Lecture 3 - Linear Equations

0. Class Business

* Prayer / Spiritual Thought

* Unit of the day

$$1 \text{ kg} = 2.20 \text{ lbm}$$

1 lbm

2.2 lbm

I. Solving systems of Equations

* In the next two lectures, we will work on solving equations of the form:

$$\underline{f}(\underline{x}) = \underline{0}$$ ← underline means vector

shorthand for:

$$f_1(x_1, x_2, \ldots x_n) = 0$$
$$f_2(x_1, x_2, \ldots, x_n) = 0$$
$$\vdots$$
$$f_n(x_1, x_2, \ldots, x_n) = 0$$

Note: There are n x_i & f_i. The system is square. We need this to guarantee a unique solution.

* Some classification:

system
- if $n=1$, we have only a single equation
- if $n>1$, we have a system

- if $f_1(x_1)=0$, $f_2(x_2)=0, \ldots, f_n(x_n)=0$
  the system is uncoupled. (n independent equations.)

coupled

- if any f_i depends on x_j, $j \neq i$, than the system is coupled.
  (In general. There are certain cases where they can be uncoupled, but ignore this detail for now

linear

- if all $f_i = a_{i1}x_1 + a_{i2}x_2 + \dots a_{in}x_n + b_i$ then the system is linear.
- If any f_i are not this form, the system is non-linear

Activity

Classify the following:

(a)
$$\left.\begin{aligned} 3x_1 + 5x_2 &= 7 \\ 5x_1^2 + x_2 &= 6 \end{aligned}\right\}$$
system, coupled, non-linear

(b)
$$\left.\begin{aligned} 4x_1 + 5x_2 + 3x_3 &= 1 \\ 7x_1 + 2x_2 + x_3 &= 10 \\ 8x_1 + 3x_2 + 4x_3 &= 0 \end{aligned}\right\}$$
system, coupled, linear

(c)
$$\frac{1}{\sqrt{x_1}} + 2\log_{10}\left(\frac{2.51}{\sqrt{x_1}}\right) = 0 \Big\}$$
single Equation, non-linear

II. Matrix Notation

* Linear systems are more easily dealt with in matrix notation (especially large systems).

$$5x_1 + 3x_2 - 14 = 0$$
$$x_1 + 2x_2 - 7 = 0$$

$$\Rightarrow \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 14 \\ 7 \end{bmatrix}$$

$$\underline{\underline{A}} \quad \underline{x} \quad = \quad \underline{b}$$

* Operations with matrices :

- addition / subtraction → +/- , e.g A + B
- multiply by scalar → *( ) , e.g 3A
- multiply by array / matrix → MMULT() (*) , e.g AB
- transpose → TRANSPOSE() (*) , e.g A^T
- determinant → MDETERM() (*) , e.g $|A|$
- inverse → MINVERSE () (*) , e.g. A^{-1}

(*) Array functions - functions that return more than one value.

(i) highlight cells for return (right size)

(ii) hit ctrl + shift + enter

Activity

* Excel sheet , practive with matrices and array formulas.

III. Solving systems of linear equations

* Given what we've said so far, solving linear systems is straightforward:

$$5x_1 + 3x_2 - 14 = 0 \qquad\qquad \Rightarrow \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 14 \\ 7 \end{bmatrix}$$
$$x_1 + 2x_2 - 7 = 0$$

$$\underset{=}{A} \quad \underline{x} = \underline{b}$$

$$\underline{x} = \underset{=}{A}^{-1}\underline{b}$$
$$= \begin{bmatrix} 2/7 & -3/7 \\ -1/7 & 5/7 \end{bmatrix}\begin{bmatrix} 14 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

* Two steps:
  (1) Matrix inverse
  (2) Matrix multiply.

* Check determinant!
  - if $|A| = 0 \rightarrow$ no unique solution
  - why? (need another independent equation)