Lecture 21 Practice - Nonlinear Equations II
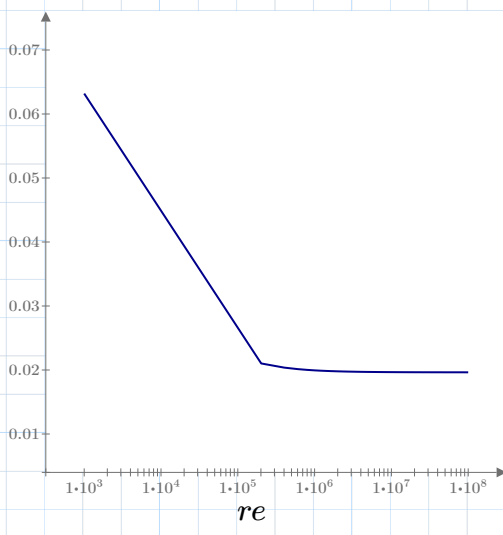
(a) **Solve block with parameters**. Plot the friction factor, f, versus the
Reynolds number, Re for the Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2 \cdot log\left(\frac{1}{3.7}\frac{\varepsilon}{D} + \frac{2.51}{Re \cdot \sqrt{f}}\right)$$

when $\dfrac{\varepsilon}{D} = 10^{-3}$

$$\varepsilon D := 10^{-3}$$

Constraints / Guess Values / Solver

$$f_g := 10^{-2}$$

$$\frac{1}{\sqrt{f_g}} = -2 \cdot log\left(\frac{1}{3.7}\varepsilon D + \frac{2.51}{re \cdot \sqrt{f_g}}\right)$$

$$f(re) := \mathbf{find}\,(f_g)$$

$f(re)$

Tip: You can make the x-axis
logarithmic by (1) Selecting the
x-axis label, (2) Clicking on the
Ribbon->Plots->Axes-
>Logarithmic scaling.

re

$$re := \text{logspace}\left(10^3, 10^8, 501\right)$$ (use 500 for nice resolution)

$$i := 0 .. 500$$ (need range variable to access the elements of the vector)

$$ff_i := f\left(re_i\right)$$

ff

re
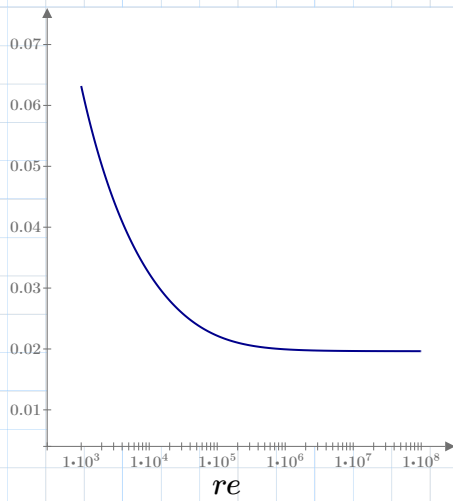
(b) **Best Practices with Solve Blocks.** Simplify the four equations to two equations for x and y, and find the two roots.

$$y + \sin(z) + x = 4$$
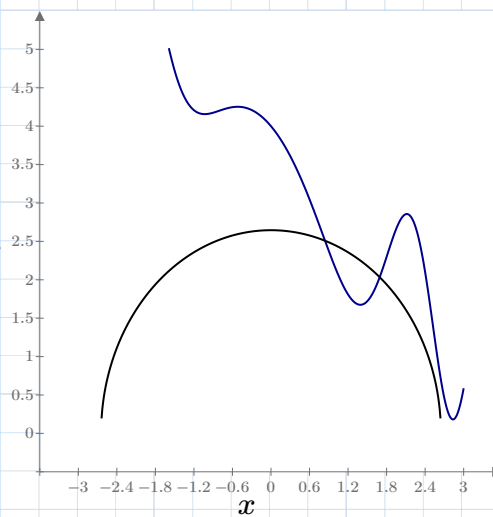$$e^{2\,w} + z = 7$$
$$z = x^2$$
$$w = \ln(y)$$

