

Practice Final Exam

Ch En 263 – Numerical Tools

Winter 2023

Instructions

- You have 3 hours to complete the exam.
- You **may** use your notes, the internet, help menus, etc. (i.e. the exam is “open book”). You may not look at another persons exam or ask them for help, but you may of course ask clarifying questions to Dr. Tree or the TAs.
- You need a computer to complete this exam. You may use either the lab computer or your own computer, but **you may not use a calculator**. You may also use scratch paper or write on your test, but neither will be accepted for credit.
- A **required** Excel template and necessary data files are provided on Learning Suite. **Enter your answers in the gray colored cells in the Excel workbook**. Also, please do not move the colored cells in the Excel workbook.
- Submit your exam to Learning Suite at the end of the time period. You will submit two files:
 - Lastname_Firstname_FinalExam.xlsx
 - Lastname_Firstname_FinalExam.py
- The Python code that you submit should complete without syntax or execution errors. Points will be taken off for codes that do not run.

Save often and make sure you submit the correct files!

Exam Contents

This exam contains:

- 24 Qualitative (T/F, Short Answer, MC) Questions (3 pts each, 72 pts)
- 8 Quantitative Multiple Choice Questions (8 pts each, 64 pts)
- 4 Free Response Questions (16 pts each, 64 pts)

I. Qualitative Questions (72 pts)

Answer the indicated question with the correct multiple choice letter or short answer as indicated. Enter your answer into the “Multiple_Choice” worksheet in the Excel workbook named “Lastname_Firstname_FinalExam.xlsx.” No partial credit will be given.

1. Recording a Macro in Excel writes a Python subroutine that can be edited in Excel’s built-in developer environment.

(a) True (b) False

2. True or False? Although it isn’t as accurate as `scipy.integrate.quad`, the function `scipy.integrate.simps` is useful for integrating data when an explicit expression for the function to be integrated is not available.

(a) True (b) False

3. True or False? Using piecewise polynomials to interpolate a data set instead of a single high-order polynomial avoids large oscillations known as “Runge’s Phenomena”.

(a) True (b) False

4. True or False? A Netwon-Coates formula is a method for solving a nonlinear equation by using an interpolating polynomial to represent the function.

(a) True (b) False

5. True or False? All computers (e.g. a smartphone, a laptop or a supercomputer) can perform the same kinds of numerical calculations, because all computers contain the same four basic elements (input, output, memory, CPU).

(a) True (b) False

6. True or False? An execution error (as opposed to a syntax or logical error) is a bug akin to a “spelling error” that the Python interpreter catches and causes execution to immediately abort.

(a) True (b) False

7. Which is equivalent to 1 lbf?

(a) $1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$ (b) $1 \frac{\text{lbf} \cdot \text{ft}}{\text{s}^2}$ (c) $1 \frac{\text{slug} \cdot \text{ft}}{\text{s}^2}$ (d) 2.2046 N

8. To use the quantity 3.14159 in a calculation in Python, it will need to be stored in memory as a:

(a) boolean data type (b) integer data type
(c) floating point data type (d) string data type

9. In general, nonlinear equations are more difficult to solve than linear equations because

- (a) There are no “direct methods” for solving nonlinear equations.
- (b) There is no guarantee of a solution for a nonlinear equation.
- (c) There may be multiple solutions for a nonlinear equation.
- (d) All of the above.

10. Identify the formula that belongs to the class of fixed-point methods.

- (a) $x^{k+1} = x^k - f(x^k)/f'(x^k)$
- (b) $y = y_0 \frac{x_1 - x}{x_1 - x_0} + y_1 \frac{x - x_0}{x_1 - x_0}$
- (c) $\frac{\Delta x}{2} [f_0 + 2f_1 + \dots + 2f_{n-1} + f_n]$
- (d) $y_{n+1} = y_n + \Delta t f(y_n, t_n)$

11. Classify the equations

$$\begin{aligned} 3x + 4y - z &= -2 \\ -5x + 3xy + 3z &= 5 \\ -x - y + z &= -1 \end{aligned}$$

