Homework 11

Ch En 263 – Numerical Tools

Due date: 21 May 2020

Instructions

- For the handwritten problems, submitted a single pdf file on Learning Suite with the name "LastName_FirstName_HW11.pdf".
- For the problems in Excel, submit a workbook named "LastName_FirstName_HW11.xlsx" where each worksheet tab is named "Problem_1", "Problem_2", etc.
- For the problems in Python, submit a separate file for each problem named "LastName_FirstName_HW11_ProblemXX.py" where XX is the problem number.
- **Please report how long it took you to complete the assignment (in hours) in the "Notes" section on Learning Suite.**

Problems

1. In this problem you will write a Python program to do forward elimination for the system of linear equations:

$$\begin{aligned} -2x_0 + x_1 - 2x_2 &= 1 \\ x_0 + x_1 - x_2 &= -6 \\ x_0 - 2x_1 - x_2 &= -3 \end{aligned}$$

Note that this is one of the systems you solved for the last homework. This can help you debug your code!

   (a) Define numpy array variables `A` and `b` and a variable for the number of rows, `n`.

   (b) Write a loop for $k = 0, 1, \ldots, n-2$ that prints out the diagonal element of each row (except the last one) of the matrix, $a_{k,k}$.

   (c) Write a nested loop for $k = 0, 1, \ldots, n-2$ and $i = k+1, k+2, \ldots, n-1$ that prints out the ratio $a_{i,k}/a_{k,k}$ where i are the rows below the k^{th} diagonal.

   (d) Write the full forward elimination algorithm using a triple nested loop where the third loop runs over the columns in row i for $j = k, k+1, \ldots, n-1$. Print the final upper-trianglar matrix and modified RHS (right-hand side) vector `b` to the console.

2. In this problem you will write a Python program to do back substitution. Consider the upper triangular system of linear equations

$$\begin{aligned} x_0 + 2x_1 + 3x_2 &= 13 \\ x_1 - x_2 &= 2 \\ -2x_2 &= -4 \end{aligned}$$

   (a) Write the matrix $\boldsymbol{A}$ and vector $\boldsymbol{b}$ for this system of equations and solve for $\boldsymbol{x}$ by hand.

   (b) Define numpy arrays for $\boldsymbol{A}$ and $\boldsymbol{b}$.

(c) Write a loop which performs the sum $\sum_{j=i+1}^{n-1} a_{i,j}x_j$ for $i = 0$ assuming $x = [0, 1, -3]$. Print the sum to the console.

(d) Write the full back substitution algorithm using a nested loop and print the solution, x, to the console. Use your hand-written solution to check your steps as necessary.

3. In this problem you will write a code to do a complete Gauss elimination algorithm in Python. *Hint: Re-use the forward elimination and backward substitution codes from above to help you do this.*

(a) Write a function called `Gauss` that takes two arguments, a 2D Numpy array `A` and a 1D numpy array `b` and returns the solution `x` obtained via the Gauss elimination algorithm.

(b) Import the data in `A.csv` into a 2D array `A` and the data in `b.csv` into a 1D array `b`. Use the function you defined in part (a) to find the solution `x` and print it to the console.