

Exam 3 Review

- * Prayer
- * ABET Evaluation (2x quiz)
- * Final Exam Cover Sheet
- * Questions about exam (TA reviews, office hours, etc.)
- * Class overview
- * Final Review Game.
- * Closing spiritual thought - missions

I. Class Overview

* Numerical Computing.

- Excel & Python
- Units, Data Types, Error
- Functions, Conditionals
- Loops & Arrays
- File I/O & Debugging.

* Numerical Algebra

- Gauss Elimination
 - Jacobi's method
 - Numpy's linear algebra tools
- } linear equations
- Fixed-Point methods - Newton's method, Picard's method
 - Systems of NLEs (Jacobian)
 - Scipy's NLE tools (roots)
 - Optimization (minimize, solver)
- } Non-linear equations

* Numerical Calculus

- Fitting data (polynomials, general curves)
 - interpolation & splines
 - Integration / quadrature (trapezoidal rule, Simpson's rule)
 - Scipy Quad / simp (+ Jupyter & symbolic math)
 - Differential equations, explicit Euler
 - Systems of ODEs & Higher order ODEs (solve_ivp)
 - VBA & XLWings
- } Integrals
- } ODEs

II. Review Game.

1. When fitting data to a curve, an R^2 near one indicates what?

(That the fit is good. Describes most of the data.)

2. Why don't I ^{want to} use a high-order polynomial to interpolate a data set?

(Runge's phenomenon. The polynomial will likely wildly oscillate.)

3. What is the fastest way to fit a polynomial to a set of data in Excel?

(use a trendline on a plot).

4. If I have noisy data, am I going to fit the data or am I going to use interpolation?

(fitting)

5. Suppose you are given a heat capacity at 298K and a heat capacity at 398K. How could you determine the heat capacity at 325K?

(via linear interpolation:

$$C_{p,T} = C_{p,298} \frac{398-T}{100} + C_{p,398} \frac{T-298}{100}$$

6. A Newton-Cotes formula uses what principle to take a numerical integral?

(polynomial interpolation)

7. Suppose I am given the integral:

$$I = \int_0^5 \sin(e^{-x^2}) dx$$

which python function will give me the most accurate estimate of I ?

(scipy.integrate.quad.)

8. When is it appropriate to use "simps" instead of "quad" (or vice versa)

(If you only have data, but no function, you can only use simps.)

9. What is the python module for performing symbolic algebra & calculus?

(sympy)

10. What order is this differential equation?

$$a \frac{d^3 y}{dt^3} + b \frac{d^2 y}{dt^2} + y^2 = 5t$$

$$y(0) = 0$$

$$y'(0) = 5$$

$$y''(0) = -2$$

(3rd order)

11. Is the prev. ODE linear or non-linear?

(non-linear)

12. Write the ODE as a system of 1st order ODEs.

$$x = \frac{dy}{dt}$$

$$z = \frac{d^2 y}{dt^2} = \frac{dx}{dt}$$

$$a \frac{dz}{dt} + bz + y^2 = 5t$$

$$z(0) = -2$$

$$\frac{dx}{dt} = z$$

$$x(0) = 5$$


$$\frac{dy}{dt} = x$$

$$y(0) = 0$$

13. Is the ODE here a BVP or an IVP?

$$\frac{d^2 y}{dx^2} = 5x \quad y(0) = 0$$

$$y(5) = 0$$

(a BVP, these are  not at the same point.)

14. Write the Explicit Euler formula for this ODE:

$$2 \frac{dy}{dt} + 3t^2 y = 0$$

$$\frac{dy}{dt} = -\frac{3}{2} y t^2$$

$$y_{n+1} = y_n + \Delta t \left(-\frac{3}{2} y_n t_n^2 \right)$$

15. When calling solve_ivp to solve the ODE in the previous example, what is the function you define?

```
def rhs(t, y):
```

```
    return (-3/2 * t**2 * y)
```

16. How many lbm are in a kg?

$$(2.2046 \text{ lbm} = 1 \text{ kg.})$$

17. What is the difference between an integer and a float?

(integer = no decimal, stored in binary)

float = decimal, stored as "scientific notation")

18. What methods would be valid for solving this problem?

$$x + 3y = 7$$

$$5x - 2y = 3$$

Gauss Elim, Jacobi's method, np.linalg.solve, np.linalg.inv.

