Chemical Engineering 612

Introductory Nuclear Engineering

Lecture 13

Fuel Temperature Profile

Spiritual Thought

"There is another way to look at your problem of crowded time. You can see it as an opportunity to test your faith. The lord loves you and watches over you. He is all-powerful, and He promised you this: 'But seek ye first the kingdom of God, and his righteousness; and all these things shall be added unto you.' This is a true promise. When we put God's purposes first, He will give us miracles. If we pray to know what He would have us do next, He will multiply the effects of what we do in such a way that time seems expanded. He may do it in different ways for each individual, but I know from long experience that He is faithful to His word."

President Henry B Eyring

Heat Transport

$$\nabla k_f \nabla T + q''' = 0$$

- Convection
  - Fluid to surface/surface to fluid
  - $\dot{Q} = hA(T_1 - T_2)$
- Conduction
  - Transport through solid material
  - $\dot{Q} = A\nabla kT = kA\frac{dT}{dr}$
- Radiation
  - Electromagnetic heat transport →two surfaces
  - $\dot{Q}_{1\to 2} = \sigma A_1 F_{1\to 2}(T_1^4 - T_2^4)$

Fuel Thermal Properties

Property	U	UO_2	UC	UN
Theoretical density at room temp (kg/m^3)	19.04×10^3	10.97×10^3	13.63×10^3	14.32×10^3
Metal density* (kg/m^3)	19.04×10^3	9.67×10^3	12.97×10^3	13.60×10^3
Melting point (°C)	1133	2800	2390	2800
Stability range	Up to 665°C†	Up to m.p.	Up to m.p.	Up to m.p.
Thermal conductivity average 200–1000°C (W/m°C)	32	3.6	23 ($UC_{1.1}$)	21
Specific heat, at 100°C (J/kg °C)	116	247	146	—
Linear coefficient of expansion (/°C)		10.1×10^{-6} (400–1400°C)	11.1×10^{-6} (20–1600°C)	9.4×10^{-6} (1000°C)
Crystal structure	Below 655°C: α, orthorhombic Above 770°C: γ, body-centered cubic	Face-centered cubic	Face-centered cubic	Face-centered cubic
Tensile strength, (MPa)	344–1380‡	110	62	Not well defined

*Uranium metal density in the compound at its theoretical density.

†Addition of a small amount of Mo, Nb, Ti, or Zr extends stability up to the melting point.

‡The higher values apply to cold-worked metal.

From Nuclear Systems I

UO2 thermal conductivity (I)

- $\bar{\bar{k}}$ is tensor
- However, typically assume isotropic $\bar{\bar{k}}$ → k
- K dependencies:
  - Temperature
    - k(T) decreases with T initially to 1750˚C
    - k(T) increases slightly with T after 1750˚C
  - Porosity (P):
    - P = $1 - \frac{\rho}{\rho_{TD}}$
    - Biancharia approximation
    - $k = \frac{(1-P)}{1+(\alpha_2-1)P} k_{TD}$, α_2 = 1.5 for spherical, larger for asymmetric

UO_2 thermal conductivity (II)

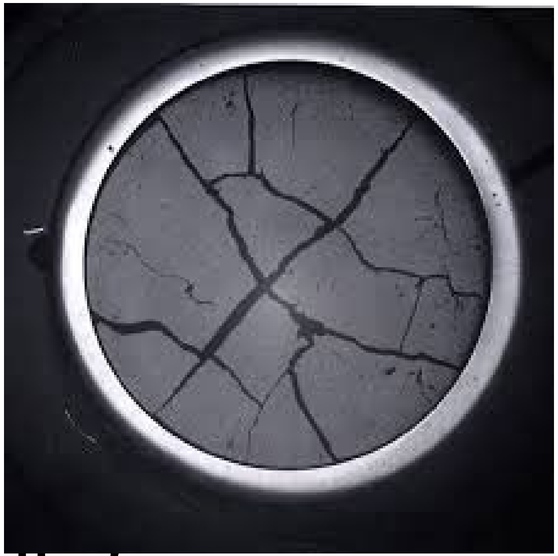
- Oxygen to metal atomic ratio
  - Theoretical (stoichiometric) = 2
  - Departure from theoretical occurs during burnup, both +/- decrease k
- Plutonium content
  - Increased Pu content decreases k
- Pellet cracking
  - INL developed relationship for k (given later)
- Burnup
  - Changes porosity, composition, stoichiometry, fission product introduction, sintering, etc.
  - Small → 3% in LWRs, larger in Fast Reactors
  - For MSR, this can be controlled by U233 addition

