Chemical Engineering 693R

Reactor Design and Analysis

Lecture 5

Introductory Neutronic Theory

Spiritual Thought

"As we consider various choices, we should remember that it is not enough that something is good. Other choices are better, and still others are best… Some of our most important choices concern family activities. Many breadwinners worry that their occupations leave too little time for their families. There is no easy formula for that contest of priorities. However, I have never known of a man who looked back on his working life and said, "I just didn't spend enough time with my job."

Elder Richard G. Scott

Neutron Interactions

- Elastic scattering (n,n) – collision with no reaction and no change in total kinetic energies. Energy neutral.
- Inelastic scattering (n,n') – collisions with energy absorption by nucleus. endoergic
- Radiative capture (n,γ) – Capture of neutron by nucleus followed by γ-ray emission. exoergic.
- Charged particle reactions (n,α) – Neutron reaction to form α particles or protons. endoergic and exoergic.
- Neutron producing reactions (n,*x*n) – Reactions with a net increase in neutrons. endoergic. (n,2n) important for ^{2}H and ^{9}Be.
- Fission (n,) forms multiple products – Nucleus forms daughters. Generally exoergic.

Flux and Current

- Neutron Flux
  - $nv = \phi$
  - Note – v is scalar speed, n is neutron density (n/cm^3)
  - Neutrons passing through area in ANY direction
  - $\widehat{R}_i = \phi N \sigma_i$

- Neutron Current
  - $J_x = -D\frac{d\phi}{dx} \rightarrow J = -D\nabla\phi$
  - Direction dependent, vector
  - $D = \frac{\lambda_{tr}}{3}$
  - $\lambda_{tr} = \frac{1}{\Sigma_{tr}} = \frac{1}{\Sigma_s(1-\bar{\mu})}$
  - $\bar{\mu} = \frac{2}{3A}$

