

## Lecture 4 - Non-linear Equations

### O. class Business

- \* prayer / spiritual thought

- \* unit of the day:

$$1 \text{ lbf} = 32.2 \frac{\text{lbm ft}}{\text{s}^2}$$

$$F = mg$$

↑

$$1 \text{ lbf} = 1 \text{ lbm} \cdot g$$

### I. Non-linear equations

- \* Last time we talked about solving equations of the form:

$$f(x) = 0$$

- \* Specifically we talked about:

- classifying equations as a system/single equation, coupled/uncoupled & as linear / non-linear

- solving linear systems,  $\underline{A} \underline{x} - \underline{b} = 0$ , using Excel.

- \* What about non-linear equations?

Non-linear equations are much harder to solve than linear equations.

- \* Non-linear isn't even a very good classification!  
(It is like saying Non-Mormon. There are many things that fit into that category!)

- \* Two common non-linear equations are

(i) Algebraic equations / polynomials.

The variables in these equations have powers.

$$\text{e.g. } x^3 + 3x^2 - 1 = 0$$

(ii) Transcendental equations

There are transcendental terms like  $\exp(x)$ ,  $\sin(x)$ , etc.

$$\text{e.g. } e^{-x} + \cos x - \frac{1}{2} = 0$$

## II. Fixed point method

- \* So how do we solve (i) & (ii)? There are many ways, but one very useful method is called the fixed point method.

Fixed point method

- Arrange equation to be in the form

$$x = f(x)$$

- guess a value  $x_0$ .

- substitute  $x_0$  on the right-hand side to solve for  $x_1$ ,

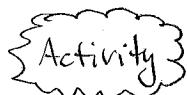
$$x_1 \leftarrow f(x_0)$$

we call this "iterating"  
 (iv) repeat until  $|x_n - x_{n-1}| < \text{error tolerance}$ .

we call this "converging"

### comments

- How get a guess? Make a plot!
- The form  $x = f(x)$  isn't unique.
- FPM won't always converge, depending on guess & form. If stuck, try changing these.



Use the fixed-point method to solve the Colebrook equation.

### III. Goal Seek.

- \* There are better ways to solve non-linear equations than using the fixed point method.  
 (But the FPM is useful because it is simple and you can do it by hand.)
- \* Excel has a tool called "goal seek" that makes solving a single non-linear equation even easier. (Makes doing multiple go faster.)



- Demonstrate Goal Seek.
- Use goal seek to solve Colebrook Equation for several Reynolds numbers.
- Still need an initial guess → use plot.

#### IV. Solver

- \* So far we have only been solving single non-linear equations. What about systems?
- \* Excel has a powerful optimization tool called "solver" that allows us to work with systems.
- \* The problem gets harder the more variables there are.



- Demonstrate solver
- Use solver to solve a system of non-linear equations.
- Again, still need an initial guess
  - ↳ Guess is harder when # of variables > 2.