Lecture 4 - Non-linear Equations

0. Class Business

* prayer / spiritual thought
* unit of the day:

$$1\ lbf = 32.2\ \frac{lbm\ ft}{s^2}$$

$$F = m g$$
$$\uparrow$$
$$1\,lbf = 1\,lbm \cdot g$$

I. Non-linear equations

* Last time we talked about solving equations of the form:

$$\underline{f}(\underline{x}) = \underline{0}$$

* Specifically we talked about:
  - classifying equations as a system/single equation, coupled/uncoupled & as linear | non-linear
  - solving linear systems, $\underline{A}\,\underline{x} - \underline{b} = \underline{0}$, using Excel.

* What about non-linear equations?

  Non-linear equations are much harder to solve than linear equations.

* Non-linear isn't even a very good classification!
(It is like saying Non-Mormon. There are many things that fit into that category!)

* Two common non-linear equations are

(i) Algebraic equations / polynomials.
The variables in these equations have powers.
e.g. $x^3 + 3x^2 - 1 = 0$

(ii) Transcendental equations
There are transcendental terms like $\exp(x)$, $\sin(x)$, etc.
e.g. $e^{-x} + \cos x - 1/2 = 0$

II. Fixed point method

* So how do we solve (i) & (ii)? There are many ways, but one very useful method is called the fixed point method.

Fixed point method

(i) Arrange equation to be in the form
$$x = f(x)$$

(ii) guess a value x_0.

(iii) Substitute x_0 on the right-hand side to solve for x_1
$$x_1 \leftarrow f(x_0)$$

(iv) repeat until $|x_n - x_{n-1}| <$ error tolerance.

↓ we call this "iterating"

↑ we call this "converging"

comments

- How get a guess? Make a plot!
- The form $x = f(x)$ isn't unique.
- FPM won't always converge, depending on guess & form. If stuck, try changing these.

Activity: Use the fixed-point method to solve the Colebrook equation.

III. Goal Seek.

* There are better ways to solve non-linear equations than using the fixed point method.
  (But the FPM is useful because it is simple and you can do it by hand.)
* Excel has a tool called "goal seek" that makes solving a single non-linear equation even easier. (Makes doing multiple go faster.)

Activity
- Demonstrate Goal Seek.
- Use goal seek to solve Colebrook Equation for several Reynolds numbers.
- Still need an initial guess → use plot

IV. Solver

* So far we have only been solving single non-linear equations. What about systems?

* Excel has a powerful optimization tool called "solver" that allows us to work with systems.

* The problem gets harder the more variables there are.

Activity

- Demonstrate solver
- Use solver to solve a system of non-linear equations.
- Again, still need an initial guess
  ↳ Guess is harder when # of variables > 2.