- (a) linear, algebraic, coupled system
- (b) nonlinear, algebraic, coupled system
- (c) linear, algebraic, uncoupled system
- (d) nonlinear, algebraic, uncoupled system
- (e) linear, differential, coupled system
- (f) nonlinear, differential, coupled system
- (g) linear, differential, uncoupled system
- (h) nonlinear, differential, uncoupled system

12. Classify the equations

$$\begin{aligned} \frac{dx}{dt} &= 5x - 9y - 4z & x(0) &= 6 \\ \frac{dy}{dt} &= 2x + 5y + 7t & y(0) &= -5 \\ \frac{dz}{dt} &= -x + z + 2t^2 & z(0) &= 6 \end{aligned}$$

- (a) 1st order ODEs, linear, ICs
- (b) 2nd order ODEs, linear, ICs
- (c) 1st order ODEs, linear, BCs
- (d) 2nd order ODEs, linear, BCs
- (e) 1st order ODEs, nonlinear, ICs
- (f) 2nd order ODEs, nonlinear, ICs
- (g) 1st order ODEs, nonlinear, BCs
- (h) 2nd order ODEs, nonlinear, BCs

Note: ICs = Initial conditions, BCs = Boundary conditoinis, ODEs = ordinary differential equations

13. Identify the output from the snippet of Python code below

```
a = 3
b = 2
c = 5
if (a == b):
    print(a-b)
elif (a >= c):
    print(a+c)
else:
    print(a*b)
```

- (a) 1
(c) 8

- (b) 6
(d) SyntaxError: invalid syntax

14. The following snippet of code

```
var = [3, 1, 4, 1, 5, 9]
```

defines a

- (a) Numpy array (b) tuple (c) list (d) dictionary

15. Identify the output from the snippet of Python code below

```
N = 10
i = 0
while (i < N):
    print(i)
```

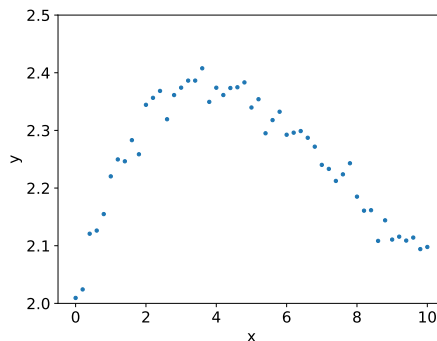
- (a) 0 0 0 ... (forever) (b) 0 1 2 3 4 5 6 7 8 9
(c) 0 1 2 3 4 5 6 7 8 9 10 (d) 1 2 3 4 5 6 7 8 9 10

For problems 16–19, match the problem, equation or system of equations with the best solution method or procedure.

	Problems	Solution Methods
16.	$5x - 4y - 2z = 5$ $x + 5y = 1$ $3x - 9y + 6z = -10$	<p>(a) Least-Square Regression</p> <p>(b) Newton's method</p>
17.	$\frac{1}{\sqrt{f}} = 4.0 \log(\text{Re} \sqrt{f}) - 0.4$	<p>(c) Trapezoidal Rule</p> <p>(d) Gauss Elimination</p>
18.	$\frac{dA}{dt} = -kAB^2$ $\frac{dB}{dt} = -2kAB^2$ $\frac{dC}{dt} = kAB^2$	<p>(e) Linear Interpolation</p> <p>(f) Explicit Euler</p>
19.	$I = \int_0^{1/2} \cos(\pi x) dx$	

20. Choose the most appropriate Python tool if you want to obtain a value for y at $x = 4.5$ that is consistent with the data in the figure below.

- (a) Use `scipy.interpolate.interp1d` with the 'linear' option
 (b) Use `scipy.interpolate.interp1d` with the 'cubic' option
 (c) Use `numpy.polyfit` and `numpy.polyval`
 (d) All of the above choices are appropriate in this case.



