Lecture 20 - Plotting and Nonlinear Equations

- Prayer/Spiritual Thought
- Announcements

Outline

1. Plotting
2. Roots of Polynomials
3. Single Nonlinear Equations
4. Systems of Nonlinear Equations

1. Plotting

A. Explanation

- Plots can be inserted in two ways:

  1. Ribbon -> Plots -> Insert Plot
  2. <ctrl><2>

- One can make a plot of a **function** by typing the independent variable into the x-axis and the function into the y-axis.
- One can make a plot of **data** by typing the name of the vector for the independent variable into the x-axis and the name of the vector of the dependent variable into the y-axis.
- Useful formatting options:

  - Many useful options are in the Ribbon
  - <shift> <enter> -- Add "trace" (another function/vector to the plot)
  - Change axes limits by directly editing the first and last tick mark labels
  - Change tick spacing by editing the second tick mark label on an axis
  - Adjust plot size in the bottom right-hand corner
  - Adjust plot axes location by dragging
  - Axis labels can be dragged to several different locations
  - Closest thing to adjusting symbol size is to change trace thickness.

More details about plotting in Mathcad can be found at the link: https://help.ptc.com/mathcad/en/#page/PTC_Mathcad_Help%2Fabout_plots.html%23

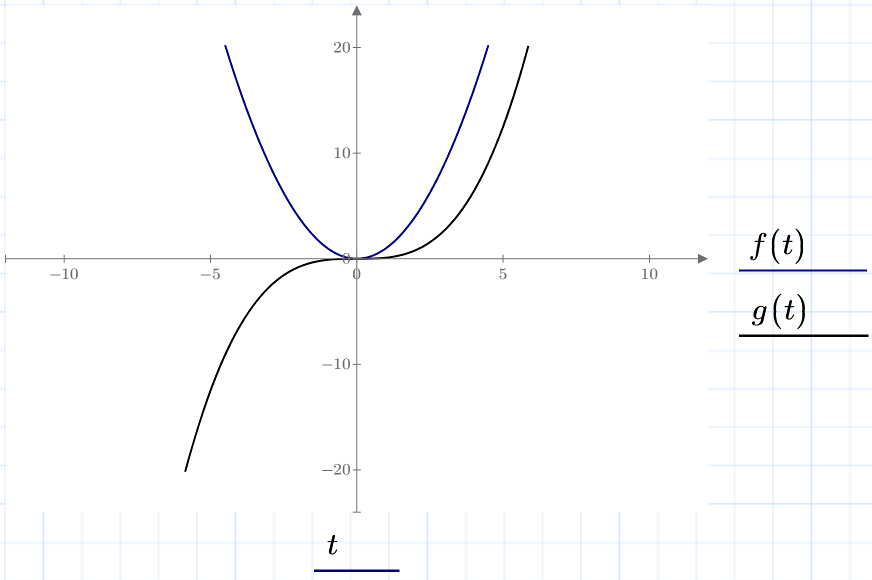
B. Example -- Plotting functions

$$f(x) := x^2$$

$$g(x) := \frac{x^3}{10}$$

Tip:

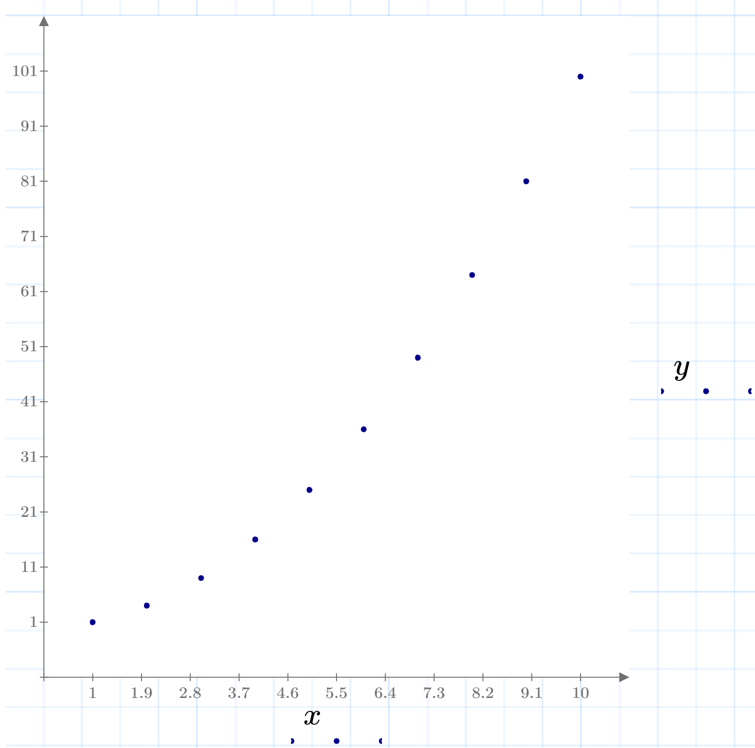
- Make sure your plot is **below** the function you have defined.
- No easy way to export plots



