Lecture 15 – Numerical ODEs

* prayer / sp. thought
* unit of the day

  $°F = °C\left(\frac{9}{5}\right) + 32$

I. Systems of Rate Equations

$$\frac{dy}{dt} = f(y, t) \qquad y(0) = y_0$$

* 1st order → 1st derivatives
* ordinary DE – $\frac{d}{dt}$ $\quad \cancel{\frac{\partial}{\partial x}} \; \cancel{\frac{\partial}{\partial y}}$
* IVP – initial value problem $y(0)$

* Integration $\neq$ solving an ODE ← harder

* method of solution: explicit Euler

$$y_{n+1} = y_n + \Delta t\, f(y_n, t_n)$$
$$y(0) = y_0$$

Systems of Rate Eqs

$$\frac{dy_1}{dt} = f_1(y_1, y_2, \cdots, y_m, t)$$

$$\frac{dy_2}{dt} = f_2(y_1, y_2, \cdots y_m, t)$$

$$\vdots$$

$$\frac{dy_m}{dt} = f_m(y_1, y_2, \cdots y_m, t)$$

m: # of eq's, # of y's

n: # of time points

IC: $y_1(0) = y_{1,0}$
$y_2(0) = y_{2,0}$
$\vdots$
$y_m(0) = y_{m,0}$

} system
* linear / non-linear
* coupled / uncoupled

y^2, $\ln y$, $y_1 y_2$ ← NL terms

$\frac{dy}{dt} = y f(t)$ ← t^2, $\ln t$, ty ok (still linear)

coupled: $\frac{dy_1}{dt} = -y_2 \quad \frac{dy_2}{dt} = -y_1$

* Explicit Euler on a system

(in vector notation)

$$\underline{y}_{n+1} = \underline{y}_n + \Delta t \, \underline{f}(\underline{y}_n, t_n)$$

(write all out)

$$y_{1,n+1} = y_{1,n} + \Delta t \, f(y_{1,n}, y_{2,n}, \dots, t_n)$$
$$y_{2,n+1} = y_{2,n} + \Delta t \, f(y_{1,n}, y_{2,n}, \dots, t_n)$$
$$\vdots$$
$$y_{m,n+1} = y_{m,n} + \Delta t \, f(y_{1,n}, y_{2,n}, \dots, t_n)$$

* Higher order ODEs?

→ turn into a system of 1st order ODEs

ex: $\frac{d^2y}{dt^2} = -gt$

$y(0) = y_0$

$\frac{dy}{dt}(t=0) = v_0$

* $\left(v = \frac{dy}{dt} \right.$ (we define)

$$\frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d}{dt}(v) = \frac{dv}{dt}$$

$$\boxed{\begin{array}{ll} \frac{dv}{dt} = -gt & \frac{dy}{dt} = v \\ \hline v(0) = v_0 & y(0) = y_0 \end{array}}$$

2nd order ODE
↓
system of 1st order ODEs

Steps

(1) Define a new variable for each order of derivative.

e.g. $\frac{d^3y}{dt^3}$ $\quad v = \frac{dy}{dt}$ $\quad a = \frac{dv}{dt} = \frac{d^2y}{dt^2}$

$$\frac{d^3y}{dt^3} = \frac{d}{dt}\left(\frac{d^2y}{dt^2}\right) = \frac{da}{dt}$$

(2) Substitute definitions into the original ODE

(3) Find ICs

(4) Solve the system

II. ODE solvers in Python

↙ quad simps

* The ODE solvers → live in scipy.integrate
* "odeint"
  - much more accurate than EE (1st order acc.)
  - method: "Runge-Kutta 45"

* Example - syntax & usage
* Practice problem.

$$\frac{dy}{dt} = \underbrace{\underline{f}(y,t)}_{\text{RHS}}$$