21. If we want to fit the model $f(x) = ax + b$ to a set of data $\{x_i, y_i\}$ via a least-square method we need to

- (a) vary a and b to minimize the sum of the squared error, $SSE = \sum [y_i - f(x_i)]^2$
 (b) vary x and y to minimize the sum of the squared error, $SSE = \sum [y_i - f(x_i)]^2$
 (c) vary a and b to minimize the total sum of squares, $SST = \sum [y_i - \bar{y}]^2$
 (d) vary x and y to minimize the total sum of squares, $SST = \sum [y_i - \bar{y}]^2$

22. The coefficient of determination, R^2 , is defined as

- (a) the total spread of the data, $R^2 = SST$
 (b) the spread of the data due to noise, $R^2 = SSE$
 (c) the fraction of the spread of the data coming only from noise, $R^2 = \frac{SSE}{SST}$
 (d) the fraction of the spread of the data not coming from noise, $R^2 = 1 - \frac{SSE}{SST}$
 (e) the ratio of the total spread of the data to the spread coming from noise, $R^2 = \frac{SST}{SSE}$

23. Which of the following is a correct re-statement of the differential equation as a system of 1st order ODEs?

$$\frac{d^3x}{dt^3} + 3\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 5x = -t^2$$

(a)
$$\begin{aligned} \frac{dx}{dt} &= -3z + 2y - 5x - t^2 \\ \frac{dy}{dt} &= x \\ \frac{dz}{dt} &= y \end{aligned}$$

(b)
$$\begin{aligned} \frac{dz}{dt} &= -3z + 2y - 5x - t^2 \\ \frac{dy}{dt} &= z \\ \frac{dx}{dt} &= y \end{aligned}$$

(c)
$$\begin{aligned} \frac{dx}{dt} &= \frac{5}{2}x + \frac{1}{2}t^2 \\ \frac{dy}{dt} &= -\frac{y}{3} \\ \frac{dz}{dt} &= -z \end{aligned}$$

(d)
$$\begin{aligned} \frac{d^3x}{dt^3} &= -3z + 2y - 5x - t^2 \\ \frac{d^2x}{dt^2} &= z \\ \frac{dx}{dt} &= y \end{aligned}$$

24. Suppose you want to solve the system of equations

$$\begin{aligned}x^2 &= \sin(y) \\ y^2 &= e^{-xy}\end{aligned}$$

in Python using an optimization method. You are given the following code:

```
1 import numpy as np
2 import scipy.optimize as opt
3
4 def f(x, y):
5     f1 = x**2 - np.sin(y)
6     f2 = y**2 - np.exp(-x*y)
7     return f1**2+f2**2
8
9 guess = np.array([1, 1])
10 soln = opt.minimize(f, guess).x
```

What should be done so that `soln` contains correct roots?

(a) Replace Line 4 by

```
def f(xy):
    x = xy[0]
    y = xy[1]
```

(b) Lines 5-6 should be replaced by

```
f1 = x**2 + np.sin(y)
f2 = y**2 + np.exp(-x*y)
```

(c) Line 7 should be replaced by

```
return np.array([f1, f2])
```

(d) None of the above. The code is correct.

II. Quantitative Multiple Choice (64 pts)

Enter your answer into the “Multiple_Choice” worksheet in the Excel workbook named “Last-name_Firstname_FinalExam.xlsx.”

- Show your work for the Excel problems in a worksheet named “ProblemXX” where XX is the problem number in the same Excel workbook.
- Show your work for the Python problems in a section labelled “Problem XX” where XX is the problem number in the file “Lastname_Firstname_FinalExam.py.”
- *You must show your work to get credit for these problems.*

25. Define a Python function that can evaluate

$$f(x, y) = \sin\left(\frac{\pi|x|y^2}{6}\right) + 1$$

where $|x|$ is the absolute value of x . Evaluate your function at (i) $x = -1$, $y = 1$, (ii) $x = 2$, $y = 4$, and (iii) $x = -4$, $y = 1$.

