Lecture 16 - Engineering Problems

* AMA / Quiz / Prayer

I. Tips for Solving Engineering Problems

* Engineering problems are a bit harder than the "pure math" problems we have been doing for the last few weeks.

* It turns out that the math is the same, but the "window-dressing" is different, so they can be confusing for students.

* I'm going to give you four tips for solving these problems, then we are going to practice.

* to give you these tips, it helps to have a concrete example, so consider the following:

what is the diameter of a 100 meter pipe where the supplied head is 32 m and the flow rate is 6 L/s? (Assume the fluid is water: $\mu = 1\,cP$, $\rho = 10^3\ kg/m^3$.)

↖ and $g = 9.8\ \frac{m}{s^2}$

Pressure lost in a pipe when the fluid flows → $$h = \frac{-v^2}{2g}\left(\frac{4Lf}{D}\right)$$

$h = -32\ m$

$L = 100\ m$

$$\frac{1}{\sqrt{f}} = 4\log_{10}\left(Re\sqrt{f}\right) - 0.4$$

← relation between fluid speed and how much friction there is. (dimensionless)

Reynold's number (dimensionless speed) → $$Re = \frac{\rho v D}{\mu}$$

$Q = 6\ L/s$

$$v = \frac{4Q}{\pi D^2}$$ ← velocity from flow rate.

Tip 1: Identify the type of equation and pick a solution method.

* Is it linear or non-linear? A single equation or a system of equations?

* what numerical tool will you need? Excel's solver? opt.root()? np.linalg.solve?

* Our example: non-linear, system → Python opt.root()
  (also ok: opt.minimize() or Excel → Solver)

Tip 2: Convert to standard form & vector notation.

* This is (in principle) the exact same process as "math-only" problems, but it is a bit less transparent.

* Identify knowns & unknowns

  knowns: h, L, Q, ρ, μ, g

  Unknowns: v, Re, f, D

* convert unknowns to $x_0, x_1, \ldots, x_{n-1}$

$$x_0 = v$$
$$x_1 = Re$$
$$x_2 = f$$
$$x_3 = D$$

* write $f_0, f_1, \ldots, f_{n-1}$ in standard form $\underline{\underline{A}} \cdot \underline{x} = \underline{b}$ (linear)

  $\underline{f}(\underline{x}) = \underline{0}$ (non linear)

$$f_0 = 0 = h + \frac{x_0^2}{2g}\left(\frac{4Lx_2}{x_3}\right)$$

$$f_1 = 0 = \frac{1}{\sqrt{x_2}} - 4\log_{10}\left(x_1\sqrt{x_2}\right) - 0.4$$

$$f_2 = 0 = x_1 - \frac{\rho x_0 x_3}{\mu}$$

$$f_3 = 0 = x_0 - \frac{4Q}{\pi x_3^2}$$

- Should use x_0, x_1, x_2 & x_3
- Only vars left should be knowns. (h, g, L, ρ, μ, Q) ✓

Tip 3: Convert to consistent units

* we talked about this at the beginning of class. This is an important time to remember this.
* Pick either SI / English units and convert at the top of the code.
* Alternatively, you can make the equations dimensionless.
* Example:

$Q = 6\ L/s = 6\times10^{-3}\ m^3/s$

$\mu = 1\,cP = 10^{-2}\ \frac{kg}{m\,s}$

$h = -32\ m$, $L = 100\ m$

$g = 9.8\ m/s^2$, $\rho = 10^3\ kg/m^3$

Lengths: m (ft)
time: s (s)
mass: kg (slug)

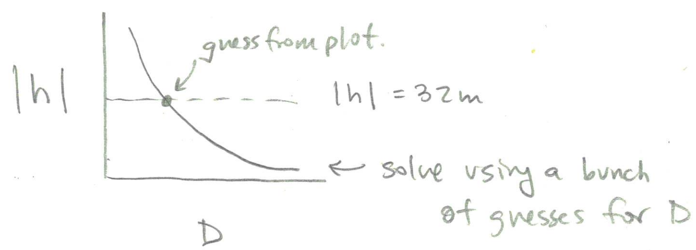
Tip 4: Make smart guesses

* To get your method to converge (for NLE) you need a guess that is "close." You can get this a couple of different ways.

  - Physical Intuition (how big are pipe diameters? cm not km)

- what units are the problem given in? A good guess is often in those units. E.g. cm or km?
- Use one consistent guess and solve for other guesses.

  E.g. D = 1 cm → find v, Re, etc.
- If 1D or 2D, make a plot:



- Use answers from related theories:

  $f = \frac{16}{Re}$ for laminar

  $p = \frac{NRT}{V}$ for ideal gas
- use mathematical / physical bounds or expert inuition.

  $v \ll 10^8$ m/s (speed of light)

  $f \approx 10^{-3}$ (from years of doing this)

  $0 < \phi < 1$ (volume fraction)

  $D > 0$ (no negative pipe diameters)

II. Case Study

* See attached handout for a practice problem related to DNA force/extension curves.