C. Example -- Plotting data

$i := 0..9 \qquad x_i := i + 1 \qquad y_i := \left(x_i\right)^2$

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{bmatrix} \qquad y = \begin{bmatrix} 1 \\ 4 \\ 9 \\ 16 \\ 25 \\ 36 \\ 49 \\ 64 \\ 81 \\ 100 \end{bmatrix}$$



Tips:

- Discrete data should be plotted using discrete points. Lines imply that something is known in between the points.
- If use a line for clarification, one should say that it is to "guide the eye."

2. Roots of polynomials

A. Explanation

- Mathcad has a built-in function for finding the roots of a polynomial, **polyroots**.

- Steps to using polyroots for the polynomial:

  p(x) = a_0 + a_1*x + a_2*x^2 + ... + a_n-1 * x^(n-1) + a_n*x^n

1. Define a vector of coefficients: coeff = [a_0, a_1, a_2, ..., a_n-1, a_n]
2. Call: polyroots(<coeff>)

More details in the help menu at this link: https://help.ptc.com/mathcad/en/#page/PTC_Mathcad_Help%2Fexample_finding_the_roots_of_a_polynomial.html%23

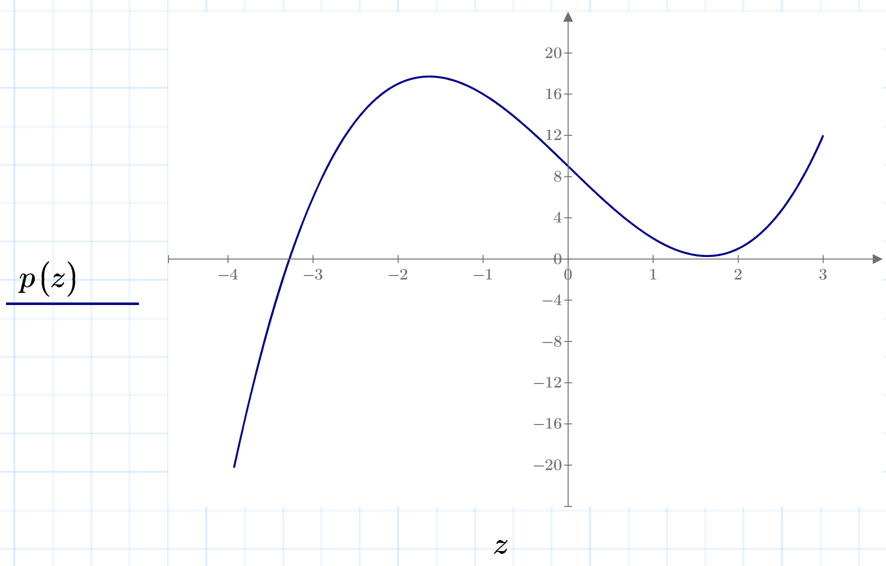
B. Example

$$coeffs := \begin{bmatrix} 9 \\ -8 \\ 0 \\ 1 \end{bmatrix} \qquad \text{polyroots}(coeffs) = \begin{bmatrix} -3.278 \\ 1.639 - 0.243i \\ 1.639 + 0.243i \end{bmatrix}$$

Tip:

- Polyroots finds **all** of the roots, even imaginary ones.

$$p(z) := \sum_{i=0}^{3} coeffs_i \cdot z^i \qquad p(z) \rightarrow z^3 - 8 \cdot z + 9 \qquad p(2.1) = 1.461$$



3. Solving A Single Nonlinear Equation

A. Explanation

- Mathcad has a built-in function for solving a single nonlinear equation called **root.**
- Steps to using root for a nonlinear equation:

f(x) = 0

1. Arrange the equation into residual form.
2. Add a guess the unknown, x
3. Call root(<function>, <guess>)

B. Example

Find the solution to the equation:

