Chemical Engineering 612

Reactor Design and Analysis

Lecture 7
Criticality Theory

Spiritual Thought

Moroni 10:5

And by the power of the Holy Ghost ye may know the truth of all things.

Reflected Reactors

$$\nabla^2\phi_c + B^2\phi_c = 0$$ core transport equation

$$B^2 = \frac{k_\infty - 1}{L_c^2}$$ core materials properties

$$\nabla^2\phi_r - \frac{1}{L_r^2}\phi_r = 0$$ reflector transport equation

$$\phi_c = A\frac{\sin Br}{r} + C\frac{\cos Br}{r}$$ general solution for core

$$\phi_c = A\frac{\sin Br}{r}$$ flux must be finite at the center

$$\phi_r = A'\frac{\exp\left(\frac{-r}{L_r}\right)}{r} + C'\frac{\exp\left(\frac{r}{L_r}\right)}{r}$$ general solution for the reflector

$$\phi_r = A'\frac{\exp\left(\frac{-r}{L_r}\right)}{r}$$ flux must be finite as *r* increases

Reflected Reactors

$$\phi_c(R) = \phi_r(R)$$

fluxes equal at core-reflector interface

$$\vec{J}_c(R) \bullet \vec{n} = \vec{J}_r(R) \bullet \vec{n}$$

$$D_c \phi_c'(R) = D_r \phi_r'(R)$$

current densities also equal

$$A \frac{\sin BR}{R} = A' \frac{\exp(-R/L)}{R}$$

equate fluxes and current densities

$$AD_c\left(\frac{B\cos BR}{R} - \frac{\sin BR}{R^2}\right) = -A' D_r \left(\frac{1}{RL_r} + \frac{1}{R}\right)\exp\left(\frac{-R}{L_r}\right)$$

$$D_c\left(B\cot BR - \frac{1}{R}\right) = -D_r\left(\frac{1}{L_r} + \frac{1}{R}\right)$$

divide current density by flux equation

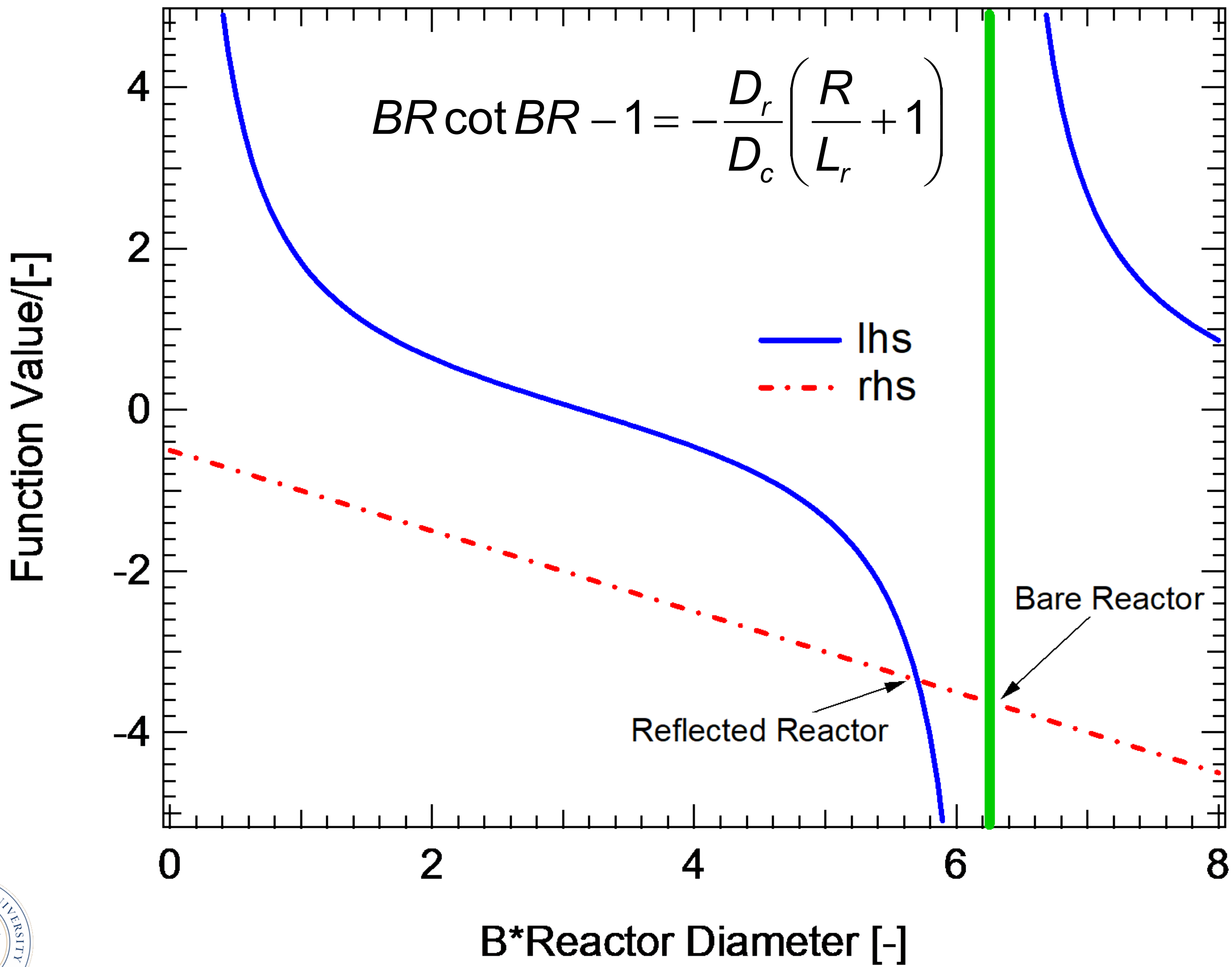
$$\boxed{BR\cot BR - 1 = -\frac{D_r}{D_c}\left(\frac{R}{L_r} + 1\right)}$$