19. If I have an ^{numpy} array x I want to find $\cos(x)$, what do I write?

$y = \cos(x)$

or

for i in $\text{range}(\text{len}(x))$:

$y[i] = \cos(x[i])$.

20. What is the output from this code?

$i = 2$

while $(i < 8)$:

print(i)

$i += 2$

(2 4 6)

Extra Questions (from Apr 2020)

* what is the purpose of interpolation?

To get values in between data that you know.

Ex. It takes 1 hr to get to downtown SLC,
" " 2 hrs " " " Logan UT.
How long to get to Ogden, UT?

* what is a spline?

A piecewise, polynomial interpolation that exactly goes through all of the data.

Higher order \rightarrow more smooth.

* Interpolation vs. fitting example.

* there is no Jupyter on the final exam.

* We learned three ways to do an integral:

(1) trapezoidal rule \rightarrow Python or Excel.

$$I = \frac{\Delta x}{2} [f_0 + 2f_1 + \dots + 2f_{n-2} + f_{n-1}] \quad \text{"Newton-Cotes Formula"}$$

Interpolation \rightarrow Integral

(2) scipy.integrate.quad.

Better than trapezoidal rule. Need a function.

(3) Scipy. integrate. simpson.

Better than trapezoidal rule. Uses Simpson's rule.

Also a Newton-Cotes formula.

Not as good as grad, but don't need function. Only data.

* Trapezoidal rule vs. Simpson's rule vs. Gaussian Quadrature.
example.

* How do you calculate the number of bytes in a variable?

Data type \rightarrow int 4 bytes } given
float 8 bytes }

Array size \rightarrow 50 x 50 matrix = 2500 variables of integers

$$2500 \times 4 \text{ bytes} = 10^4 \text{ bytes} = 10 \text{ kilobytes} \\ \text{or} \\ 10 \text{ KB}$$

* How do you calculate the asymptotic running time of a code?

- Principle: each line that executes costs something.

- Example: Fill a 50 x 50 matrix, $n=50$

for i in range(n):

for j in range(n):

$A[i, j] = i + j$

\leftarrow will happen 50×50
or n^2 times.

So, the total cost is $T = c \cdot 50^2$ where c might be 10^{-2} millisec.

- For big n , only the largest power matters.
- we can measure $T(n)$ at small n & then use the fact that $T(n) \propto n^2$ (or whatever power) to guess $T(n)$ at a big value of n .

* what is Newton's method used for?

- solving a non-linear equation.

$$x^{(k+1)} = x^{(k)} - \frac{f(x)}{f'(x)} \quad \leftarrow \text{a "fixed-point" method.}$$

$$x^{(k+1)} = g(x^{(k)})$$

- make a guess for $x^{(0)} \rightarrow$ find $x^{(1)}$.

- Example: $f(x) = \sin(x)$ (zero @ $0, \pi, \dots$)

$$f'(x) = \cos(x)$$

k	x	$\frac{f(x)}{f'(x)} = \tan(x)$
0	3	$\tan(3) = -0.1425$
1	3.1425	$\tan(3.1425) = 0.00095$
2	3.141593	\leftarrow pretty close to π !

* what is Picard's method?

- A worse fixed point method where you just add x to both sides

$$x^{k+1} = x^k + \sin(x).$$

<u>k</u>	<u>x</u>	<u>sin(x)</u>	← didn't need to find $f'(x)$.
0	3	$\sin(3) = 0.14112$	
1	3.14112	$\sin(3.14112) \approx 0.00047$	
2	3.141593		← pretty close to π !

* what did we learn in the last lecture?

- A macro is a repeated set of instructions you can record.
- Excel has its own language called "VBA"
- There is a package in Python called xlwings for talking to Excel.
- Be familiar with this.

* Gauss Elimination:

- Forward Elimination
 - Back substitution
- } Review Lecture 11
practic problem.
- If I don't ask you to do Gauss Elim, use np.linalg.solve instead!