Homework 10

Ch En 263 – Numerical Tools

Due date: 14 May 2020

Instructions

- For the handwritten problems, submitted a single pdf file on Learning Suite with the name "LastName_FirstName_HW10.pdf".
- For the problems in Excel, submit a workbook named "LastName_FirstName_HW10.xlsx" where each worksheet tab is named "Problem_1", "Problem_2", etc.
- For the problems in Python, submit a separate file for each problem named "LastName_FirstName_HW10_ProblemXX.py" where XX is the problem number.
- **Please report how long it took you to complete the assignment (in hours) in the "Notes" section on Learning Suite.**

Problems

1. Do the following by hand. Consider the system of linear equations

$$\begin{aligned} -2x_0 + x_1 - 2x_2 &= 1 \\ x_0 + x_1 - x_2 &= -6 \\ x_0 - 2x_1 - x_2 &= -3 \end{aligned}$$

   (a) Find the $\boldsymbol{A}$ matrix and $\boldsymbol{b}$ vector when this problem is written in the form $\boldsymbol{A} \cdot \boldsymbol{x} = \boldsymbol{b}$

   (b) Use Gauss Elimination to solve this problem by hand.

2. You can confirm that your solution to the previous problem is correct by calculating a quantity called the *residual*, which is defined as the norm

$$\epsilon = |\boldsymbol{A} \cdot \boldsymbol{x} - \boldsymbol{b}| .$$

   Write a Python function called `residual` that takes arguments of an $n \times n$ matrix `A`, an $n \times 1$ vector `b`, and an $n \times 1$ vector `x` and returns the (scalar) value of the residual. Use loops and arrays to execute the matrix algebra. Use your answer to Problem 1 as a test case to confirm that you have the correct solution for Problem 1 and that your residual code works.

3. Do the following by hand. Use the first step of the Gauss Elimination algorithm (forward elimination) to reduce the linear system

$$\begin{aligned} 4x_0 - 3x_2 - 4x_3 &= 4 \\ x_0 - 4x_1 + 4x_2 + 2x_3 &= -21 \\ x_0 - 5x_1 - 3x_2 + 4x_3 &= 5 \\ -4x_0 - 3x_1 + x_2 + 2x_3 &= 6 \end{aligned}$$

   to an upper triangular matrix and a modified right-hand-side (RHS) vector. Your answer should consist of the upper triangular matrix and RHS vector that results from the procedure described in class and in the notes. Other versions will not get full credit; the point of this problem is to become familiar with the forward elimination algorithm to help you program it on the next homework. You *do not* need to report the solution, $\boldsymbol{x}$.

4. Using the function that you developed in Problem 2, decide which is the correct solution to the linear system in Problem 3:

a) $x = \begin{bmatrix} -3 \\ 0 \\ -4 \\ -1 \end{bmatrix}$ b) $x = \begin{bmatrix} 5 \\ -1 \\ 4 \\ 7 \end{bmatrix}$ c) $x = \begin{bmatrix} 3 \\ 0 \\ 5 \\ -1 \end{bmatrix}$ d) $x = \begin{bmatrix} 2 \\ -1 \\ -4 \\ 7 \end{bmatrix}$

You should turn in a Python code that uses the `residual` function to test each of the potential answers (a-d). Your code should print something to the console that identifies the correct solution.