critical equation for reflected reactor
(transcendental equation)

$$B\cot BR = -\frac{1}{L_r}$$

critical equation when $D_r = D_c$
(not transcendental in R)

Reflected Reactors < Bare Reactors



Determine Remaining Unknown

$$A' = A\exp\left(\frac{R}{L_R}\right)\sin BR$$

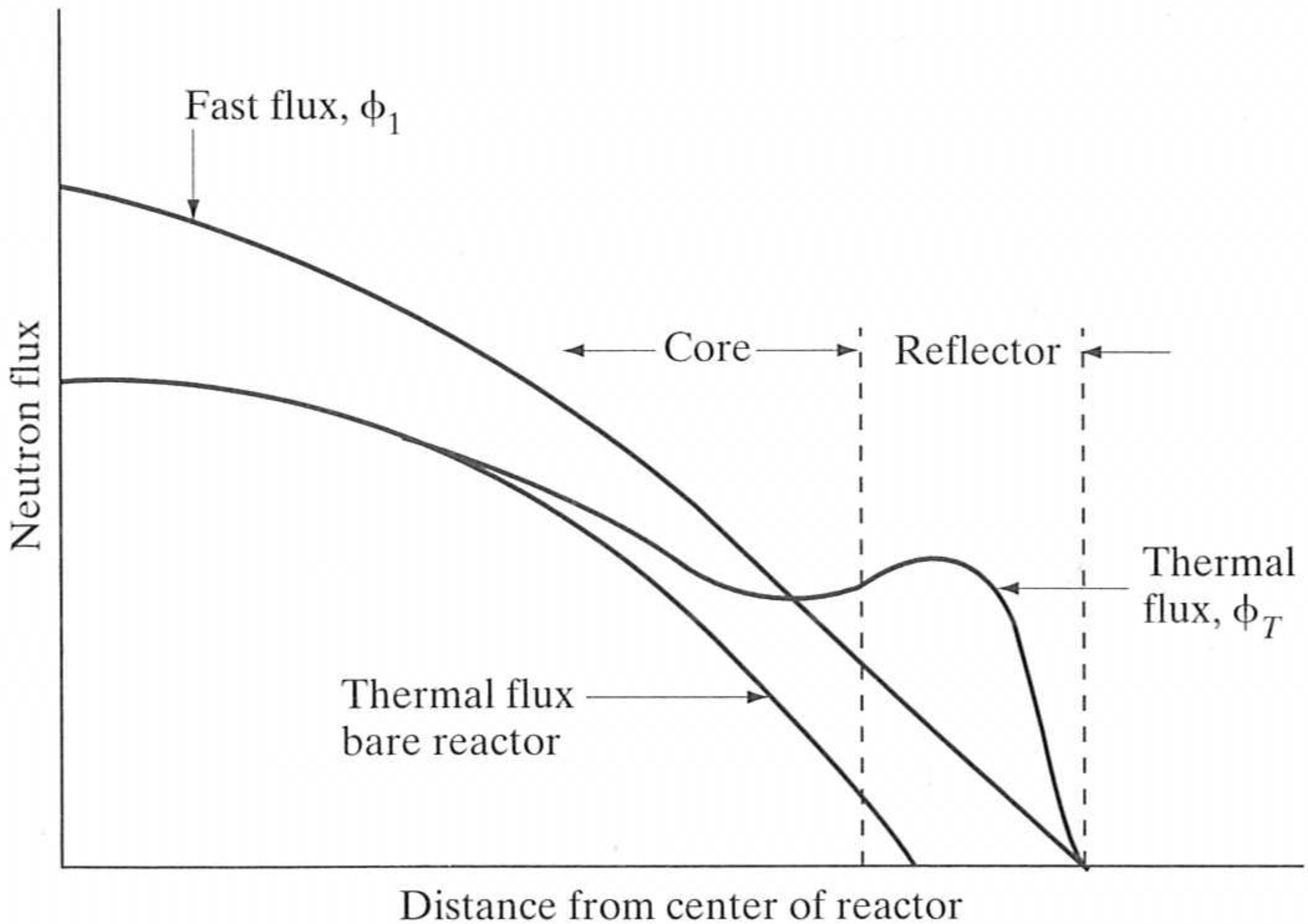
$$P = E_R\Sigma_f\int_0^R \phi_c \, d\forall$$

