Chemical Engineering 612

Reactor Design and Analysis

Lecture 6

Criticality Theory

Spiritual Thought

D&C 101: 16

Therefore, let your hearts be comforted concerning Zion; for all flesh is in mine hands; be still and know that I am God.

Bare Slab Reactor Solution

$$\frac{d^2\phi}{dx^2} = -B^2\phi$$ reactor equation

$$\phi\left(\frac{\tilde{a}}{2}\right) = \phi\left(-\frac{\tilde{a}}{2}\right) = \phi'(0) = 0$$ boundary conditions

$$\phi(x) = A\cos(Bx) + C\sin(Bx)$$ general solution

$$C = 0$$ from symmetry or by substitution

$$\phi(x) = A\cos(Bx)$$

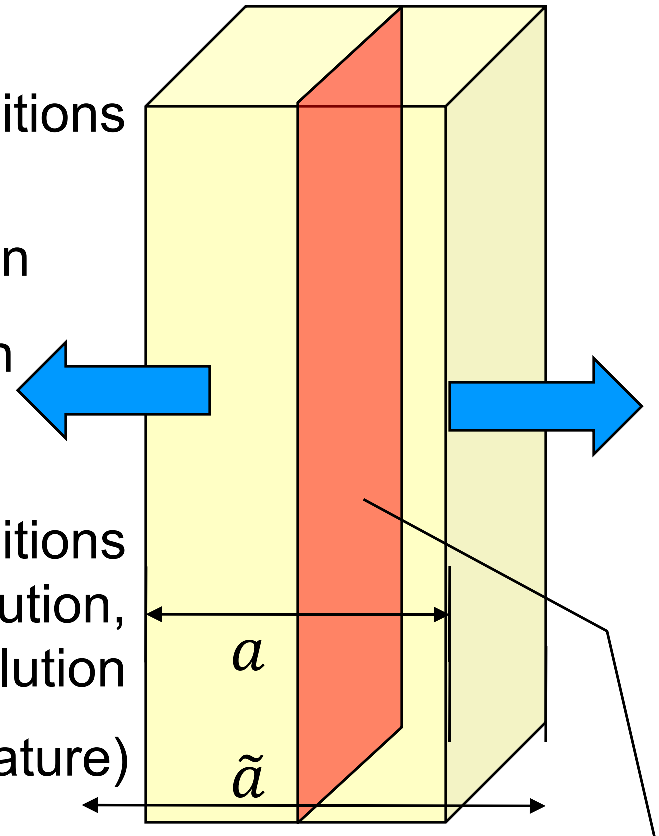
$$\phi(\tilde{a}/2) = 0$$

$$\Rightarrow B_n = \frac{n\pi}{\tilde{a}}$$

Eigenvalues from boundary conditions – all n important in transient solution, only n=1 important for steady solution

$$B_1^2 = -\frac{1}{\phi}\frac{d^2\phi}{dx^2}$$

B_1^2 is buckling (prop. to flux curvature)



The constant A is as yet undetermined and relates to the power. There are different solutions to this problem for every power level.

