

1. Classify the following systems of differential equations as either linear/non-linear and coupled/uncoupled.

(a)

$$\begin{aligned}\frac{dy_1}{dt} &= -t^2 \\ \frac{dy_2}{dt} &= -y_2\end{aligned}$$

Linear, uncoupled

(c)

$$\begin{aligned}\frac{dy_1}{dt} &= -y_1 y_2 \\ \frac{dy_2}{dt} &= -y_1 \sin(t)\end{aligned}$$

Non-linear, coupled

(b)

$$\frac{dy}{dt} = -y^2$$

Non-linear

(d)

$$\begin{aligned}\frac{dy_1}{dt} &= -y_1 + 2y_2 \\ \frac{dy_2}{dt} &= -y_1 t^2\end{aligned}$$

Linear, coupled

2. Re-arrange the differential equation to express it as a system of first-order rate equations.

$$\frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + ty = \cos(t)$$

$$\text{Let } v = \frac{dy}{dt} \Rightarrow \frac{dv}{dt} = \frac{d^2 y}{dt^2}$$

$$\frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + ty = \cos t$$

$$\frac{dv}{dt} + 3v + ty = \cos t \Rightarrow$$

$$\begin{aligned}\frac{dv}{dt} &= -3v - ty - \cos t \\ \frac{dy}{dt} &= v\end{aligned}$$