$$P = 4\pi E_R\Sigma_f A\int_0^R r\sin Br\,dr$$

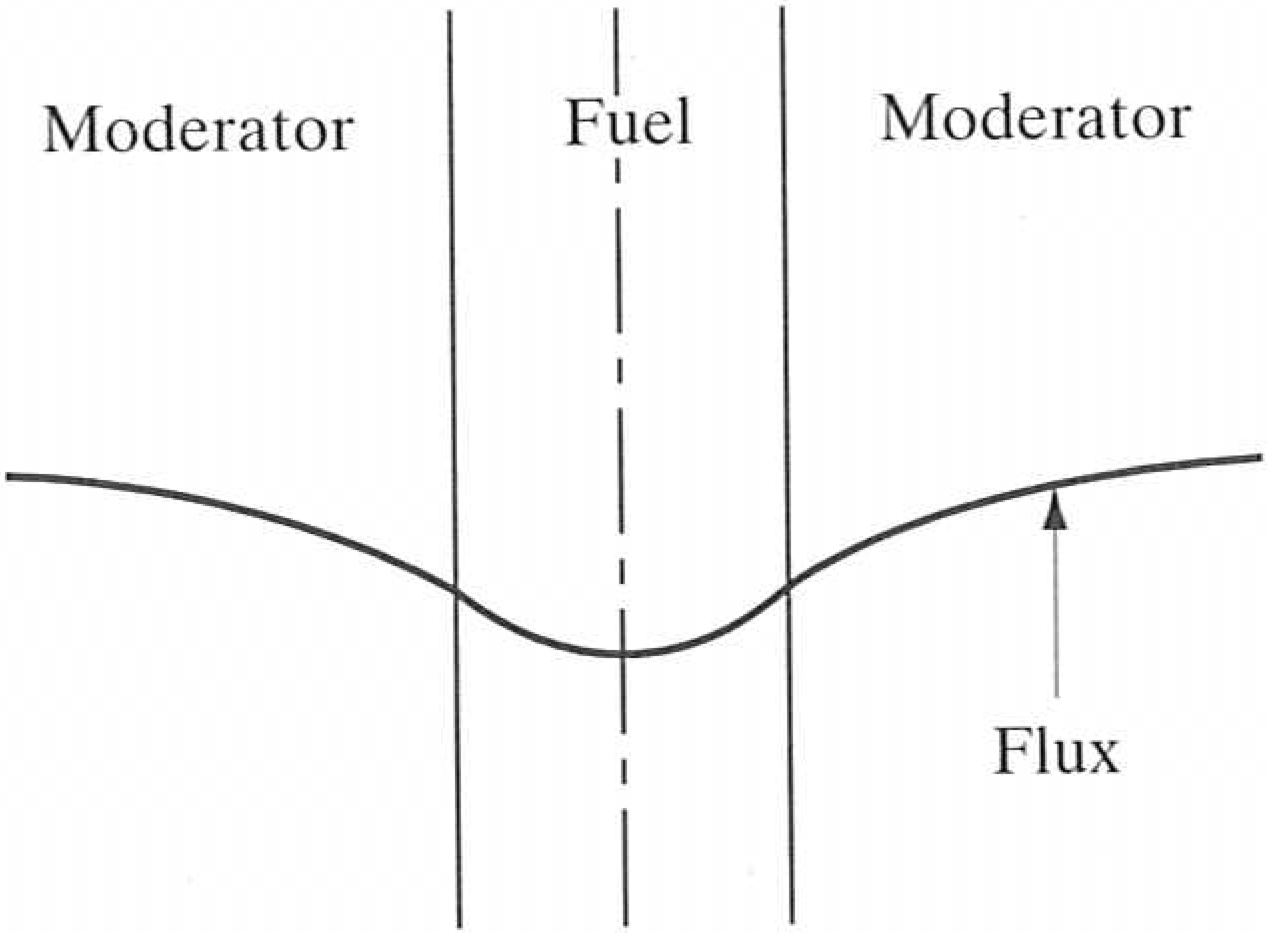
$$= \frac{4\pi E_R\Sigma_f A}{B^2}\left(\sin BR - BR\cos BR\right)$$

$$A = \frac{PB^2}{4\pi E_R\Sigma_f\left(\sin BR - BR\cos BR\right)}$$

Flux Comparisons



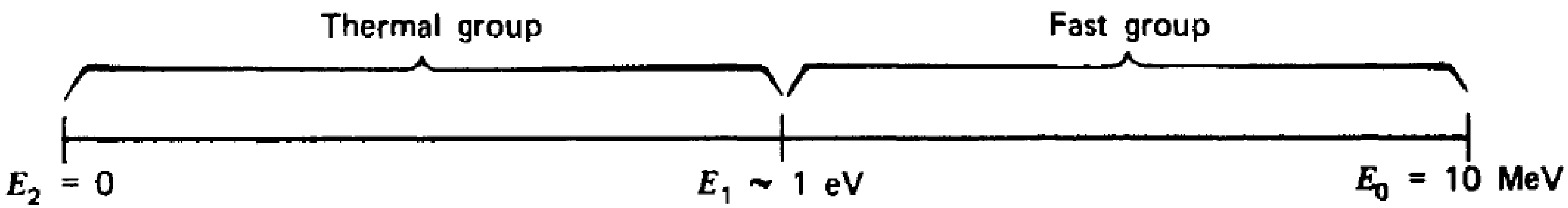
Thermal Flux Variations



Heterogeneous vs. homogeneous

- Heterogeneous cores change the reactor parameters :
  - $\boldsymbol{p}$ resonance escape probability
    - increases significantly
    - neutrons slow primarily in the moderator
    - no (or controlled amounts of) highly absorbing nuclides.
  - $\boldsymbol{\epsilon}$ fast fission
    - Increases slightly
    - fast neutrons are primarily surrounded by fissionable and fissile nuclides
  - $\boldsymbol{f}$ thermal utilization at fixed fuel loading ($\boldsymbol{N^F/N^{NF}}$)
    - Lower in heterogeneous reactor
    - Thermal neutron flux in fuel rod is less than that in moderator
  - $\boldsymbol{\eta}$ thermal fission factor
    - unchanged
    - depends only on the type of fuel
  - P_{NL}^{f}, P_{NL}^{th} Leakage probabilities
    - Unchanged
    - Depend primarily on reactor shape and size