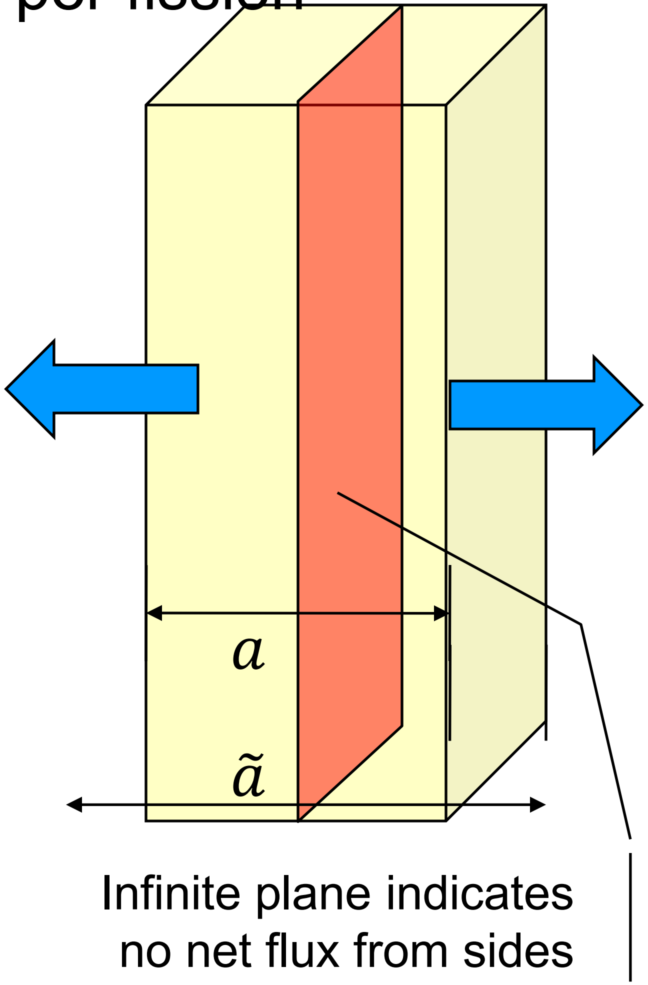
Bare Slab Reactor Power

$$P = E_R \Sigma_f \int_{-a/2}^{a/2} \phi(x)\,dx$$

E_R is the recoverable energy per fission

$$P = \frac{2A\tilde{a}E_R\Sigma_f}{\pi}\sin\left(\frac{\pi a}{2\tilde{a}}\right)$$

$$\phi(x) = \frac{\pi P}{2\tilde{a}E_R\Sigma_f \sin\left(\frac{\pi a}{2\tilde{a}}\right)}\cos\left(\frac{\pi x}{\tilde{a}}\right)$$



Power Scales with flux !

Infinite plane indicates no net flux from sides

Spherical Reactor

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = -B^2\phi$$

reactor transport equation

$$\phi(\tilde{R}) = \phi'(0) = 0$$

boundary conditions

$$\phi(r) = A\frac{\sin(Br)}{r} + \mathrm{C}\frac{\cos(Br)}{r}$$

general solution

$$C = 0$$

from symmetry or by substitution

$$\phi(r) = A\frac{\sin(Br)}{r}$$

specific solution

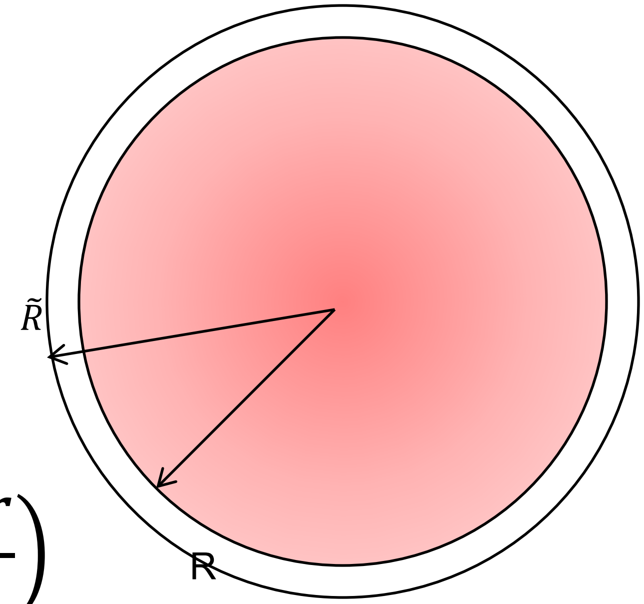
$$B_n = \frac{n\pi}{\tilde{R}}$$

Eigen values

$$B_1^2 = \left(\frac{\pi}{\tilde{R}}\right)^2$$

buckling

$$\phi(r) = A\frac{\sin\left(\frac{\pi r}{\tilde{R}}\right)}{r}$$



Spherical Reactor Power

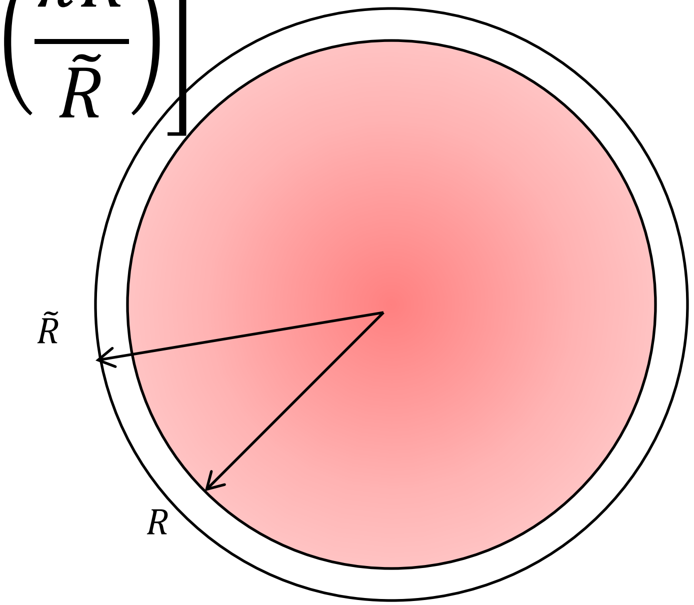
Integrate over 2 symmetric dimensions –
transform volume integral to radial integral

