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#!/usr/bin/env python3
# -*- coding: utf-8 -*-

import numpy as np

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# Lecture 11 - Gauss Elimination
# (A) Forward Elimination
# -----
#
# - Reduce  $A.x = b$  to  $U.x = b'$ 
#

A = np.array([[-3., -5., -5., -2.],
              [-5., -3., 0., -2.],
              [-3., 2., -1., 2.],
              [ 1., -2., -4., -5.]])

b = np.array([6., 13., 3., -4.])
n = len(b)

print('Beginning...\n')
print('-'*20)
print('A:')
print(A)
print('b:')
print(b)
print('-'*20)

# Forward Elimination algorithm:
#  $a_{ij} = a_{ij} - (a_{ik}/a_{kk}) * a_{kj}$ 

print('\nForward Elimination...')

for k in range(0, n-1): # loop over the diagonals
    for i in range(k+1, n): # loop over the rows
        ratio = A[i,k]/A[k,k]
        for j in range(k, n): # loop over the elements (cols) in the row
            A[i,j] -= ratio*A[k,j]

            print('(k: ', k, ', i: ', i, 'j: ', j, ') ', 'ratio: ', ratio)
        print(A)

        b[i] -= ratio*b[k] # don't forget the b vector

print('Finished...\n')
print('-'*20)
print('A:')
print(A)
print('b:')
print(b)
print('-'*20)

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