Other UO_2 Fuel Properties

- Fission Gas Release
  - Plenum included in fuel rod
  - Initially in pellet, released to plenum based on temperature (see Nuclear Systems I)
- Melting Point
  - 2840˚C, starts at solidus temperature moves up to liquidus temperature. (see text)
- Specific Heat
  - Varies greatly over temperature
  - Plays key role in accident behavior

Fuel Irradiation

- Changes k dramatically
- $k_e = k_{UO_2} - (0.0002189 - 0.050867X + 5.6578X^2)$
- $X = (\delta_{hot} - 0.014 - 0.014\delta_{cold})\left(\frac{0.0545}{\delta_{cold}}\right)\left(\frac{\rho}{\rho_{TD}}\right)^8$

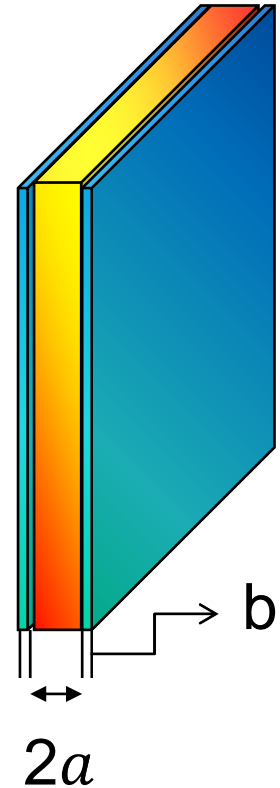


- Where
- δ_{hot} = calculated hot gap witdh for uncracked fuel (mm)
- δ_{cold} = calculated cold gap width (mm)

Temperature Profiles in a Plate (I)

In fuel

$$\nabla k_f \nabla T + q''' = 0 \qquad -k_f \frac{d^2T}{dx^2} = q'''$$



BC1: $T(0) = T_{max}$

BC2: $\left.\dfrac{dT}{dx}\right|_{x=0} = 0$

$$\Rightarrow T = -\frac{q'''}{2k_f}x^2 + C_1 x + C_2 = T_{max} - \frac{q''' x^2}{2k_f}$$

$$T_s = T_{max} - \frac{q''' a^2}{2k_f}$$

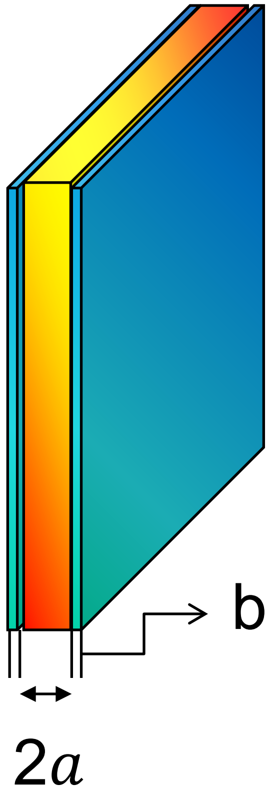
Temperature Profiles in a Plate (II)