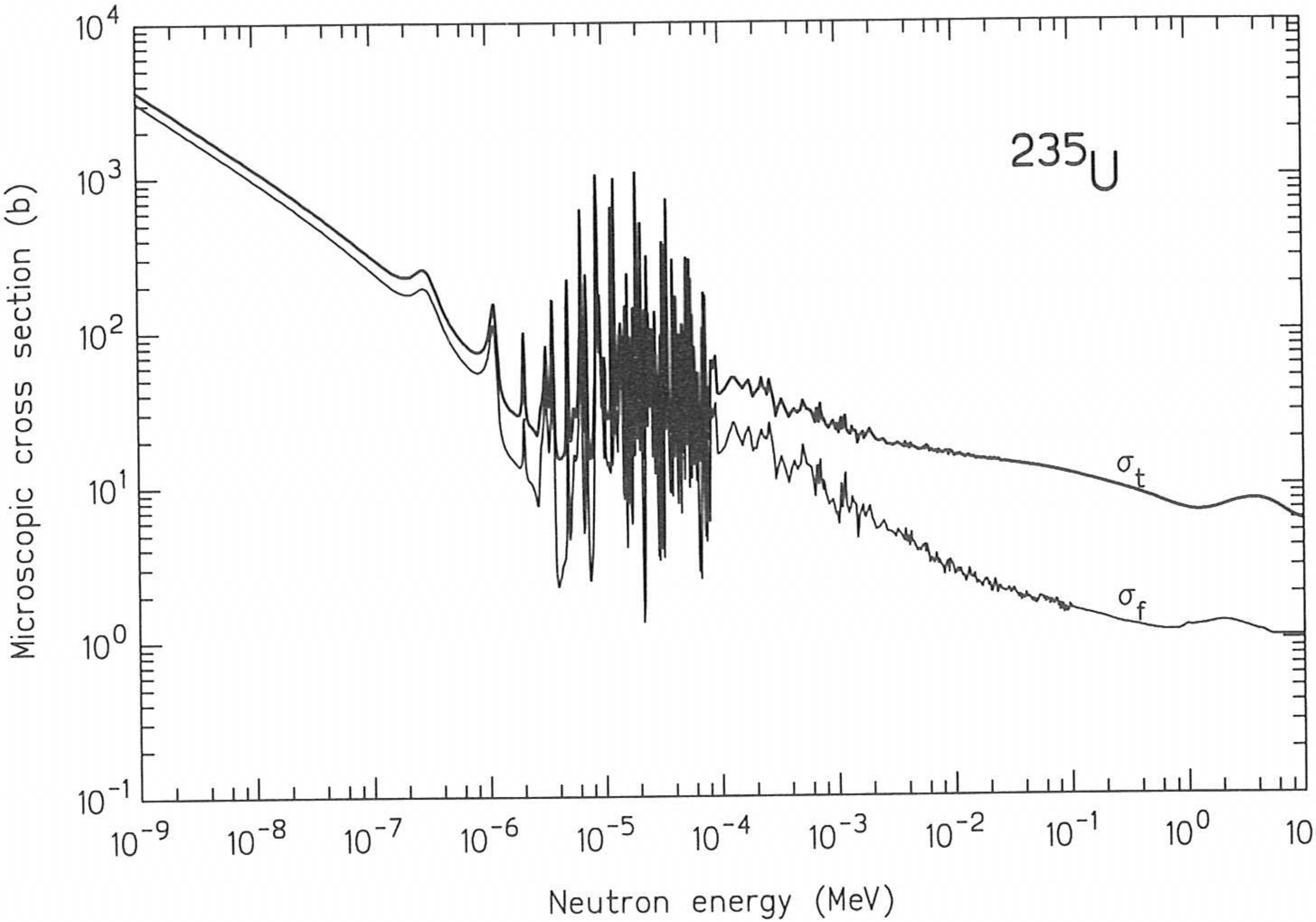
Microscopic Cross Section

- Probability of interaction is proportional to the concentration of interaction sites/atoms

