Homework 21

Ch En 263 – Numerical Tools

Due date: 11 Jun. 2020

Instructions

- For the handwritten problems, submitted a single pdf file on Learning Suite with the name "LastName_FirstName_HW21.pdf".
- For the problems in Excel, submit a workbook named "LastName_FirstName_HW21.xlsx" where each worksheet tab is named "Problem_1", "Problem_2", etc.
- For the problems in Python, submit a separate file for each problem named "LastName_FirstName_HW21_ProblemXX.py" where XX is the problem number.
- **Please report how long it took you to complete the assignment (in hours) in the "Notes" section on Learning Suite.**

Problems

1. Solve the IVP

$$\frac{dv}{dt} = g - cv^2$$
$$v(0) = v_0$$

for $t \in [0, 2]$ in Excel using the Explicit Euler method with paramters $g = 9.81$ m/s^2, $c = 1$ m^{-1} and $v_0 = 0$ m/s. Use $\Delta t = 0.1$ and plot the solution.

2. Solve the IVP

$$\frac{dy}{dx} = x^2 y^{1/2}$$
$$y(0) = 1$$

in Python using the Explicit Euler method using $\Delta x = 0.1$ from $x \in [0, 2]$ The exact solution of this ODE is

$$y = \left(\frac{x^3}{6} + 1\right)^2$$

Plot the numerical solution and the exact solution on the same plot. Make sure to include a legend that labels the different curves.

3. Solve the IVP from Problem 2

$$\frac{dy}{dx} = x^2 y^{1/2}$$
$$y(0) = 1$$

in Python using the Explicit Euler method using four different values of $\Delta x = [10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}]$. (*Hint: This is probably easiest by done writing a function to solve the IVP that takes Δx as an argument.*) Calculate the error $\epsilon = ||y_{\text{Euler}} - y_{\text{exact}}||$ between each of your four numerical solutions and the exact solution. Make a plot of $\log(\epsilon)$ versus $\log(\Delta x)$. What do you notice about the plot?

4. A continuous stirred tank reactor reacts chemical species A according to the following rate equation:

$$\frac{dA}{dt} = \frac{A_{in} - A}{\tau} - kA^2$$
$$A(0) = A_0.$$

   (a) Solve this equation in Excel using the Explicit Euler method. Plot $A(t)$ out to $t = 15$ using $A_{in} = 1$, $A_0 = 0$, $\tau = 2$, and $k = 0.1$. Use $\Delta t = 0.1$.

   (b) What is the long-time (steady state) value of A from your solution?

   (c) In the above equation, set $dA/dt = 0$ and solve for A. This is the analytic steady state value of A. How does it compare to the long-time solution from the rate equation?