Lecture 16 - Tips for Engineering Problems.

Outline

1. Identify
2. Standard Form
3. Units
4. Guesses

I. Tips for Solving Engineering Problems.

* Engineering Problems are harder than "pure math" problems
  - math is the same
  - different "window dressing" leads to confusion

* Example:

  What is the diameter of a 100 meter pipe where the supplied head is 32 m, and the flow rate is 6 L/s?

  (assume water, i.e. $\mu = 1cP$, $\rho = 10^3 \, kg/m^3$)

EQs:

head, velocity, friction factor, pipe length

(1) $h = \frac{v^2}{2g}\left(\frac{4Lf}{D}\right)$ ← pipe diam. Pressure lost in a pipe when fluid flows

(2) $\frac{1}{\sqrt{f}} = 4 \log_{10}(Re\sqrt{f}) - 0.4$ Relation between fluid velocity and how much friction there is.

(3) $Re = \frac{\rho v D}{\mu}$ Dimensionless velocity (Reynold's number)

(4) $v = \frac{4Q}{\pi D^2}$ ← flow rate velocity from flow rate ($v = \frac{Q}{A}$)

Tips

I. Identify the types of equations & pick a solution method.

* Is it linear? Non-linear?
* what numerical tool do I need.? Excel? Python?

Solver?

np.linalg.solve?

scipy.optimize.root?

Our Example:

- Nonlinear system
- Scipy. opt. root → Python.

II. Convert to standard form / vector notation

* Identify knowns & unknowns
* Otherwise same process as "math" problem.

knowns

Q

L

h

μ

ρ

g

unknowns

$D = x_3$

$v = x_0$

$Re = x_1$

$f = x_2$

convert to vector notation

$$\underline{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} v \\ Re \\ f \\ D \end{bmatrix}$$

* std form.

$$\underline{\underline{A}} \cdot \underline{x} = \underline{b} \qquad \text{(linear)}$$

$$\underline{f}(\underline{x}) = \underline{0} \qquad \text{(non linear)}$$

$$h = \frac{-v^2}{2g}\left(\frac{4Lf}{D}\right)$$

$$\frac{1}{\sqrt{f}} = 4\log_{10}(Re\sqrt{f}) - 0.4$$

$$Re = \frac{\rho v D}{\mu}$$

$$v = \frac{4Q}{\pi D^2}$$

$$f_0 = 0 = h + \frac{x_0^2}{2g}\left(\frac{4Lx_2}{x_3}\right)$$

$$f_1 = 0 = -\frac{1}{\sqrt{x_2}} + 4\log_{10}(x_1\sqrt{x_2}) - 0.4$$

$$f_2 = 0 = x_1 - \frac{\rho x_0 x_3}{\mu}$$

$$f_3 = 0 = x_0 - \frac{4Q}{\pi x_3^2}$$

→ should use all x_0, x_1, x_2, x_3 (get rid of all unknowns)
→ only knowns should be left.

III. Convert to consistent units

* We did this at the beginning of class.
(not new, but important!)

- Pick SI / Imperial.
  - SI: kg, m, s, K
  - Imperial: slug, ft, s, °R

- Alternate (advanced technique): Make it dimensionless

* Example: (SI)

$Q = 6\ L/s = 6 \times 10^{-3}\ m^3/s$

$\mu = 1\text{cP} = 10^{-2}\ \frac{kg}{m \cdot s}$

$h = -32\ m, \quad L = 100\ m$

$g = 9.8\ m/s^2, \quad \rho = 10^3\ kg/m^3$

m, kg, s ✓

← knowns!

Unknowns:

$v \to m/s$
$Re \to 1$
$f \to 1$
$D \to m$

Tip 4: Make smart Guesses.

* (For nonlinear systems & big linear systems)
× not for this class

- There are no guarantees!! (convergence could be hard!)
- Physical Intuition → D – cm? m? ~~km~~ ~~mm~~
- Use units as a guide → cm
"natural" units

• use a consistent guess.

$$\underline{x} = \begin{bmatrix} v \\ Re \\ f \\ D \end{bmatrix}$$

D = 5 cm

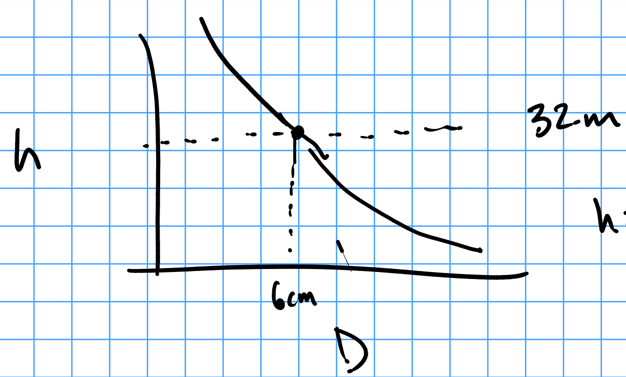
↓

1 guess to "rule them all"

$D \to v \to Re \to f$

– Don't have a good scale, guess different one?

• If 1D or 2D → make a plot!



$$h = -\frac{v^2}{2g}\left(\frac{4Lf}{D}\right)$$

↑

guess a lot

$D \in [1, 100]$ cm

• Use a related, but simpler theory

$$f = \frac{16}{Re}$$ ← laminar flow (not correct theory)

↑ $Re < 2100$

correct O.O.M?

$$P = \frac{NRT}{V}$$ ← ideal gas

• Basic common sense

$v << 10^8$ m/s (speed of light)

$3x^2 - 7x + 2 = 8$ guesses $\pm O(1) - O(10)$

$f \approx 10^{-3}$ (expert intuition)

$Re \approx 10^5$

$0 < \phi < 1$ (vol. fraction) $\phi = \frac{v_1}{v_1 + v_2}$

$D > 0$

* Case Study