Homework 18

Ch En 263 – Numerical Tools

Due date: 4 Jun 2020

Instructions

- For the handwritten problems, submitted a single pdf file on Learning Suite with the name "LastName_FirstName_HW18.pdf".
- For the problems in Excel, submit a workbook named "LastName_FirstName_HW18.xlsx" where each worksheet tab is named "Problem_1", "Problem_2", etc.
- For the problems in Python, submit a separate file for each problem named "LastName_FirstName_HW18_ProblemXX.py" where XX is the problem number.
- **Please report how long it took you to complete the assignment (in hours) in the "Notes" section on Learning Suite.**

Problems

1. Answer the following in a text box in an Excel worksheet.
   (a) Explain in your own words what a spline is.
   (b) In class we talked about using data from a fitbit to get your speed (fitting) and determining how much it will cost to fill your gas tank (interpolation). Come up with your own example of a situation where you might need to use either fitting or interpolation. Explain why fitting or interpolation is the appropriate choice for the situation you chose.
2. In this problem, we are going to explore how well an interpolation approximates a function. Use the data in the table to the right to answer the questions in this problem.
   (a) Use Excel to calculate the linear interpolate y_{linear} at $x = 0.54$.
   (b) Use Python to generate a cubic spline interpolation y_{cubic} at $x = 0.54$. Print the value of y_{cubic} to the console.
   (c) The data in the table to the right comes from the function $y = \exp(4x)$. The relative error of an interpolation is given by $\epsilon = |(y_{\text{interpolate}} - y_{\text{exact}})/y_{\text{exact}}|$. Find the relative error between your interpolates and the exact value (from the function). How accurate is the linear interpolation compared to the cubic spline?

x	y
0	1.0
0.2	2.2255409
0.4	4.9530324
0.6	11.0231764
0.8	24.5325302
1.0	54.5981500

3. In this problem we are going to explore how noisy data can affect an interpolation. The data set "HW18-Prob3-data.csv" contains three sets of x and y pairs based on an underlying function
$$y = \exp(-2x)\sin(4\pi x). \tag{1}$$
One of the data sets has no noise, one has a small amount of noise and one has a large amount of noise.

(a) Use Python functions to generate a cubic spline for each data set (no noise, small noise, large noise). Using the original function in Eq. 1, calculate the relative error of your interpolation at $x = 1.37$ for each data set. (See Problem 2 for the definition of the relative error.) Print the values of the relative error to the console.

(b) Make three separate plots—one for each data set—and compare your cubic spline to the original function in Eq. 1. Each plot should be formatted using:

- A solid line for the original function.
- Points for the data.
- A dashed line for the cubic spline.
- A legend labelling the interpolating function as "no noise", "small noise" or "large noise".

(c) What happens to the interpolation as the data gets noisier? Is the interpolation better or worse for noisy data when the y is close to zero? Speculate about why. Print your comments as a string to the console.