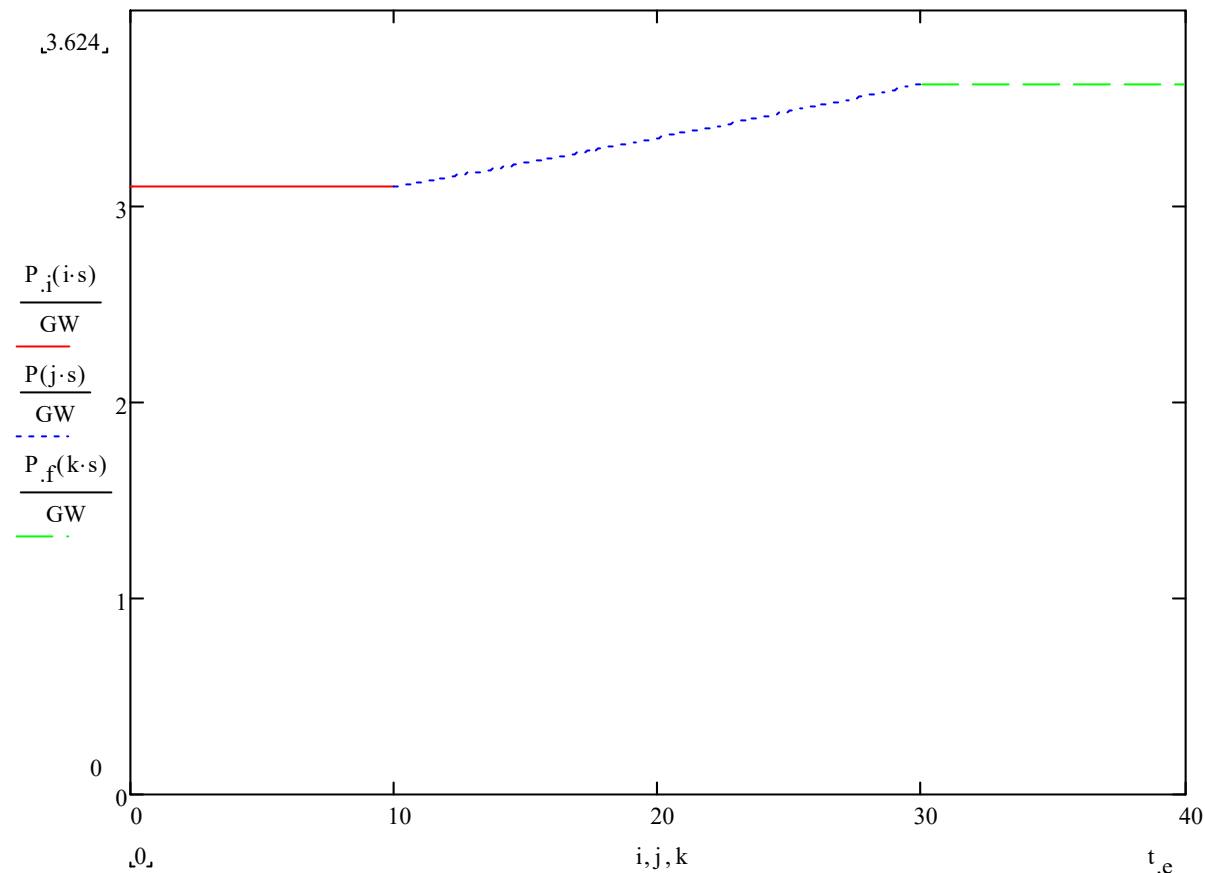
Exploration 2

- What if we add \$0.1 to AP1000 core?



Exploration 3

- What if we add \$0.1 to AP1000 core, then after 10 seconds we add -\$0.1?



Reality