In cladding

$$-k_c \frac{d^2T}{dx^2} = 0$$

BC1: $T(a) = T_s$

BC2: $T(a+b) = T_c$



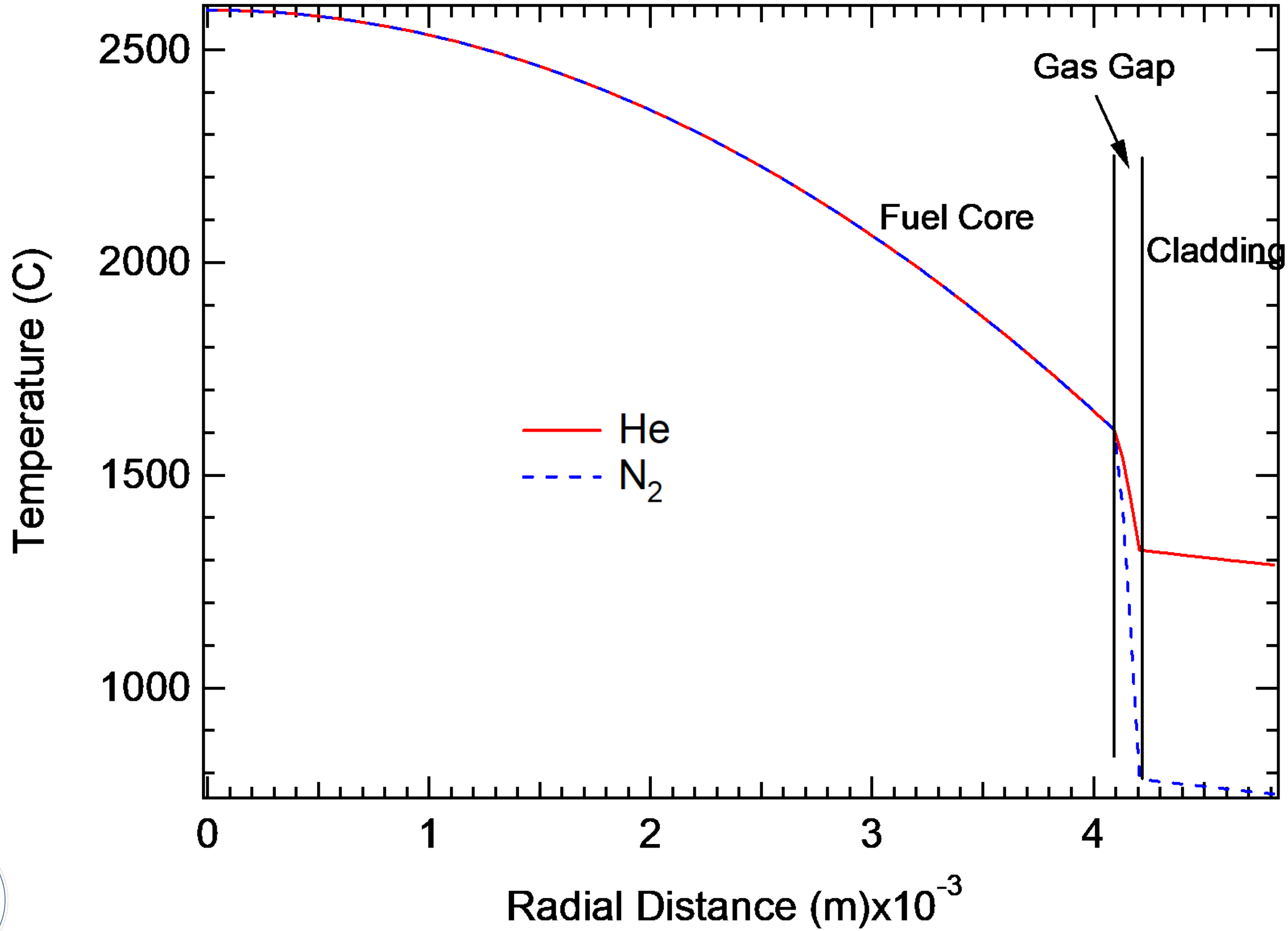
$$\Rightarrow T = C_1 x + C_2 = T_s - \frac{x-a}{b}(T_s - T_c)$$

$$q''' = \frac{T_{max} - T_c}{\frac{q}{2k_f A} + \frac{b}{k_c A}} = \frac{T_{max} - T_c}{\sum R}$$

Thermal Resistances

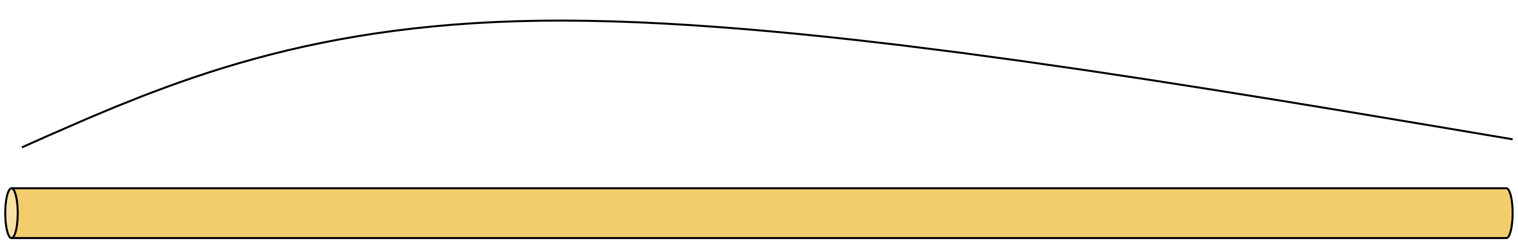
- Conduction
  - Plane Wall – $R_{cond} = \frac{x}{kA}$
  - Cylindrical Wall – $R_{cond} = \frac{\ln\left({}^{r_2}/_{r_1}\right)}{2\pi Lk}$
  - Spherical Wall – $R_{cond} = \frac{r_2 - r_1}{4\pi r_1 r_2 k}$
- Convection – $R_{conv} = \frac{1}{hA}$
- Radiation – $R_{rad} = \frac{1}{h_{rad}A}$
  - $h_{rad} = \varepsilon\sigma(T_1^2 + T_2^2)(T_1 + T_2)$

