Lecture 19 - Functions, Arrays and Linear Equations

- Prayer/Spiritual Thought
- Announcements

Outline

1. Built-in Mathcad functions
2. User-defined functions
3. Arrays and Matrices
4. Range Variables
5. Systems of Linear Equations

1. Built-in Mathcad functions

A. Explanation

Mathcad has many built-in functions for math and other purposes. Here is a non-comprehensive list:

- exp(x), e^(x) -- The number 'e' raised to the power of x
- sin(x), cos(x), etc. -- Trig functions: Sine of x, Cosine of x
- asin(x), acos(x), etc. -- Inverse trig functions: arcsine of x, arccos of x
- cosh(x), sinh(x), etc. -- Hyperbolic functions
- log(x, [b]) -- Base b logarithm of x (b=10 by default)
- ln(x) -- Natural logarithm of x
- x! -- Factorial of x
- max(a, b, c, ...) -- Value of the largest argument
- min(a, b, c, ...) -- Value of the smallest argument
- mean(a, b, c, ...) -- The average of a, b, c, etc.
- floor(x), ceil(x) -- Largest/smallest integer less than or equal to x
- round(x, n) -- The number x rounded to n decimal places (n=0 by default, i.e. round to integer).
- trunc(x, n) -- The integer part of x
- mod(x, y) -- The remainder of dividing x by y
- if(<condition>, <if true>, <if false>) -- If <condition> is true, return <if true>, else return <if false>
- $\sum_{n=1}^{m} f(n)$ -- Summation of f(n) from n = 1 to m (shortcut: <ctrl><shift>4)
- $\prod_{n=1}^{m} f(n)$ -- Product of f(n) from n=1 to m (shortcut: <ctrl><shift>3)

A comprehensive list of built-in functions can be found at the link: https://help.ptc.com/mathcad/en/index.html#page/PTC_Mathcad_Help/about_built-in_functions.html

B. Examples

Basic function evaluation

$$\sin\left(\frac{2\pi}{3}\right)=0.866 \qquad \ln(0.5)=-0.693147$$

$$\cos(2\pi)=1 \qquad \frac{\log(0.5)}{\log(e)}=-0.693147$$

$$\text{mean}(4,5,6,7,8)=6$$

$$\text{round}(\ln(0.5),2)=-0.690000$$

Tips:

- You can type in the name of the function or you can use the Functions menu bar.

C. The 'if' function

Just like python, there are conditional operators (equals, less than, greater than) and logical operators (and, or, not)

Conditional operators

- $a=b$, Equal to (<ctrl>=)
- $a<b$, less than (<)
- $a>b$, greater than (>)
- $a\leq b$, less than or equal (<=)
- $a\geq b$, greater than or equal (>=)
- $a\neq b$, not equal (<>)

Tips:

- Both types of operators can be accessed via Math -> Operators or via the keyboard shortcuts given here.

Logical operators

- $a\wedge b$, and (<ctrl><shift>7)
- $a\vee b$, or (<ctrl><shift>2)
- $\neg a$, not (<ctrl><shift>1)

D. Examples of the 'if' function

$$x:=2 \qquad \text{if}(x>2,3,1)=1 \qquad y:=\text{if}(x<2,0,2) \qquad y=2$$

$$\text{if}((5=5)\wedge(3=2),\text{“true”},\text{“false”})=\text{“false”}$$

$$\text{if}((5=5)\vee(3=2),\text{“true”},\text{“false”})=\text{“true”}$$

$$\text{if}(\neg(5=5),\text{“true”},\text{“false”})=\text{“false”}$$

2. User-defined functions

A. Explanation

- One of the powerful features of Mathcad is the ability to define your own functions. This is very helpful when doing engineering calculations and will be used often.

Syntax to create a function:

1. Type the desired function name
2. Type (x) where x is the variable of the function.

3. Type : to give you :=
4. Define the expression in terms of x.

- Note: To make a multivariable function, simply list more than one variable inside the parentheses, separated by a comma.

B. Examples

Function of one variable

$f(x) := 3\ x^2$ $\qquad$ $f(5) = 75$

Function of multiple variables

$g(x, y) := 3\ x^2\ \sin(2 \cdot \pi \cdot y)$ $\qquad$ $g(5, 1) = -1.837 \cdot 10^{-14}$

$g\left(2, \frac{1}{4}\right) = 12$

Piecewise function

$my\_abs(x) := \mathbf{if}(x < 0, -x, x)$ $\qquad$ $my\_abs(-4) = 4$

$my\_abs(4) = 4$

3. Arrays, Vectors and Matrices

A. Defining arrays

Arrays, vectors and matrices are all input using the "Matrices/Tables" tab in the Ribbon or with keyboard shortcuts.

