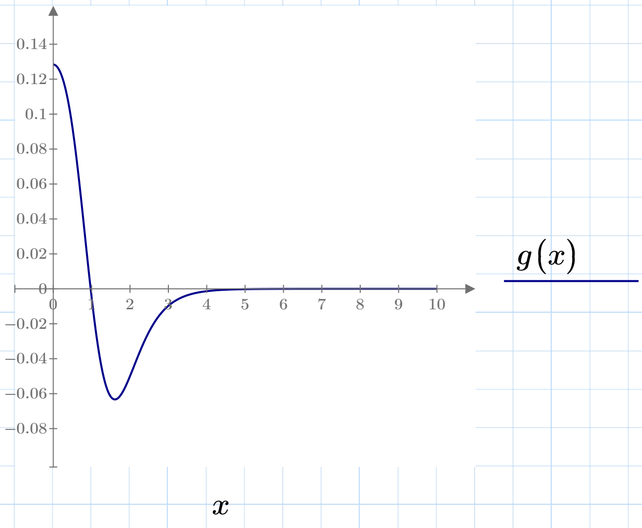
Lecture 23 Practice - Calculus and ODEs

(a) Let $f(x) = erf(x) - tanh(x)$ where erf(x) is the error function. Symbolically evaluate the derivative of f(x) and plot the resulting function for x = 0 to 10.

$$f(x) := \mathrm{erf}(x) - \tanh(x)$$

$$g(x) := f'(x) \rightarrow \tanh(x)^2 + \frac{2 \cdot e^{-x^2}}{\sqrt{\pi}} - 1$$



(b) Symbolically evaluate the Laplace transform of sin(at) and sinh(at), where a Laplace transform is given by:

$$F(s) = \int_0^\infty f(t)\, e^{-s \cdot t}\, \mathrm{d}t$$

$$\int_0^\infty \sin(a \cdot t)\, e^{-s \cdot t}\, \mathrm{d}t \rightarrow \frac{a}{a^2 + s^2} + \lim_{t \to \infty^-} -\frac{e^{-(s \cdot t)} \cdot (a \cdot \cos(a \cdot t) + s \cdot \sin(a \cdot t))}{a^2 + s^2}$$

$$\int_0^\infty \sinh(a \cdot t)\, e^{-s \cdot t}\, \mathrm{d}t \rightarrow \lim_{t \to \infty^-} \left(\frac{e^{a \cdot t - s \cdot t}}{2 \cdot a - 2 \cdot s} + \frac{e^{-(a \cdot t) - s \cdot t}}{2 \cdot a + 2 \cdot s} \right) - \frac{1}{2 \cdot a + 2 \cdot s} - \frac{1}{2 \cdot a - 2 \cdot s}$$

(c) Use numerical integration to find the value of y at t=3, given that,

$$y'(t) = -3 \cdot t^2$$
$$y(0) = 4$$

$$4 + \int_0^3 -3 \cdot t^2 \, dt = -23$$

(d) Use a solve block to solve the ODE for y(t) in the range of t=0,3

$$y'(t) = -3 \cdot t^2$$
$$y(0) = 4$$

Plot your answer. Does it agree with part c?

Constraints

$$y'(t) = -3 \cdot t^2$$

$$y(0) = 4$$

Solver

$$y := \text{odesolve}(y(t), 3)$$

$$y(3) = -23$$

Yep, agrees with (c).