Temperature Profile in Cylinder



Flux Profile

geometry	Buckling (B^2)	Flux	A	$\Omega = \dfrac{\phi_{max}}{\phi_{av}}$
$plate - 1D$	$\left(\dfrac{\pi}{a}\right)^2$	$A\cos\dfrac{\pi x}{a}$	$1.57P / aE_R\Sigma_f$	1.57
$plate - 3D$	$\left(\dfrac{\pi}{a}\right)^2 + \left(\dfrac{\pi}{b}\right)^2 + \left(\dfrac{\pi}{c}\right)^2$	$A\cos\dfrac{\pi x}{a}\cos\dfrac{\pi y}{b}\cos\dfrac{\pi z}{c}$	$3.85P / VE_R\Sigma_f$	3.88
$cylinder - 1D$	$\left(\dfrac{2.405}{R}\right)^2$	$A\,\mathrm{J}_0\left(\dfrac{2.405}{R}\right)$	$0.738P / R^2E_R\Sigma_f$	2.32
$cylinder - 3D$	$\left(\dfrac{2.405}{R}\right)^2 + \left(\dfrac{z}{H}\right)^2$	$A\,\mathrm{J}_0\left(\dfrac{2.405}{R}\right)\cos\dfrac{\pi z}{H}$	$3.63P / VE_R\Sigma_f$	3.64
$sphere$	$\left(\dfrac{\pi}{R}\right)^2$	$\dfrac{A}{r}\cos\dfrac{\pi r}{R}$	$P / 4R^2E_R\Sigma_f$	3.29

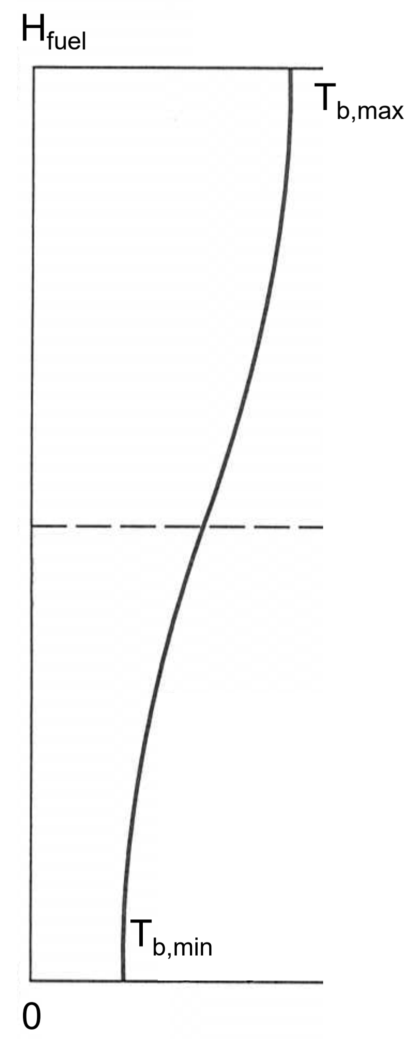


Power Distribution

- In reality, not perfect curve… depends on:
  - Geometry (not perfect cylinder)
  - Irradiation Cycle (enrichment distribution)
  - Refueling Schemes
  - Control Rods
- Interested in highest q''' →max power
  - Power peaking factor
    - Radial - $\frac{Peak\ FA\ power}{Average\ FA\ power}$
    - Local - $\frac{Peak\ pin\ power\ (in\ FA)}{Average\ pin\ power\ (in\ FA)}$
    - Axial - $\frac{Peak\ linear\ power}{Average\ linear\ power}$

Bulk Flow and Boundary T

- Previous problem → oversimplified!
  - Tc is rarely constant!
  - q'' is more likely to be constant
  - Coolant is ultimate heat sink
  - Coolant temperature changes!
- 2 ways to relax assumptions
  1. Assume coolant removes heat from fuel rod cladding (4th resistance)
  2. Assume non-linear temperature profile in coolant



Example

- Calculate the peak temperature in a sodium (Cp=1.23J/g*K, μ = 0.072cp, ρ=927kg/m^3) fast reactor core (1000 MW) with 10,000 fuel rods (UO2, Zr metal clad) in which the radial factor is 1.1 and the local peaking factor is 1.2 assume a 1.0 cm fuel pellet, a 0.01cm gap, and a 0.1 cm clad thickness. For simplicity, assume no axial peaking, and coolant inlet temperature of 400C with a flow rate of 5,000 kg/s. The core flow area is roughly 1.5 m^2, while the equivalent diameter of each channel is 1.5 cm.