Shortcuts:

- <ctrl> M -- input a matrix
- <shift> <space> -- add a column
- <shift> <enter> -- add a row

B. Examples

Use the "Insert Matrix" option on the ribbon to create a 3x3 matrix

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Use keyboard shortcuts to create a 4x1 array:

$$b := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

C. Accessing individual elements

- Individual elements of an array are accessed using subscript notation.
- Subscripts are accessed with a left square brace: [
- Mathcad indices start at ZERO (like python)
- This can be changed with a variable named ORIGIN

D. Examples

$A_{1,2} = 6$ $b_2 = 3$

$\mathbf{ORIGIN} := 1$

$A_{1,2} = 2$ $b_2 = 2$

Tip:

- Don't confuse the array subscript $A_{1,2}$ ([ ) with the text subscript you use when naming a variable A_{12} (<ctrl> <minus>)

4. Range Variables

A. Explanation

Range variables are sequences that you can use as indices to arrays. These are kind of like substitutes for using loops. You can also think of them like the slicing operator in python, e.g. A[2:].

Definition:
i = <start>, <start+step> .. <end>

- To define a range variable, you need to type the range operator, ".."
- If you don't specify a step, Mathcad will assume <step>=1
- Range variables *are not vectors*.

B. Examples

Define a range variable

$\mathbf{ORIGIN} := 0$

$i := 0..2 \quad i = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ $j := 1, 1.1..1.5 \quad j = \begin{bmatrix} 1 \\ 1.1 \\ 1.2 \\ 1.3 \\ 1.4 \\ 1.5 \end{bmatrix}$

Use a range variable to get part of a matrix

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $A_{i,0} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$ $A_{0,i} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Use a range variable to define a vector

$j := 0..5$ $\quad x_j := 0.2 \cdot j + 2$

$$x = \begin{bmatrix} 2 \\ 2.2 \\ 2.4 \\ 2.6 \\ 2.8 \\ 3 \end{bmatrix}$$

Tip:

- When defining a vector, the range variable needs to be an integer with a <start> that is greater than or equal to ORIGIN.

5. Systems of Linear Equations

A. Matrix Operations

Matrix operations are in the ribbon: Matrices/Tables -> Vector/Matrix Operations

- Addition, subtraction, scalar multiplication, matrix multiplication are all done with the same operators as scalars (+, -, *)
- A^{T} -- Transpose, <ctrl><shift><T>
- A^{-1} -- Inverse, ^-1
- $\|A\|$ -- Norm, <ctrl> <shift> |
- $b \times c$ -- Cross product, <ctrl> 8

B. Examples

$$A := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B := \begin{bmatrix} 3 & 5 \\ 2 & 8 \end{bmatrix} \quad x := \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad y := \begin{bmatrix} 8 \\ 5 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 4 & 7 \\ 5 & 12 \end{bmatrix} \quad x + y = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

$$A - B = \begin{bmatrix} -2 & -3 \\ 1 & -4 \end{bmatrix} \quad x - y = \begin{bmatrix} -6 \\ -4 \end{bmatrix}$$

$$A^{\mathrm{T}} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \quad x^{\mathrm{T}} = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

$$\|A\| = -2 \quad \|x\| = 2.236$$

$$A \cdot x = \begin{bmatrix} 4 \\ 10 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

Tip:

- Vectors must be 3x1 to be able to use the cross product.

C. Matrix Functions

Matrix functions are in the ribbon: Matrices/Tables -> Vector/Matrix Functions. There are many functions, some examples include:

- eigenvals(<matrix>) -- gives eigenvalues of the matrix
- eigenvecs(<matrix>) -- gives the eigenvectors of the matrix as columns
- lsolve(A, b) -- solves A x = b for x (this is faster/better than multiplying by the inverse)

D. Examples

$$A := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad b := \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\text{eigenvals}(A) = \begin{bmatrix} 5.372 \\ -0.372 \end{bmatrix}$$

$$\text{eigenvecs}(A) = \begin{bmatrix} -0.416 & -0.825 \\ -0.909 & 0.566 \end{bmatrix}$$

$$\text{lsolve}(A, b) = \begin{bmatrix} -3 \\ 2.5 \end{bmatrix} \qquad A^{-1} \cdot b = \begin{bmatrix} -3 \\ 2.5 \end{bmatrix}$$