The equations re-arrange to:

$$y + \sin(x^2) + x = 4$$
$$y^2 + x^2 = 7$$

$$\frac{4 - x - \sin(x^2)}{}$$

$$\frac{\sqrt{7 - x^2}}{}$$

Root 1

Solver Guess Values

$$x := 0.5 \qquad y := 2.4$$

$$y + \sin(x^2) + x = 4 \qquad y^2 + x^2 = 7$$

$$\mathbf{find}(x, y) = \begin{bmatrix} 0.841 \\ 2.508 \end{bmatrix}$$

Root 2

Solver Guess Values

$$x := 1.5 \qquad y := 2.4$$

$$y + \sin(x^2) + x = 4 \qquad y^2 + x^2 = 7$$

$$\mathbf{find}(x, y) = \begin{bmatrix} 1.692 \\ 2.034 \end{bmatrix}$$

For reference: without simplification

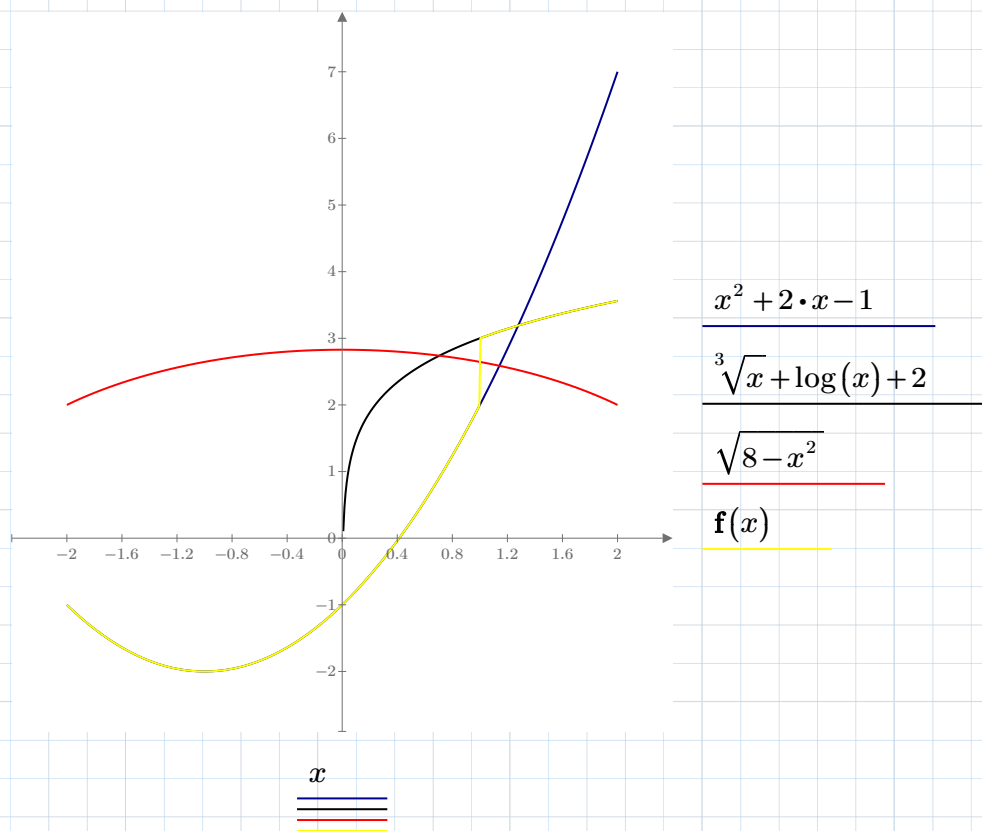$$w := 4 \qquad x := 0.5 \qquad y := 2.4 \qquad z := -5$$

$$y + \sin(z) + x = 4 \qquad e^{2w} + z = 7 \qquad z = x^2 \qquad w = \ln(y)$$

$$\mathbf{find}(w, x, y, z) = \begin{bmatrix} 0.92 \\ 0.841 \\ 2.508 \\ 0.708 \end{bmatrix}$$

(c) **Debugging Solve Blocks.** Identify why this this solve block does not return a value for x and y.

$x := 2 \qquad y := 1$

$\mathbf{if}\left(x < 1, x^2 + 2 \cdot x - 1, \sqrt[3]{x} + log(x) + 2\right) = y$

$x^2 + y^2 = 8$

$ans := \boxed{\mathbf{find}\,(x, y)} = \,?$

Constraints — Guess Values (left margin labels)
Solver (left margin label)

$\mathbf{f}(x) := \mathbf{if}\left(x < 1, x^2 + 2 \cdot x - 1, \sqrt[3]{x} + log(x) + 2\right)$

$x^2 + 2 \cdot x - 1$

$\sqrt[3]{x} + log(x) + 2$

$\sqrt{8 - x^2}$

$\mathbf{f}(x)$

x

As one can see in the plot, the function is discontinuous where they should intersect. Thus the solve block is actually correct, but the problem is not solvable.