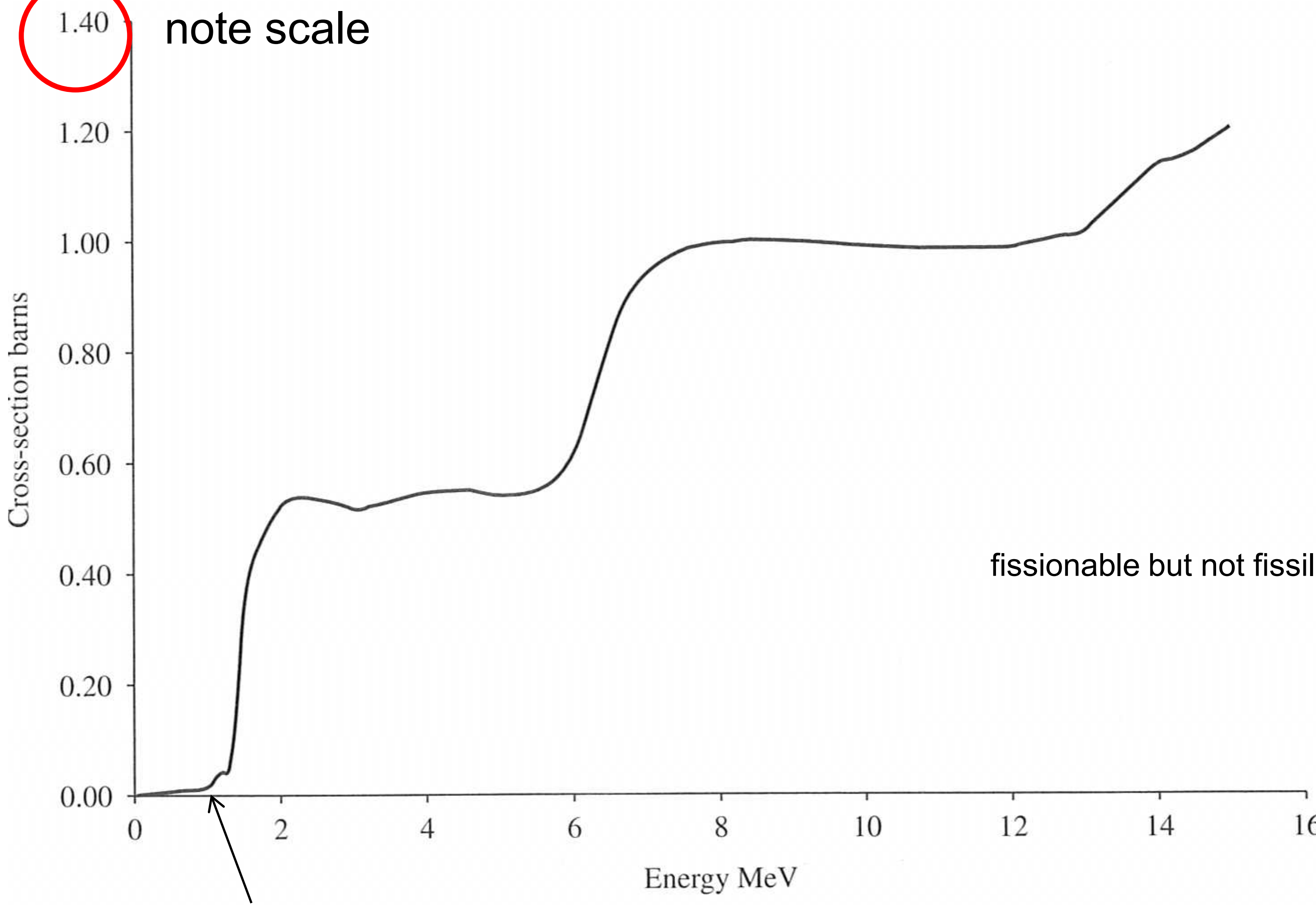
$$\sum_i = N\sigma_i = \sigma_i \frac{\rho N_a}{A}$$

- σ_i = microscopic cross section, has units of L^2
- $\boldsymbol{N}$ = Number/atom density
- $\boldsymbol{\rho}$ = Mass density
- $\boldsymbol{N_a}$ = Avagadro's number
- $\boldsymbol{A}$ = Atomic mass of the medium

