Homework 22

Ch En 263 – Numerical Tools

Due date: 11 Jun. 2020

Instructions

- For the handwritten problems, submitted a single pdf file on Learning Suite with the name "LastName_FirstName_HW22.pdf".
- For the problems in Excel, submit a workbook named "LastName_FirstName_HW22.xlsx" where each worksheet tab is named "Problem_1", "Problem_2", etc.
- For the problems in Python, submit a separate file for each problem named "LastName_FirstName_HW22_ProblemXX.py" where XX is the problem number.
- **Please report how long it took you to complete the assignment (in hours) in the "Notes" section on Learning Suite.**

Problems

1. The temperature change of a coal particle in a furnace is given by the following rate equation

$$\frac{dT}{dt} = \frac{hA}{mc_p}(T_f - T) + \frac{\sigma A}{mc_p}(T_f^4 - T^4).$$

The initial particle temperature is $T_0 = 500$ K. In addition, the following data are given

Variable	Value	Units
D	100	μm
ρ_p	1000	kg/m^3
c_p	1380	J/kg·K
k	0.1	W/m·K
Nu	2	–
σ	5.67E-8	W/m^{2}K^4
T_f	1500	K
t_{end}	0.05	s

Also, the area, mass and Nusselt number Nu are given, respectively, by:

$$A = \frac{\pi}{4}D^2,$$

$$m = \frac{\pi}{6}D^3\rho_p,$$

$$\text{Nu} = \frac{hD}{k}.$$

Solve the rate equation for the particle temperature as a function of time using `solve_ivp`. Plot your solution and label and format your plot (including units).

2. A harmonic oscillator obeys the second-order differential equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

where m is the mass, k is the spring constant and c is a constant characterizing a damping force. Re-arrange the harmonic oscillator equation as a system of first order rate equations. Solve the resulting system of equations using `solve_ivp` with $m = 1$ kg, $k = 1$ kg/s^2 and $c = 0.5$ kg/s. Additionally assume that the initial velocity is zero, $x_0 = 1$ m and $t_{\text{end}} = 10$ s. Plot (and label) the velocity and the position as a function of time.

3. Solve the set of rate equations,

$$\begin{aligned}\frac{dx}{dt} &= \sigma(y - x)\\ \frac{dy}{dt} &= x(\rho - z) - y\\ \frac{dz}{dt} &= xy - \beta z\end{aligned}$$

over the range $t \in [0, 35]$ where $x(0) = 5$, $y(0) = -2$, $z(0) = 1$ and $\sigma = 10$, $\rho = 28$ and $\beta = 8/3$. Make two plots: $y(t)$ versus $x(t)$ and $z(t)$ versus $x(t)$. Note that time is implicit in these plots. *Hint: you will need a large number of time points ($N_{pts} > 1000$) to get a well resolved plot.*