- | | |
|----------------------|----------------------|
| (a) $f(-1, 1) = 1.5$ | (b) $f(-1, 1) = 1.5$ |
| $f(2, 4) = 0.13398$ | $f(2, 4) = 1.86603$ |
| $f(-4, 1) = 1.86603$ | $f(-4, 1) = 1.86603$ |
| (c) $f(-1, 1) = 0.5$ | (d) $f(-1, 1) = 0.5$ |
| $f(2, 4) = 0.13398$ | $f(2, 4) = -0.86603$ |
| $f(-4, 1) = 0.13398$ | $f(-4, 1) = 0.86603$ |

26. Use Python to evaluate the formula

$$S = \sum_{i=1,3,5,\dots,23} \frac{i^3}{3}$$

- | | | | |
|--------------|-----------|---------------|-----------|
| (a) 9720.333 | (b) 13776 | (c) 21336.333 | (d) 25392 |
|--------------|-----------|---------------|-----------|

27. Use Python to find the value of the integral:

$$I = \int_0^1 x^3 e^{-x} dx$$

- | | | | |
|------------|------------|------------|------------|
| (a) 0.1139 | (b) 0.1839 | (c) 0.3679 | (d) 0.5634 |
|------------|------------|------------|------------|

28. Use Python to solve the system of equations:

$$\begin{aligned} y &= \cosh(x) \\ 2 &= x^2 + y^2 \end{aligned}$$

- | | |
|--|--|
| (a) root: $x = 1.24897, y = 1.88678$ | (b) root: $x = -0.00087, y = -0.19862$ |
| (c) root 1: $x = 0.00920, y = 0.10802$ | (d) root 1: $x = 0.67981, y = 1.24011$ |
| root 2: $x = -0.00920, y = 0.10802$ | root 2: $x = -0.67981, y = 1.24011$ |

29. Use Python to solve the system of equations:

$$5v - 4w - 4x - 3y + z = 41.5$$

$$-3v + 3x + 2y - 6z = -48.5$$

$$-6v - 4w - 4x - 2y = 41$$

$$2v - 3w + 5x + 2y - z = -29$$

$$-v - 6w + x - 4y - z = 11.5$$

- | | | | |
|-----------------|--------------|--------------|----------------|
| (a) $v = 2.506$ | (b) $v = -1$ | (c) $v = -3$ | (d) $v = -0.5$ |
| $w = -1.429$ | $w = -4$ | $w = -6$ | $w = -3$ |
| $x = -13.404$ | $x = -6$ | $x = 5$ | $x = -6$ |
| $y = 1.647$ | $y = 4$ | $y = -1$ | $y = -1$ |
| $z = -25.423$ | $z = 10$ | $z = -0.5$ | $z = 5$ |

30. The table to the right gives the internal energy U at a given temperature T of saturated steam. Use Excel to find a second order polynomial $U(T)$ that fits the data. The best fit equation is:

T (°F)	U (BTU/lbm)
32	1021.3
34	1022.0
36	1022.6
38	1023.3
40	1023.9
42	1024.6
44	1025.2
46	1025.9
48	1026.6
50	1027.2

- (a) $\frac{T}{^\circ\text{F}} = 3.049 \left(\frac{U}{\text{BTU/lbm}} \right) - 3082$
- (b) $\frac{U}{\text{BTU/lbm}} = 0.3279(T/^\circ\text{F}) + 1011$
- (c) $\frac{U}{\text{BTU/lbm}} = 1.894 \times 10^{-4}(T/^\circ\text{F})^2 + 0.3123(T/^\circ\text{F}) + 1011$
- (d) $\frac{U}{\text{BTU/lbm}} = 6.136 \times 10^{-4}(T/^\circ\text{F})^2 + 0.5840(T/^\circ\text{F}) + 1021$

31. In Excel, solve the ODE

$$\frac{dy}{dt} = 2 + \frac{y^5}{1 + y^5} - y$$

using the Explicit Euler method with $y(0) = 4$, $\Delta t = 0.1$ in the range $t \in [0, 5]$. What is the value of $y(5)$?