Heat Transfer Correlations

- For convective cooling:
  1. Find coolant flow regime (laminar vs. turbulent)
  2. Determine appropriate Pr number of fuel $\left(\mathrm{Pr} = \frac{\mu C_p}{k}\right)$
  3. Pick a correct heat transfer correlation (samples given below)
  4. Calculate h $\left(Nu = \frac{hD_{eq}}{k}\right)$
  5. Evaluate the heat transfer resistance and add to sum of resistances in calculation $\left(q'' = \frac{\Delta T}{\sum resistances}\right)$

Seider and Tate: $Nu = 0.023Re^{0.8}Pr^{0.4}\left(\frac{\mu_w}{\mu}\right)^{.014}, 0.7 < Pr < 120, Re > 10{,}000,$

Dittus-Boelter: $Nu = 0.023Re^{0.8}Pr^{0.4}, 0.7 < Pr < 100, Re > 10{,}000, \frac{L}{D} > 60, heated, \mu_w \sim \mu$

$Nu = 0.023Re^{0.8}Pr^{0.4}, 0.7 < Pr < 100, Re > 10{,}000, \frac{L}{D} > 60, cooled, \mu_w \sim \mu$

Colburn: $Nu = 0.023Re^{0.8}Pr^{0.333}, High\ \mu,\ 0.7 < Pr < 100, Re > 10{,}000, \frac{L}{D} > 60, \mu_w \sim \mu$

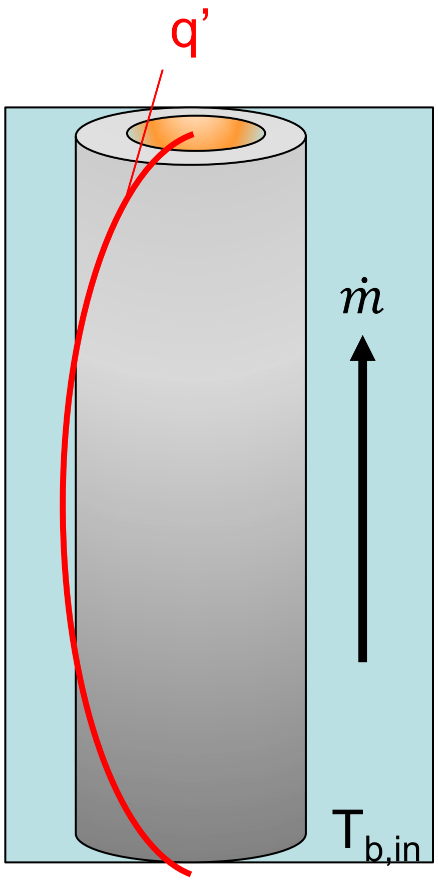
Metallic Fluids: $Nu = 7 + 0.025Pe^{0.8}, constant\ q''\ \&\ q',\ circular\ tube, fully\ developed$

$Nu = 5.0 + 0.025Pe^{0.8}, constant\ T_{w,axial}\ \&\ q''\ circular\ tube, developed$

$Nu = 5.25 + 0.0188Pe^{0.8}, concentric\ annuli, \frac{D_1}{D_2} > 1.4, developed, q'' = C$

Rod Analysis with non-constant q'