Cross section over entire range



Fission Cross Section of ^{238}U

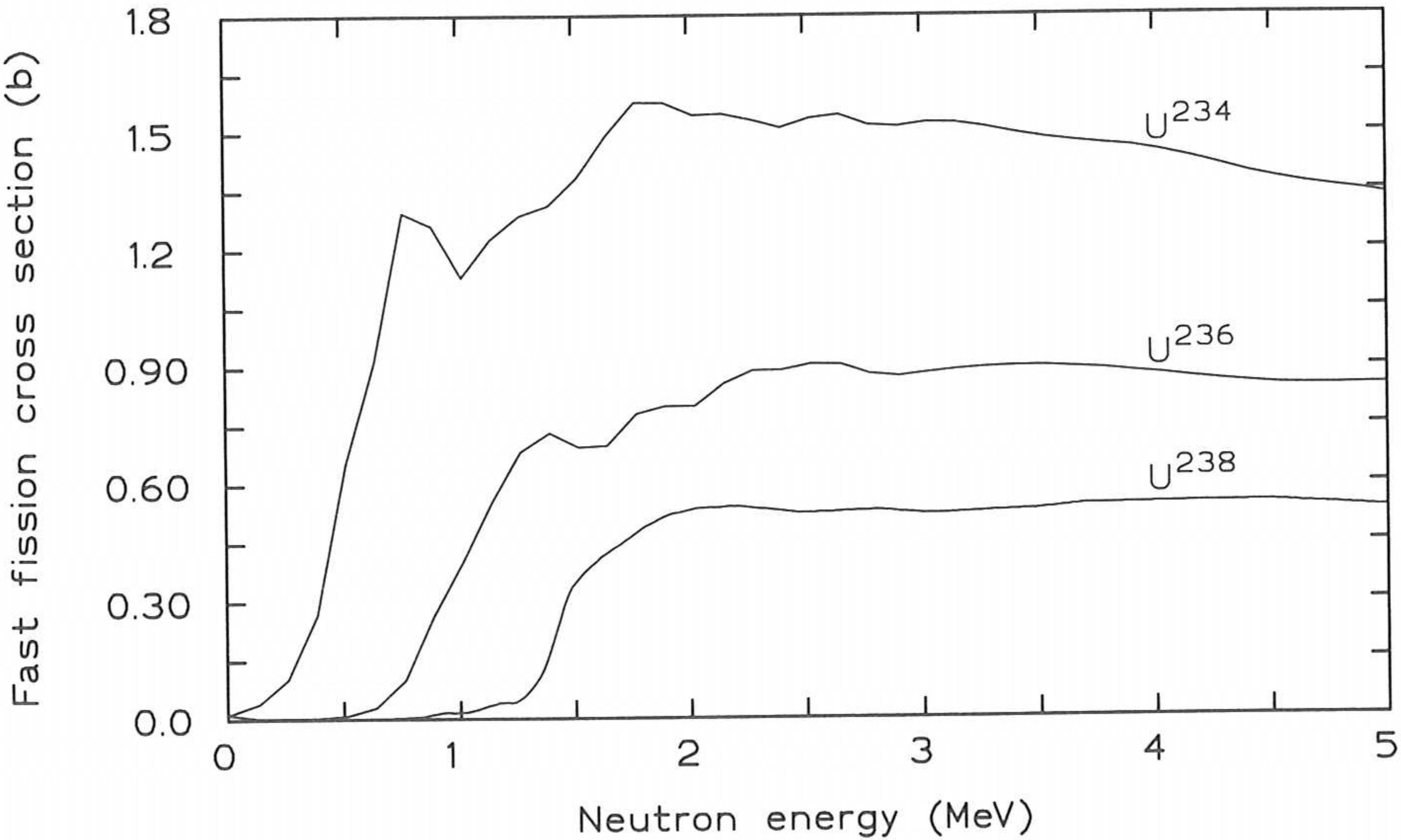


note scale

fissionable but not fissile

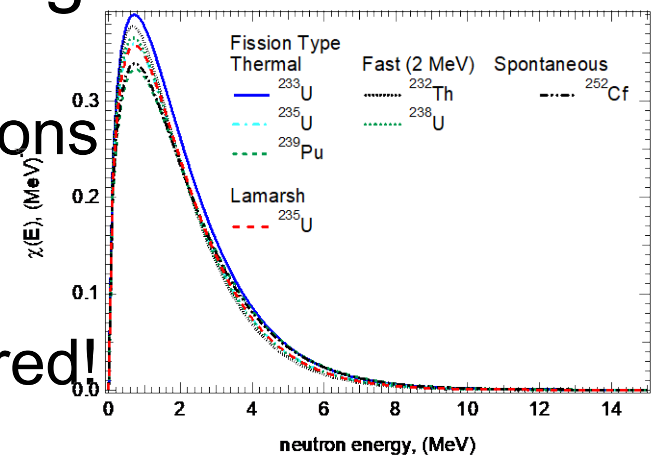
threshold energy (> resonance region energy)