- (a) 2.9736 (b) 3.0012 (c) 3.7001 (d) 9.0010

32. In Excel, find the quadrature

$$\gamma = \int_{-10}^{10} \left[\text{sech} \left(\frac{x}{2} \right) \right]^2 dx.$$

using the composite trapezoidal rule with $\Delta x = 0.5$.

Hint: The hyperbolic secant function sech is given by SECH() in Excel.

- (a) 9.0792×10^{-5} (b) 3.9996 (c) 7.9994 (d) 1008.6

III. Free Response (64 pts)

33. Do the following in an Excel Worksheet named “Problem33” within the file “Lastname.Firstname.FinalExam.xlsx.”

(a) Use Newton’s method to find the solution(s) to the equation

$$x^2 = \frac{1}{3} \sin(x).$$

(b) Using any valid method, find the value(s) of y and z that solves the system of equations

$$\begin{aligned} 2y^{1/3} + z^2 &= 5 \\ \ln y + 0.25z^2 &= 2 \end{aligned}$$

In both (a) and (b), to get full credit, you must find all of the solutions if the system possesses more than one.

34. Do the following in the Python file named “Lastname.Firstname.FinalExam.py” in a section labelled “Problem 34.”

(a) Use loops to create the 50×50 matrix \mathbf{A} and the 50×1 vector \mathbf{b} ,

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & \dots & 0 \\ -1 & 2 & -1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 2 & -1 & 0 & \dots & 0 \\ 0 & 0 & -1 & 2 & -1 & \dots & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & \dots & 0 & -1 & 2 & -1 & 0 \\ 0 & \dots & 0 & 0 & -1 & 2 & -1 \\ 0 & \dots & 0 & 0 & 0 & -1 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 51 \end{bmatrix}.$$

Print them both to the console.

- (b) Solve for \mathbf{x} in the expression $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$, and print it to the console.
- (c) Using the value of \mathbf{x} that you found in part (b), evaluate the residual $\|\mathbf{A} \cdot \mathbf{x} - \mathbf{b}\|$ and print it to the console.

35. Do the following in the Python file named “Lastname_Firstname_FinalExam.py” in a section labelled “Problem 35.”

The data in the table to the right gives the concentration of ethanol produced by *E. coli* as a function of time in a bioreactor. Use Python to fit the data to the logistic equation

$$C(t) = \frac{a}{1 + \exp(-b(t - c))}$$

where a , b and c are parameters in the model.

- (a) Fit the curve to the data and print the coefficients a , b , and c to the console. In addition, calculate the value of R^2 , and print its value to the console.
- (b) Using the same data, calculate a cubic spline that interpolates the data in the table. Plot the data (as points), the best fit $C(t)$ from part (a) (as a solid blue line) and the cubic spline (as a red dashed curve). Label the x - and y -axes and include a legend.

t (min)	C (mM)
0	0.051
0.4	0.055
0.8	0.174
1.2	0.507
1.6	1.235
2.0	1.709
2.4	1.882
2.8	1.920
3.2	1.933
3.6	1.903
4.0	1.951

36. Do the following in the Python file named “Lastname_Firstname_FinalExam.py” in a section labelled “Problem 36.”

When fluid flows near a wall, a boundary layer develops. The velocity profile, u , is zero at the wall, and transitions to the free-stream velocity far away from the wall. Three rate equations describe the flow:

$$\begin{aligned}\frac{dk}{dx} &= -\frac{1}{2}gk \\ \frac{dg}{dx} &= u \\ \frac{du}{dx} &= k\end{aligned}$$

Here, x is distance from the wall instead of time. The “initial” ($x = 0$ or wall) values are $u(0) = 0$, $g(0) = 0$ and $k(0) = 0.3152$.

Solve the rate equations from $x = 0$ to $x = 10$ and plot $u(x)$. Format the plot so that $u(x)$ appears as a line. Label the axes and increase the font size of the plot to at least 12 points.