$$P = E_R \Sigma_f \iiint \phi(r) dV = 4\pi E_R \Sigma_f \int_0^R r^2 \phi(r) dr$$

$$P = 4\pi E_R \Sigma_f A \frac{\tilde{R}}{\pi} \left[\frac{\tilde{R}}{\pi} \sin\left(\frac{\pi R}{\tilde{R}}\right) - R \cos\left(\frac{\pi R}{\tilde{R}}\right) \right]$$

again, power is proportional to
flux and highest at center

$$\phi(r) = \frac{P \sin\left(\frac{\pi r}{\tilde{R}}\right)}{4\, E_R \Sigma_f R^2 r}$$



Infinite Cylindrical Reactor

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right) = -B^2\phi = \frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr}$$

reactor transport equation

$$\phi(\tilde{R}) = \phi'(0) = 0; |\phi(r)| < \infty$$

boundary conditions

$$\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} + \left(B^2 - \frac{m^2}{r^2}\right)\phi = 0$$

zero-order (m=0) Bessel equation

$$\phi(r) = AJ_0(Br) + CY_0(Br)$$

general solution involves Bessel functions of first and second kind

$$\phi(r) = AJ_0(Br)$$

flux is finite

$$B_n = \frac{x_n}{\tilde{R}}$$

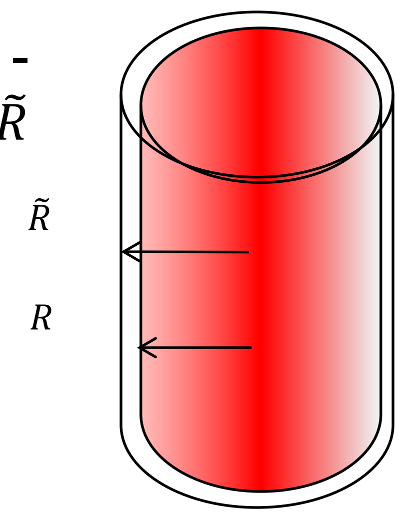
roots of Bessel functions - ϕ is zero at boundary $\tilde{R}$

$$B_1^2 = \left(\frac{x_1}{\tilde{R}}\right)^2 = \left(\frac{2.405}{\tilde{R}}\right)^2$$