Fissionable Cross Sections



Energy Distribution

- Neutrons have widely varying energies
  - $10 MeV+$, Fast neutrons
  - 2200 m/s or .025 eV, Thermal neutrons
  - Everything in between
- $\sigma = \sigma(E)$
- Thus, neutron energy must be considered!
  - $\widehat{R}_i = \phi N \sigma_i \rightarrow \mathrm{d}\widehat{R}_i(E) = \phi(E) N \sigma_i(E)$
  - $\widehat{R}_i(E) = \int_0^V \int_0^\infty \phi(E,r) N(r) \sigma_i(E,r)\, dE dV$
  - i= $\mathrm{ab}sorption\ (a), fission\ (f), scatter\ (s), etc.$

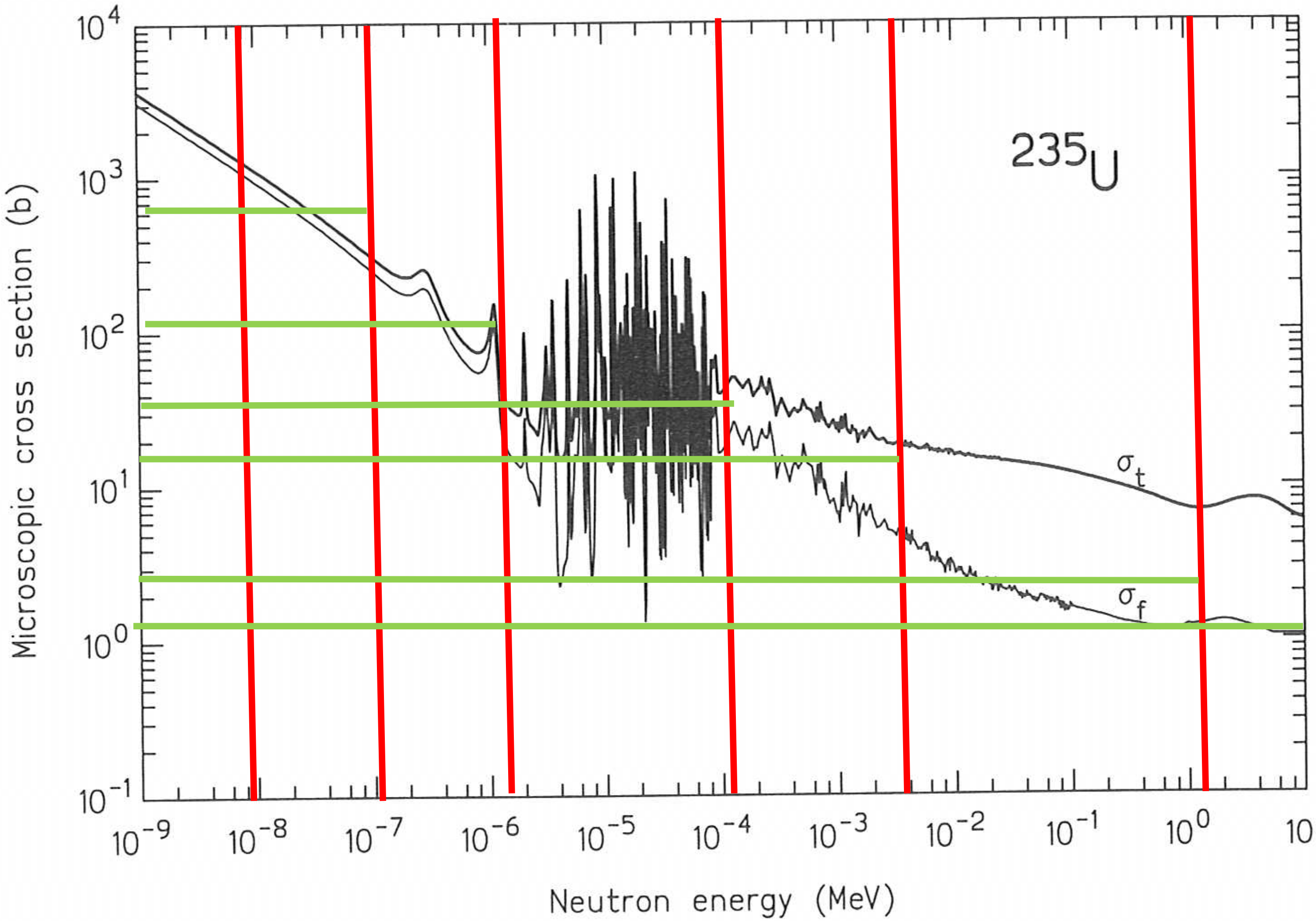


- Heart of neutronics is two-fold:

1. Find the cross section of any material at any particle energy for all particles
2. Find the neutron flux at any given location in the core

#1 has been done and is catalogued in ENDF/B-VII.1 tables

Cross section – multiple energies



One-group Reactor Equation

Mono-energetic neutrons (Neutron Balance)

$$D\nabla^2\phi - \Sigma_a\phi + s = -\frac{1}{v}\frac{\partial\phi}{\partial t}$$

v is neutron speed

For reactor, $s = \nu\Sigma_f\phi$

ν is neutrons/fission

In eigenfunction form and at steady state

$$D\nabla^2\phi - \Sigma_a\phi + \frac{\nu}{k}\Sigma_f\phi = 0$$

$$\Rightarrow \nabla^2\phi - \frac{\Sigma_a - \frac{\nu}{k}\Sigma_f}{D}\phi = \nabla^2\phi + B^2\phi = 0$$

Material Buckling

$$\nabla^2 \phi = -B^2 \phi$$ eigenfunction form of reactor equation

$$\nabla^2 \phi + B^2 \phi = 0$$ one-group, steady-state, reactor equation

$$B^2 = \frac{\frac{\nu}{k}\Sigma_f - \Sigma_a}{D}$$ material buckling = (neutron generation – absorption)/diffusion

$$k = \frac{\nu \Sigma_f \phi}{D B^2 \phi + \Sigma_a \phi} = \frac{\nu \Sigma_f}{D B^2 + \Sigma_a}$$ reactor multiplication factor

multiplication factor = neutron generation rate/(leakage + absorption)

Perspective

- Previous equations show
  - how to solve for neutron flux profile ϕ as a function of space
- First find solutions to the reactor equations
  - 1D, 2D, or 3D
  - Then find dimensions for a critical reactor
- Assumptions:
  - Bare, homogeneous reactors
  - Constant (special and temporal) properties
  - None are valid but, but help to develop insight into reactor operations
- Because source terms are proportional to the flux, the generally inhomogeneous differential equations are now homogeneous equations.

Bare Slab Reactor Solution

$$\frac{d^2\phi}{dx^2} = -B^2\phi$$ reactor equation

$$\phi\left(\frac{\tilde{a}}{2}\right) = \phi\left(-\frac{\tilde{a}}{2}\right) = \phi'(0) = 0$$ boundary conditions

