A.1 for an incompressible fluid $\rightarrow \nabla \cdot V = 0$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \longrightarrow \frac{3}{2} t y^2 + 3 a y^2 t = 0$$

$$\boxed{a = -\frac{1}{2}}$$

$$\left.\begin{aligned} \frac{\partial V_x}{\partial x} &= \frac{3}{2} t y^2 \\ \frac{\partial V_y}{\partial y} &= 3 a y^2 t \end{aligned}\right\}$$

A.2 at $t=2 \longrightarrow V_x = 3y^2 x$ and $V_y = -y^3$

$$|\underline{V}| = \sqrt{V_x^2 + V_y^2} = \sqrt{9y^4x^2 + y^6} = y^2\sqrt{9x^2 + y^2}$$

$$V_x = \frac{\partial \psi}{\partial y}$$

$$3y^2x = \frac{\partial \psi}{\partial y}$$

$$\int \partial\psi = \int 3y^2 x \, dy$$

$$\psi = \underline{y^3 x} + f(x) + C$$

$$V_y = -\frac{\partial \psi}{\partial x}$$

$$+y^3 = +\frac{\partial \psi}{\partial x}$$

$$\int \partial\psi = \int y^3 \, dx$$

$$\psi = \underline{y^3 x} + f(y) + C$$

$$\boxed{\psi = y^3 x + C}$$

A.3 $\quad a_{tot} = a_{loc} + a_{conv.}$

$$\frac{D\underline{V}}{Dt} = \frac{\partial \underline{V}}{\partial t} + \underline{V} \cdot \nabla \underline{V} \qquad \text{or}$$

$$\begin{bmatrix} V_x & V_y \end{bmatrix} \cdot \begin{bmatrix} \dfrac{\partial \underline{V}}{\partial x} \\ \dfrac{\partial \underline{V}}{\partial y} \end{bmatrix}$$

$$\underline{V} = \begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} \frac{3}{2} t y^2 x \\ -\frac{1}{2} y^3 t \end{bmatrix}$$

$$a_{loc} = \frac{\partial \underline{V}}{\partial t} = \begin{bmatrix} \partial V_x / \partial t \\ \partial V_y / \partial t \end{bmatrix} = \begin{bmatrix} \frac{3}{2} y^2 x \\ -\frac{1}{2} y^3 \end{bmatrix}$$

$$a_{conv} = \underline{V} \cdot \nabla \underline{V} = V_x \cdot \frac{\partial \underline{V}}{\partial x} + V_y \cdot \frac{\partial \underline{V}}{\partial y}$$

$$\frac{\partial \underline{V}}{\partial x} = \begin{bmatrix} \dfrac{\partial V_x}{\partial x} \\ \dfrac{\partial V_y}{\partial x} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} t y^2 \\ 0 \end{bmatrix} \rightarrow V_x \cdot \frac{\partial \underline{V}}{\partial x} = \begin{bmatrix} \frac{9}{4} t^2 y^4 x \\ 0 \end{bmatrix}$$

$$\frac{\partial \underline{V}}{\partial y} = \begin{bmatrix} \dfrac{\partial V_x}{\partial y} \\ \dfrac{\partial V_y}{\partial y} \end{bmatrix} = \begin{bmatrix} 3 t y x \\ -\frac{3}{2} y^2 t \end{bmatrix} \rightarrow V_y \cdot \frac{\partial \underline{V}}{\partial y} = \begin{bmatrix} -\frac{3}{2} t^2 y^4 x \\ \frac{3}{4} y^5 t^2 \end{bmatrix}$$

$$a_{conv} = \begin{bmatrix} \frac{3}{4} t^2 y^4 x \\ \frac{3}{4} y^5 t^2 \end{bmatrix}$$

$$a_{tot} = \begin{bmatrix} \frac{3}{2} y^2 x \left(\frac{1}{2} t^2 + 1 \right) \\ \frac{1}{2} y^3 \left(\frac{3}{2} y^2 t^2 - 1 \right) \end{bmatrix}$$

B No, it depends on time.

→ t appears explicitly in $\underline{V}$

→ we have an acceleration

C find $\omega = \nabla \times V$ → if $\omega = 0$ → irrotational

→ if $\omega \neq 0$ → rotational

$$W = \begin{vmatrix} e_x & e_y & e_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) e_z$$

← since we don't have V_z and $\frac{\partial}{\partial z}$

$$W = (0 - 3tyx)\, e_z = -3tyx\, e_z \neq 0$$

→ rotational ✓