10*sin(x) = -x

$$f(x) := 10 \cdot \sin(x) + x$$

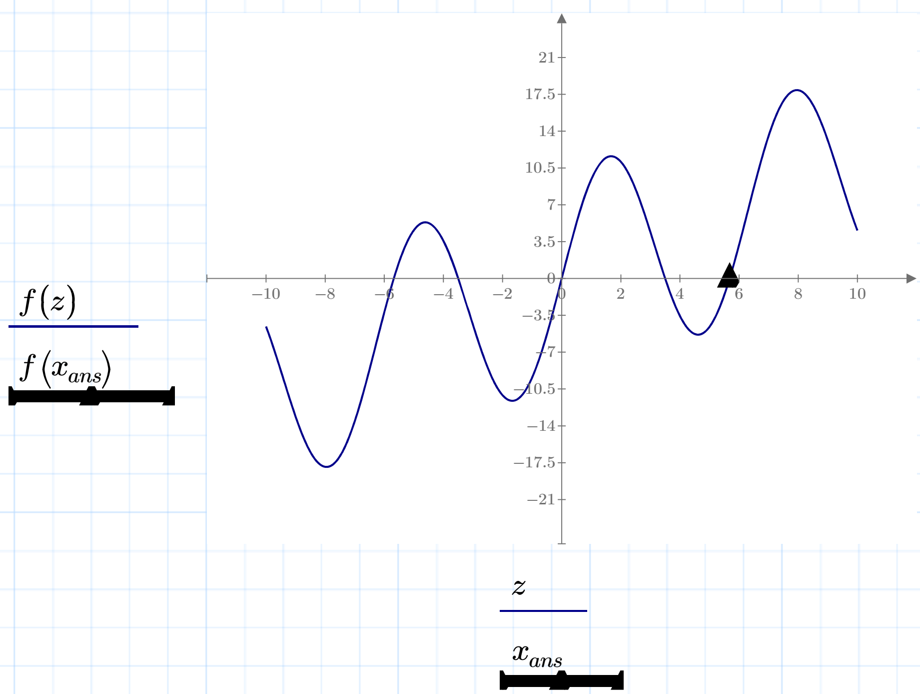
$$x_g := 5$$

$$\mathbf{root}\left(10 \cdot \sin\left(x_g\right) + x_g, x_g\right) = 5.679$$

$$x_{ans} := \mathbf{root}\left(f\left(x_g\right), x_g\right)$$

Tips:

- You need to use the names of the variable that you use for your guess as your unknown.
- You can define a separate function or explicitly type the function inside root.



4. Solving Systems of Nonlinear Equations

A. Explanation

- Systems of nonlinear equations are solved using **solve blocks.**
- A solve block can be inserted:

    1. Via Ribbon -> Math -> Solve Block
    2. <ctrl> 1

- Steps to using a solve block for a system of equations:

    f1(x1, x2, ..., xn) = 0
    f2(x1, x2, ..., xn) = 0
    ...
    fn(x1, x2, ..., xn) = 0

1. Add a solve block to the sheet
2. Add a guess for each variable (x1, x2, ..., xn)
3. Add equations ("constraints") f1, f2, ..., fn using the **conditional equals** (<ctrl>=).
4. Use the built-in function find(x1, x2, ..., xn) to solve for the unknowns

B. Examples

1. Find the roots to:
   x1^2 + x2^2 = 0.9
   x2 = cos(pi*x1/2)

Guess Values

$$x_1 := 6 \qquad x_2 := 6$$

Constraints

$$x_1^{\,2} + x_2^{\,2} - 0.9 = 0$$

$$x_2 - cos\left(\frac{\pi \cdot x_1}{2}\right) = 0$$

Solver

$$\mathbf{ans} := \mathbf{find}\left(x_1, \mathbf{x}_2\right) = \begin{bmatrix} 0.945 \\ 0.087 \end{bmatrix}$$

Tips:

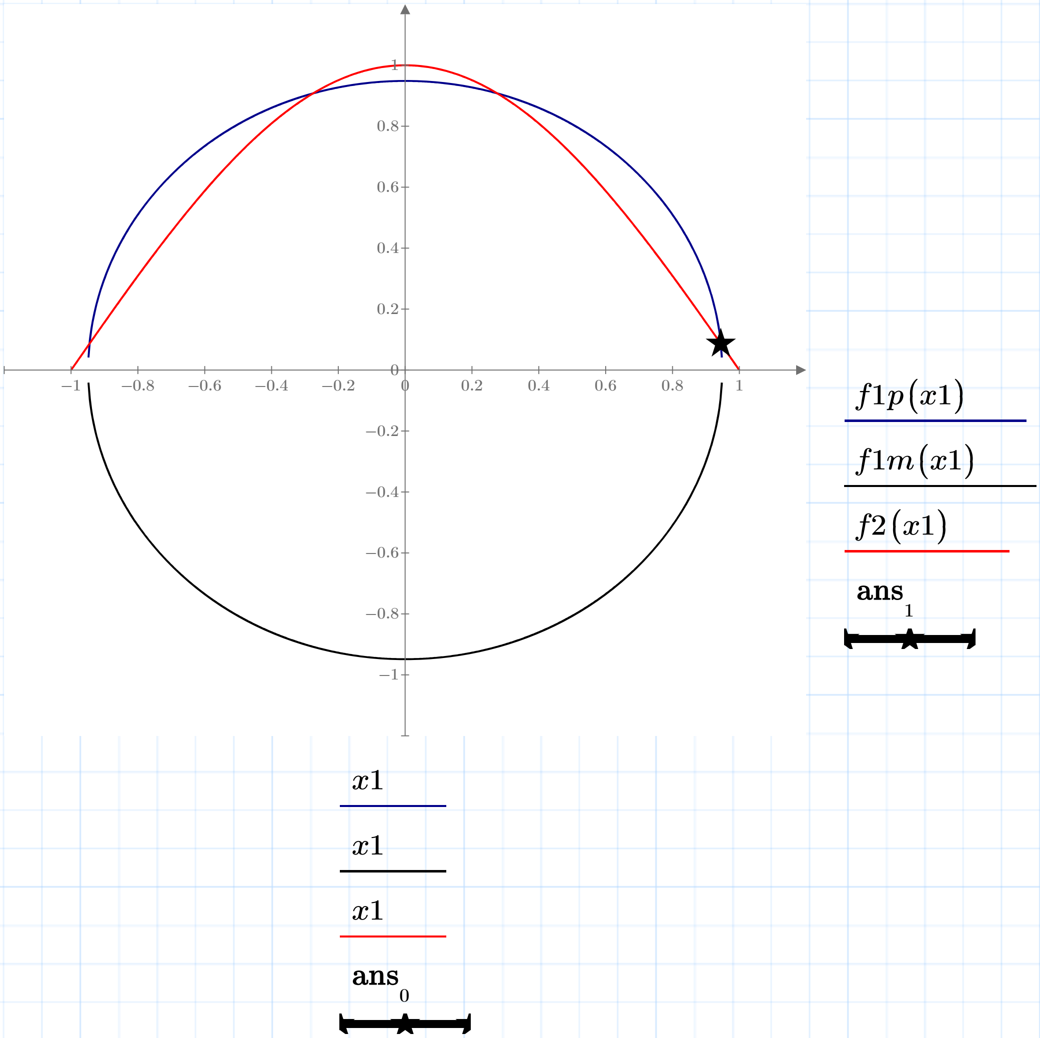
- Like with root, you need to use the names of the variables that you used for the guesses as your unknowns.
- You do not need residual form for solve blocks, but it is a good habit anyway

$$f1p(x1) := \sqrt{\left(0.9 - x1^2\right)}$$

<-- For the plot on the next page.

$$f1m(x1) := -\sqrt{\left(0.9 - x1^2\right)}$$

$$f2(x1) := \cos\left(\frac{\pi \cdot x1}{2}\right)$$


1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
f1p(x1)
f1m(x1)
f2(x1)
ans 1
x1
x1
x1
ans 0

2. Use a function in a solve block.

The heat, q, needed to change the temperature of a gas from T1 to T2 is given by,

$$q = n\left(\int_{T_1}^{T_2} C_p(T)\,\mathrm{d}T\right)$$

where Cp(T) is the heat capacity of the gas. If 10,000 BTU is removed from 500 moles of ethane initially at 700K, calculate the final temperature of the gas. Cp(T) is given below.

Given in the problem statement:

$$C_p(T) := \left(5.01696 \cdot 10^8 + 1.08753 \cdot 10^6 \cdot \left(\frac{T}{K} - 273.15\right) + 863.337 \cdot \left(\frac{T}{K} - 273.15\right)^2 - 1.9525 \cdot \left(\frac{T}{K} - 273.15\right)^3 + 8.58038 \cdot 10^{-4}\left(\frac{T}{K} - 273.15\right)^4\right) \cdot \frac{erg}{mol \cdot K}$$

$T_1 := 700\ K$ $\quad q := -10 \cdot 10^3 \cdot BTU$ $\quad n := 500\ mol$

Solve block:

Guess Values

$T_2 := 500\ K$

Constraints

$$q = n \cdot \int_{T_1}^{T_2} C_p(T)\,\mathrm{d}T$$

Solver

$\text{find}(T_2) = 455.522\ K$