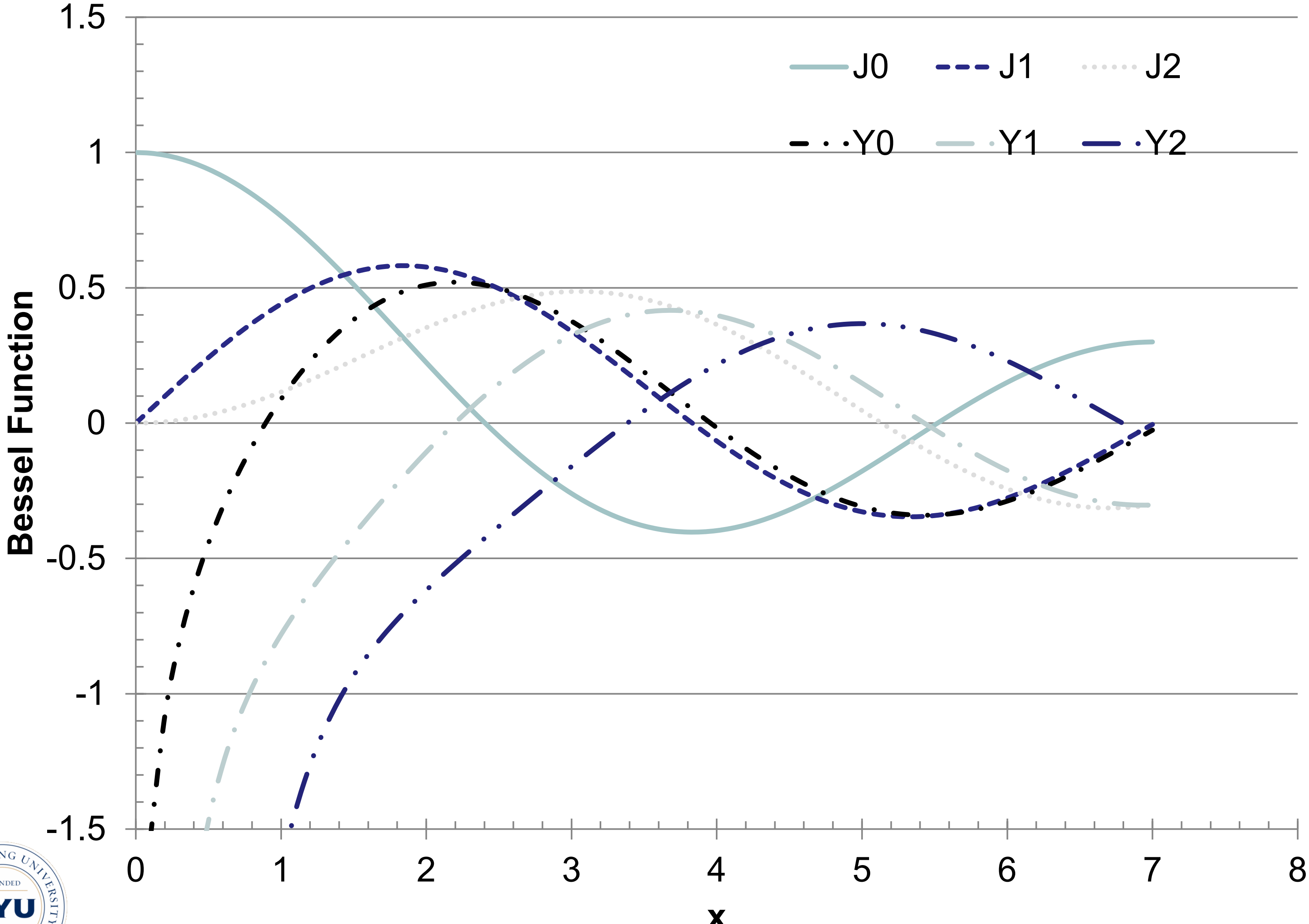
first root

$$\phi(r) = AJ_0\left(\frac{2.405\,r}{\tilde{R}}\right)$$

solution (power production determines A)



Bessel Functions



Infinite Cylindrical Reactor Power

$$P = E_R \Sigma_f \iiint \phi(r) dV = 2\pi E_R \Sigma_f \int_0^R r\phi(r) dr$$

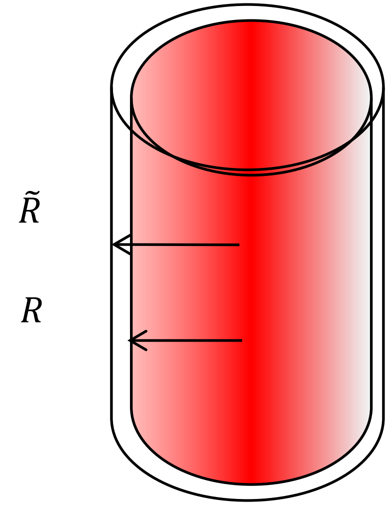
transform volume integral to radial integral – becomes power per unit length

$$P = 2\pi E_R \Sigma_f \int_0^R r J_0\left(\frac{2.405r}{R}\right) dr$$

$$\int_0^R x' J_0(x') dx' = x J_1(x)$$

$$P = \frac{2\pi E_R \Sigma_f R^2 A J_1(2.405)}{2.405}$$

$$\phi(r) = \frac{0.738P}{E_R \Sigma_f R^2} J_0\left(\frac{2.405r}{R}\right)$$



again, power is proportional to power and highest at center

Finite Cylindrical Reactor

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial \phi}{\partial r}\right)+\frac{\partial^2 \phi}{\partial z^2}=-B^2\phi$$

reactor transport equation

$$\phi\left(\tilde{R},z\right)=\phi'(0,z)=\phi\left(r,\frac{\widetilde{H}}{2}\right)=\phi\left(r,-\frac{\widetilde{H}}{2}\right)=0$$