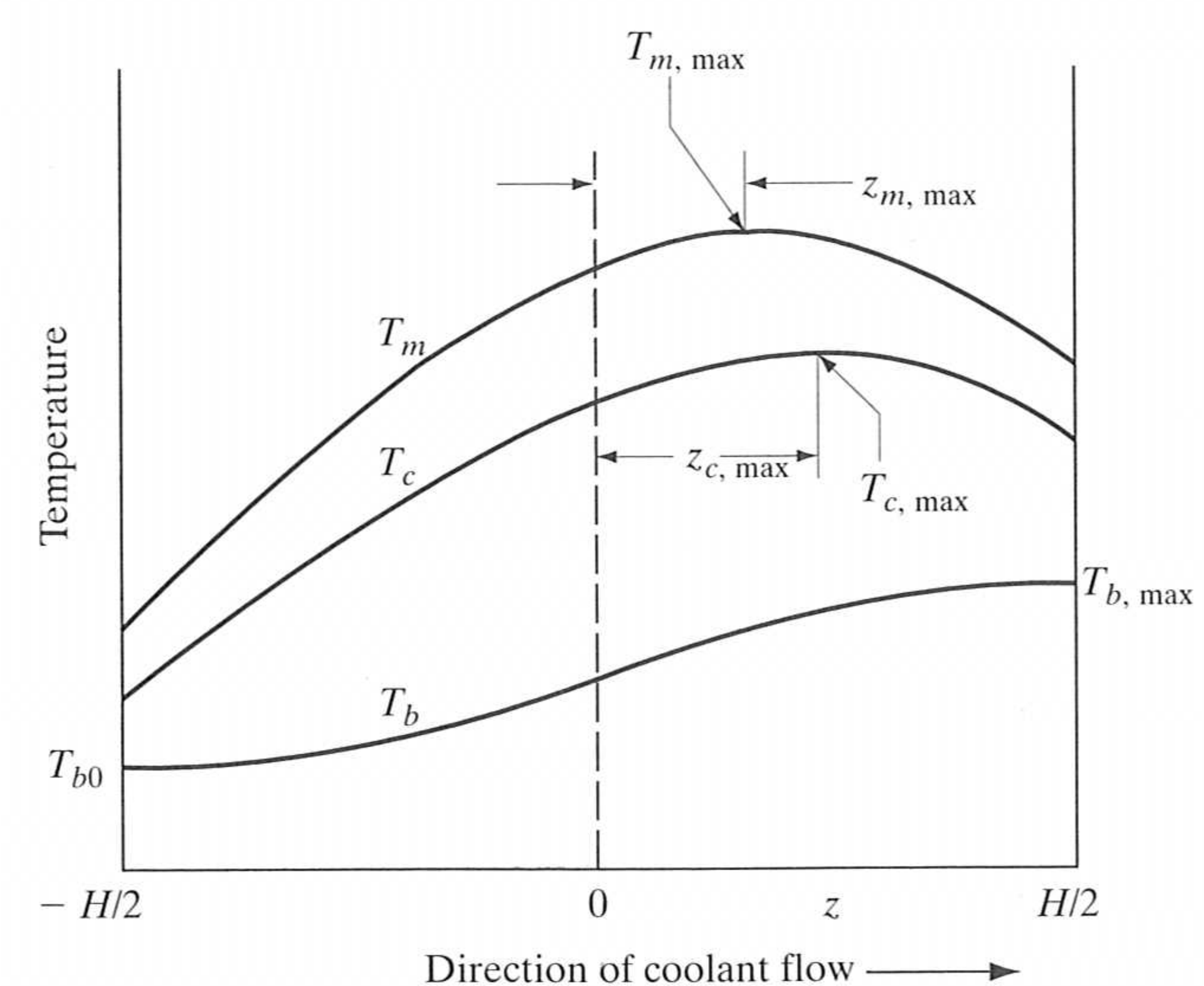
- Now q' = $q'(z) = q'_{max} \sin\left(\frac{\pi z}{L}\right)$
  - Steady state
  - Know $\dot{m}$, $T_{b,in}$
- Develop expressions for:
  - $T_{max}(z)$
  - $T_{fuel}(z)$
  - $T_{clad}(z)$
  - $T_{bulk}(z)$



Bulk Flow

- Energy increase in Flow:
  - $q'(z) = \dot{m}\frac{dh}{dz}$
    - If no phase change, $dh = C_p dT_b$
  - $\dot{m}\frac{dC_pT_b}{dz} = q'(z) = q'_{max}\sin\left(\frac{\pi z}{L}\right)$
  - Solve for T_b
- $T_b(z) = \frac{q'_{max}}{\dot{m}C_p}\frac{L}{\pi}\left[1 - \cos\left(\frac{\pi z}{L}\right)\right]$
- Now apply same method, but $T_b = T_b(z)$

Typical Temperature Profiles



Subscripts *b*, *c*, and *m* represent bulk, cladding, and middle, respectively.

Decay Heat

- Decay of fission products
  - Fuel
  - Moderator
  - time reactor is in operation
  - power
  - time reactor has been shut down, etc
    - For UO_2 fuel, water-moderated reactor
- $\frac{P}{P_o} = 0.066[(\tau - \tau_s)^{-0.2} - \tau^{-0.2}]$