ChE 475

Statistics Review Problem

We are measuring the density of a cylinder and want the 95% confidence interval. Write the equation for the density of a cylinder in terms of mass (m), diameter (D), and length (L):

$$\rho = \frac{m}{V}, \quad V = \frac{\pi D^2}{4} L \quad \text{so} \quad \rho = \frac{m}{\pi D^2 L/4}$$

Now write the equation for propagation of error in terms of partial derivatives of m, D, and L:

$$\delta_\rho = \left|\frac{\partial \rho}{\partial m}\right| \delta_m + \left|\frac{\partial \rho}{\partial D}\right| \delta_D + \left|\frac{\partial \rho}{\partial L}\right| \delta_L$$

The trick is now to get δ_m, δ_D, and δ_L.

m is read from a scale that reads 967.2 g. The uncertainty for m (i.e., δ_m) is then 0.1 g, or 10^{-4} kg.

D is from calipers, with a digital scale that has a minimum reading of 0.1 mm. Therefore, the uncertainty for D (i.e., δ_D) is 0.1 mm, or 10^{-4} m.

L is from a tape measure, with markings to 1/32 inch. Therefore, δ_L = 1/32 inches, or 7.94×10^{-4} m.

The measurements were D = 2.00 cm and L = 30.00 cm. What is the mean value of the density?

$$\rho = \frac{.9672 \text{ kg}}{\pi (.02 \text{ m})^2 (0.3 \text{ m})/4} = 10{,}262 \text{ kg/m}^3$$

Now compute the values of the partial derivatives:

$$\frac{\partial \rho}{\partial m} = \frac{1}{\pi D^2 L/4} = \frac{4}{\pi (.02)^2 (0.3)} = 10{,}610 \text{ m}^{-3} \text{ (units?)}$$

$$\frac{\partial \rho}{\partial D} = \frac{4m}{\pi L}(-2D^{-3}) = -1.026 \times 10^{+6} \text{ (units?) } \frac{\text{kg}}{\text{m}^4}$$

$$\frac{\partial \rho}{\partial L} = \frac{4m}{\pi D^2}(-L^{-2}) = -3.42 \times 10^{+4} \text{ (units?) } \frac{\text{kg}}{\text{m}^4}$$

Now put everything together:

$$\delta_\rho = (10{,}610 \text{ m}^{-3})(10^{-4} \text{ kg}) + \left|-1.026 \times 10^6 \frac{\text{kg}}{\text{m}^4}\right|(10^{-4} \text{ m}) + \left|-3.42 \times 10^4 \frac{\text{kg}}{\text{m}^4}\right|(7.94 \times 10^{-4} \text{ m}) = 130.84 \frac{\text{kg}}{\text{m}^3}$$

This is the maximum error. We want the 95% confidence interval. First back out the standard deviation.

$s_\rho = \delta_\rho/2.5$ = 52.33 kg/m^3

Then, the 95% uncertainty is 1.96 s = 102.6 kg/m^3.

Part 2

What if the cylinder was not perfectly smooth? Here is a set of diameter measurements at different locations along the cylinder:

Meas #	D (cm)
1	2.03
2	2.01
3	1.98
4	1.99
5	2.00

Find D_{mean} and standard deviation (s_D): $D_{mean} = 2.002$ cm

$s_D = 0.0192$ cm

Remember $\delta_D = 2.5\ s_D =$ 4.81×10^{-4} m.

Comment: we propagate maximum error, and then correct at last for 95% confidence interval.