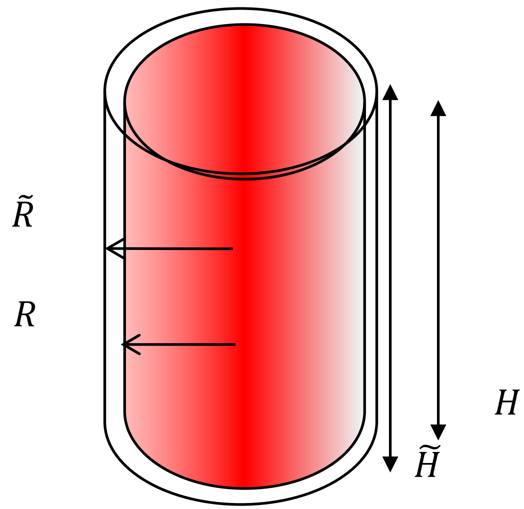
boundary conditions

$$\phi\left(r,z\right)=R(r)Z(z)$$

separation of variables

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial \phi}{\partial r}\right)+\frac{\partial^2 \phi}{\partial z^2}=\frac{Z}{r}\frac{\partial}{\partial r}\left(\frac{\partial R}{\partial r}\right)+R\frac{\partial^2 Z}{\partial z^2}R=-B^2R(r)Z(z)$$

$$\frac{1}{R}\frac{\partial}{\partial r}\left(\frac{\partial R}{\partial r}\right)+\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}R=-B^2$$



since R and Z vary independently, both portions of the equation must equal (generally different) constants, designated as B_R and B_Z, respectively

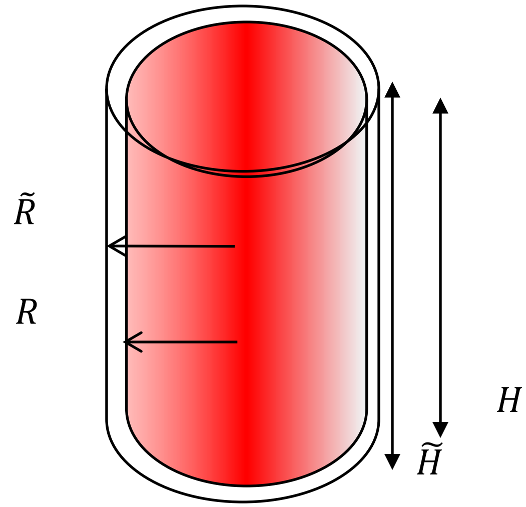
Finite Cylinder Solution

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) = -B_R^2 R$$ a problem we already solved, w/ same bcs

$$\Rightarrow R(r) = A\, J_0\left(\frac{2.405}{\tilde{R}} r\right)$$

$$\frac{\partial^2 Z}{\partial z^2} = -B_Z^2 Z$$ again a problem we already solved, w/ same bcs

$$\Rightarrow Z(z) = A\cos\frac{\pi z}{\tilde{H}}$$



$$\phi(r,z) = A\, J_0\left(\frac{2.405}{\tilde{R}} r\right)\cos\frac{\pi z}{\tilde{H}}$$

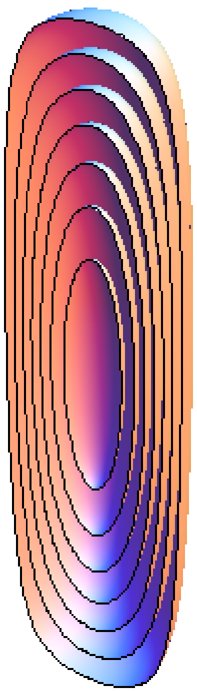
$$B^2 = B_R^2 + B_H^2$$

solution is the product of the infinite cylinder and infinite slab solutions

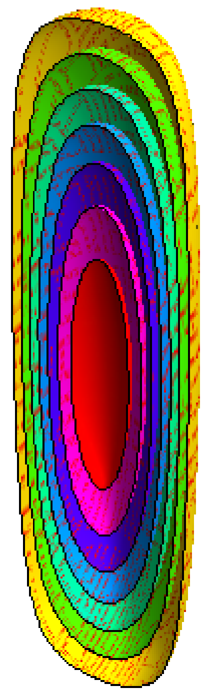
Buckling is higher than for either the infinite plane or the infinite cylinder. Buckling generally increases with increasing leakage, and there are more surfaces to leak here than either of the infinite cases.