Two Group Multiplication Factor (I)



$$\phi_2(\mathbf{r},t) = \int_{E_2}^{E_1} dE\,\phi(\mathbf{r},E,t) \equiv \text{thermal flux.}$$

$$\phi_1(\mathbf{r},t) = \int_{E_1}^{E_0} dE\,\phi(\mathbf{r},E,t) \equiv \text{fast flux,}$$

$$\chi_2 = \int_{E_2}^{E_1} dE\,\chi(E) = 0$$

$$\chi_1 = \int_{E_1}^{E_0} dE\,\chi(E) = 1,$$

$$S_{f_1} = \nu_1 \Sigma_{f_1}\phi_1 + \nu_2 \Sigma_{f_2}\phi_2 \qquad \text{(fast),}$$

$$S_{f_2} = 0 \qquad \text{(thermal).}$$

Two Group Multiplication Factor (I)

$$-\nabla\cdot D_1\nabla\phi_1+\Sigma_{R_1}\phi_1=\frac{1}{k}\left[\nu_1\Sigma_{f_1}\phi_1+\nu_2\Sigma_{f_2}\phi_2\right],$$

$$-\nabla\cdot D_2\nabla\phi_2+\Sigma_{a_2}\phi_2=\Sigma_{s_{12}}\phi_1.$$

$$\nabla^2\psi+B^2\psi(\mathbf{r})=0,\quad \psi(\tilde{\mathbf{r}}_s)=0.$$

$$\phi_1(\mathbf{r})=\phi_1\psi(\mathbf{r}),\qquad \phi_2(\mathbf{r})=\phi_2\psi(\mathbf{r})$$

$$\boxed{\begin{aligned}\left(D_1B^2+\Sigma_{R_1}-k^{-1}\nu_1\Sigma_{f_1}\right)\phi_1-k^{-1}\nu_2\Sigma_{f_2}\phi_2&=0,\\ -\Sigma_{s_{12}}\phi_1+\left(D_2B^2+\Sigma_{a_2}\right)\phi_2&=0.\end{aligned}}$$

Is there Solution?

Yes, if determinant of coefficient matrix is zero:

$$\left(D_1B^2+\Sigma_{R_1}-\frac{\nu_1\Sigma_{f_1}}{k}\right)\left(D_2B^2+\Sigma_{a_2}\right)-\frac{\nu_2\Sigma_{f_2}\Sigma_{s_{12}}}{k}=0.$$

Solve for k:

$$\boxed{k=\frac{\nu_1\Sigma_{f_1}}{\Sigma_{R_1}+D_1B^2}+\frac{\Sigma_{s_{12}}}{\left(\Sigma_{R_1}+D_1B^2\right)}\frac{\nu_2\Sigma_{f_2}}{\left(\Sigma_{a_2}+D_2B^2\right)}}$$

Multigroup Reactor Equations (4 group)

Derive a 4-group reactor equation (using diffusion theory) for neutrons in a bare, homogenous, spherical reactor. This reactor is at steady-state but is not necessarily critical. The following assumptions should be used in this derivation:

a. Fission neutrons are only born in the top two groups, i.e. groups 1 and 2.
b. The fast neutron generation distribution is as follows: X1 = 0.75, X2 = 0.25
c. Thermal neutrons only exist in the bottom group, i.e. group 4.
d. Only thermal neutrons induce fissions.
e. There are no up-scatterings in the thermal group (i.e. neutrons only lose energy)
f. The absorption and scattering cross sections can be combined to form a "removal" cross section, ΣR.
g. Scattered neutrons will only drop to adjacent energy levels. This means that the scattering cross section for group 1 represents neutrons scattered to group 2 only, or Σs1,2, etc. h. Because cross sections are energy dependent, there is a separate cross section of each type for each energy group, indicated by the appropriate subscript. i. v is specific to each energy group… however, only one energy group undergoes fission…