$$\phi(x) = A\cos(Bx) + C\sin(Bx)$$ general solution

$$C = 0$$ from symmetry or by substitution

$$\phi(x) = A\cos(Bx)$$

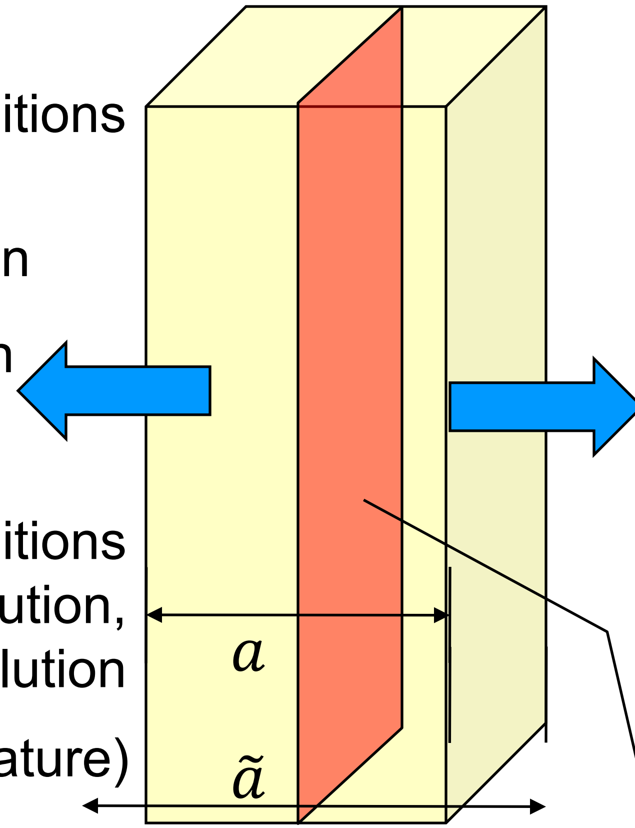
$$\phi(\tilde{a}/2) = 0$$

$$\Rightarrow B_n = \frac{n\pi}{\tilde{a}}$$

Eigenvalues from boundary conditions – all n important in transient solution, only n=1 important for steady solution

$$B_1^2 = -\frac{1}{\phi}\frac{d^2\phi}{dx^2}$$

B_1^2 is buckling (prop. to flux curvature)



Infinite plane indicates no net flux from sides

The constant A is as yet undetermined and relates to the power. There are different solutions to this problem for every power level.

Bare Slab Reactor Power

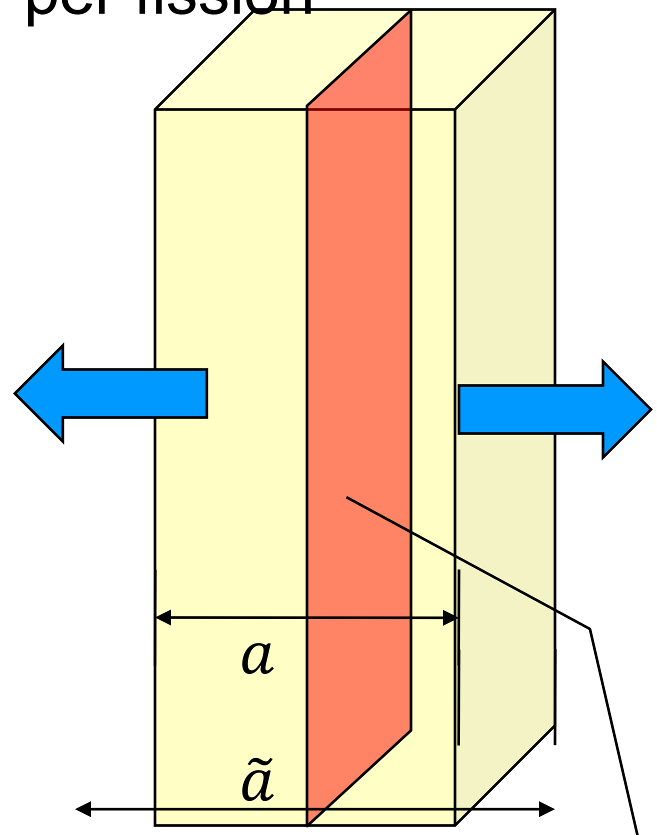
$$P = E_R \Sigma_f \int_{-a/2}^{a/2} \phi(x)\,dx$$

E_R is the recoverable energy per fission

$$P = \frac{2A\tilde{a}E_R\Sigma_f}{\pi}\sin\left(\frac{\pi a}{2\tilde{a}}\right)$$

$$\phi(x) = \frac{\pi P}{2\tilde{a}E_R\Sigma_f \sin\left(\frac{\pi a}{2\tilde{a}}\right)}\cos\left(\frac{\pi x}{\tilde{a}}\right)$$

Power Scales with flux !



Infinite plane indicates no net flux from sides