Neutron Flux Contours

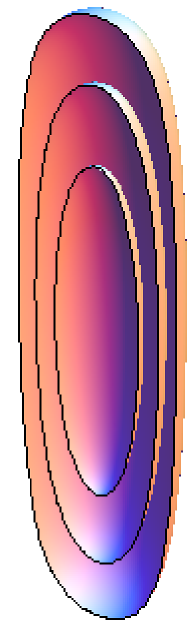
Neutron flux in finite cylindrical reactor



3D contours of neutron flux at high power



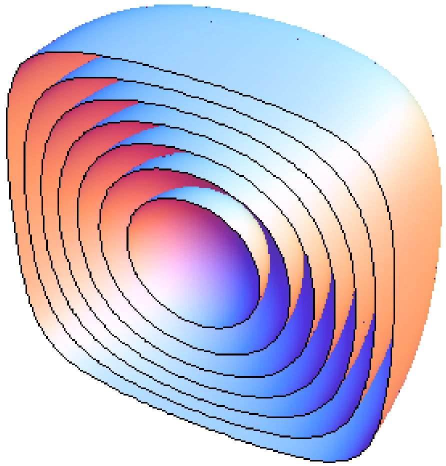
3D contours w color scaled to magnitude – intermediate power



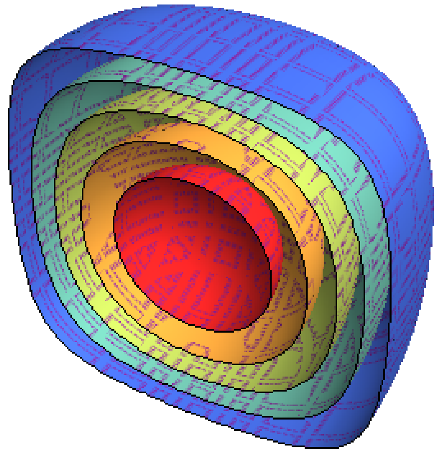
3D contours of neutron flux at low power

Neutron Flux Contours

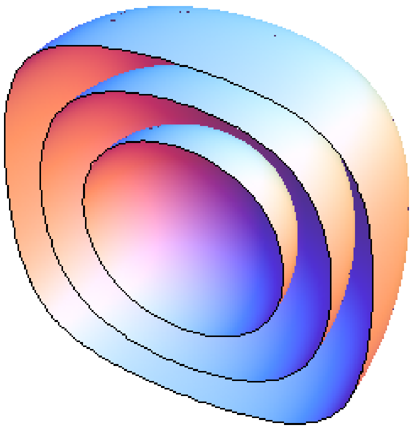
Neutron flux in finite parallelepiped (cubical) reactor



3D contours of neutron flux at high power



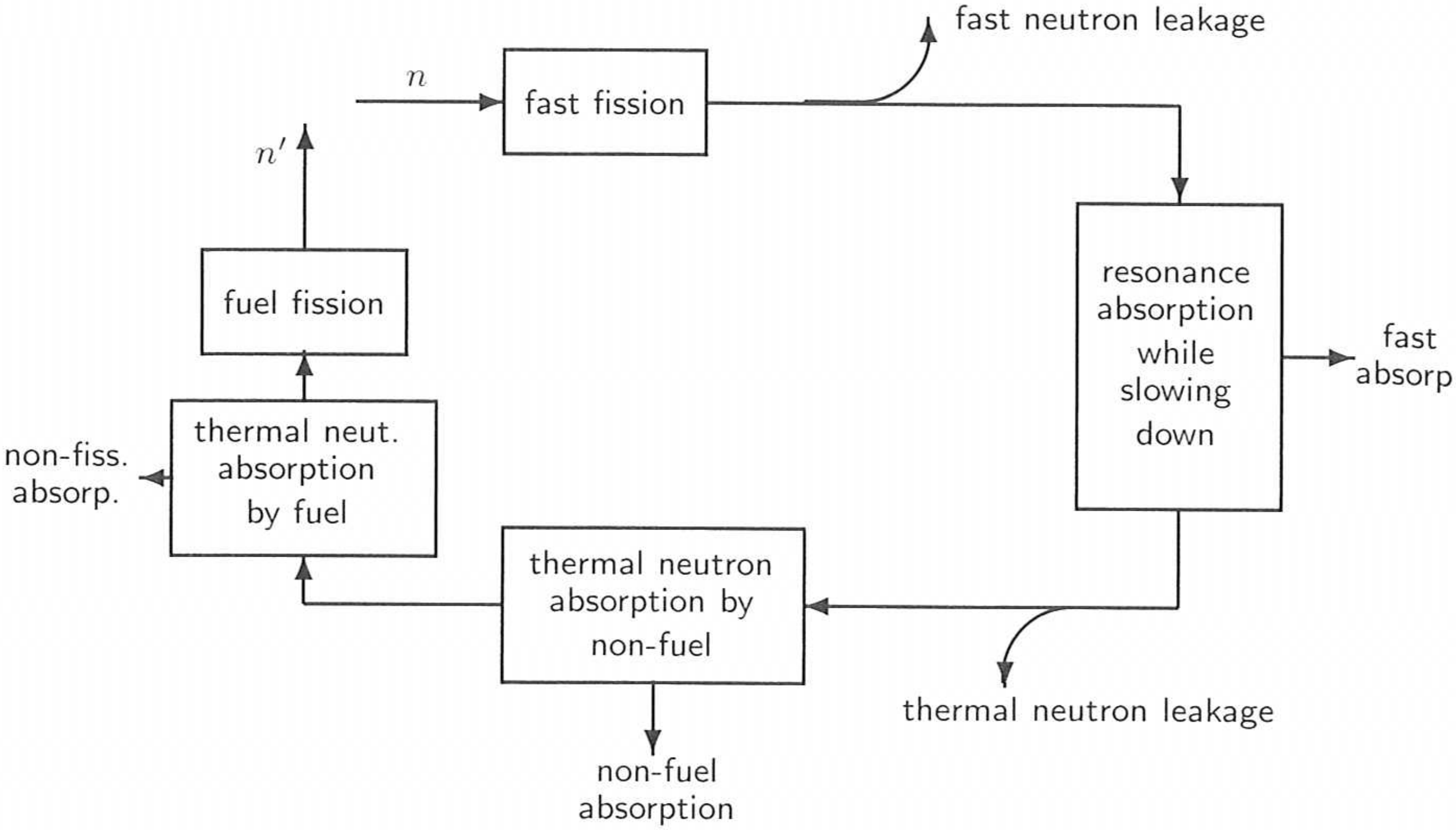
3D contours w color scaled to magnitude – intermediate power



3D contours of neutron flux at low power

Fast Neutron Life Cycle

- What happens to fast neutrons?



Multiplication Factor

$$k_{eff} \equiv \frac{neutrons\ at\ point\ in\ cycle}{neutrons\ at\ same\ point\ in\ previous\ generation}$$

$$k_{eff} = \frac{n'}{n}$$

$$k_{eff} = \epsilon p \eta f P_{NL}^{f} P_{NL}^{th}$$

$$k_{\infty} = \epsilon p \eta f$$

Reactor Considerations

- Increase Power?

$$k_{eff} > 1$$

- Decrease Power?

$$k_{eff} < 1$$

- Most Reactors have K_{eff}>1, but cancel excess out with absorptive “poisons”, which are removed with time.
- Reactors designed to not reach prompt supercriticality
- If k_{eff} increases, “feedback” effects resist increase
- What if we want to change amount of fuel or moderator?
  - Impacts various “six factor” parameters
  - Changes k_{eff}

One-Group Six Factor Formula

$$n' \equiv n\epsilon p\eta f P_{NL}^{f} P_{NL}^{th}$$

n'	next generation neutrons
n	neutrons produced per thermal fission
ϵ	ratio of total neutrons to thermal neutrons (1.0-1.08)
p	resonance escape probability (0.8-0.9)
f	thermal utilization = $\frac{\Sigma_a^F \phi^F V^F}{\Sigma_a^F \phi^F V^F + \Sigma_a^{NF} \phi^{NF} V^{NF}}$ (0-1)
η	fission factor $= \nu \frac{\Sigma_f^F}{\Sigma_a^F}$ (2.0-2.2)
P_{NL}^{f}	non-leakage of fast neutrons = $\exp(-B_c^2 \tau)$ (near 1)
P_{NL}^{th}	non-leakage of thermal neutrons = $\frac{1}{1+L^2 B_c^2}$ (near 1)

Bare Reactor Summary

geometry	Buckling (B^2)	Flux	A	$\Omega = \dfrac{\phi_{max}}{\phi_{av}}$
$plate - 1D$	$\left(\dfrac{\pi}{a}\right)^2$	$A\cos\dfrac{\pi x}{a}$	$1.57P / aE_R\Sigma_f$	1.57
$plate - 3D$	$\left(\dfrac{\pi}{a}\right)^2 + \left(\dfrac{\pi}{b}\right)^2 + \left(\dfrac{\pi}{c}\right)^2$	$A\cos\dfrac{\pi x}{a}\cos\dfrac{\pi y}{b}\cos\dfrac{\pi z}{c}$	$3.85P / VE_R\Sigma_f$	3.88
$cylinder - 1D$	$\left(\dfrac{2.405}{R}\right)^2$	$A\,\mathrm{J}_0\left(\dfrac{2.405}{R}\right)$	$0.738P / R^2E_R\Sigma_f$	2.32
$cylinder - 3D$	$\left(\dfrac{2.405}{R}\right)^2 + \left(\dfrac{\pi}{H}\right)^2$	$A\,\mathrm{J}_0\left(\dfrac{2.405}{R}\right)\cos\dfrac{\pi z}{H}$	$3.63P / VE_R\Sigma_f$	3.64
$sphere$	$\left(\dfrac{\pi}{R}\right)^2$	$\dfrac{A}{r}\sin\dfrac{\pi r}{R}$	$P / 4R^2E_R\Sigma_f$	3.29

Critical Buckling

$$k = \frac{\nu\Sigma_f}{\Sigma_a + DB^2}$$

value of k for critical reactor

$$\Rightarrow B^2 = \frac{\nu\Sigma_f - \Sigma_a}{D}$$

value of *B* when *k* = 1

$$B_c^2 = \frac{\nu\Sigma_f - \Sigma_a}{D}$$

critical material buckling

$$B_l^2 = \frac{k_\infty - 1}{L^2}$$

geometric buckling

$$\frac{\nu\Sigma_f - \Sigma_a}{D} = \frac{k_\infty - 1}{L^2}$$

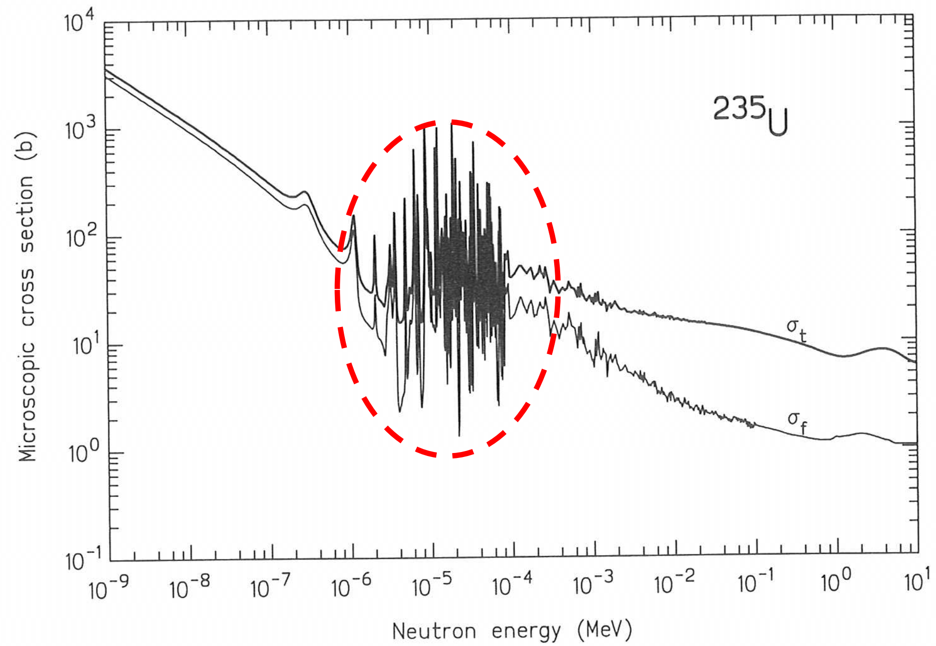
geometric and material buckling must be equal for a critical reactor

Resonance Escape Probability

- Probability of neutron slowing down without being absorbed
- $p = \frac{thermalized\ neutrons}{neutrons\ that\ slow}$
- $p < 1$
- Highly dependent on resonance region
- $p = \prod_i p_i = e^{\left(-\frac{N_F V_F}{\xi \Sigma_{sM} V_M} \sum_i \int \frac{\sigma_a(E)}{1+\sigma_a(E)/\sigma_o} \frac{dE}{E}\right)}$

$$\approx e^{\left(-\frac{N_F V_F}{\xi \Sigma_{sM} V_M} I_{eff}\right)}$$

$UO_2 \rightarrow I_{eff} = 4.45 + 26.6\sqrt{4/\rho D}$



$$\xi = \Delta u = 1 - \frac{(A-1)^2}{2A} \ln \frac{A+1}{A-1} = 1 + \frac{\alpha}{1-\alpha} \ln \alpha \cong \frac{2}{A + \frac{2}{3}}$$