Exam 2 Review

* Prayer / Thought / Quiz
* Questions
* Review Game.

1. What mathematical quantity tells you about rotation in a fluid?

vorticity: $\underline{\omega} = \underline{\nabla} \times \underline{v}$

2. What is the relationship between the viscous stress tensor τ the rate of deformation tensor for a Newtonian Fluid?

$$\tau = 2\mu\Gamma$$

3. What assumptions do I need to go from the Cauchy momentum equation to the Navier-Stokes Equation?

- Incompressibility
- Newtonian Fluid

4. If you are given a velocity field (v_x, v_y, v_z) what equation can you use to find the pressure?

Navier-Stokes. It will give you ∇P.

How find P? Integrate.

5. If I assume unidirectional flow in the y-direction is this term zero?

$v_z \frac{\partial v_y}{\partial z}$ → yes, zero

$v_y \frac{\partial v_y}{\partial y}$ → yes! unidirectional implies fully developed.

Unidirectional: $v_z = v_x = 0$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

$\frac{\partial v_x}{\partial x}$: ↑ 0 ; $\frac{\partial v_z}{\partial z}$: ↑ 0

6. which pipe flow velocity profile is more narrow: laminar or turbulent?

laminar

7. What equation is this? when does it apply?

$0 = -\nabla p + \mu \nabla^2 v$: Stokes Equation, small Re

8. what Equation is this, when apply?

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu \frac{\partial^2 v_x}{\partial y^2}$$

- Boundary Layer Equation,

  Applies at high Re, very close to surface.

✓ 9. How do you calculate a friction factor from a velocity profile?

(i) calculate u: $u = \frac{1}{A}\int v_x \, dA$

(ii) calculate τ_w: $\tau_w = \left| \mu \frac{\partial v_x}{\partial r}\Big|_{r=R} \right|$

(iii) put together: $f = \frac{2\tau_w}{\rho u^2}$

10. what is the first step to solving this poiseuille flow problem?

$$0 = -\frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right)$$

$$\downarrow$$

$$\frac{\partial P}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right) = \text{const}$$

11. why is CFD useful?

Bridge between analytical theory & experiment.

12. What do you need as inputs to calculate a drag force on a surface?

normal vector, $\underline{n}$

pressure field, P

velocity field, $v \rightarrow \tau$ (derivatives)

$$F_D = -\int e_z \cdot \underline{n} \, P \, dS + \int n \cdot \tau \cdot e_z \, dS$$

13. What is Couette flow?

Simple Shear, Parallel Plates

* No pressure drop, moving boundary

14. When does $\nabla^2 \phi$, Laplace's equation apply?

irrotational flow. → when high Re, often useful

*Remember Kelvin's theorem.
An irrotational flow "stays" irrotational @ high Re.

15. How do I get a force on a surface if I know the total stress?

$$F = A \, \underline{n} \cdot \underline{\underline{\sigma}}$$

16. How do I get the total stress?

$$\sigma = -SP + \tau$$

↑ need velocity derivatives

- See sheet!

17. What is the physical meaning of $\frac{Dv}{Dt}$?

acceleration of a "chunk" of fluid moving with the flow.

18. If I'm given v_x & v_z, how do I find v_y?

use continuity: $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$

19. Does D'Alembert's paradox apply at high Re, low Re or both?

high Re

20. Evaluate dP/dx using finite differences:

$$\frac{P_{i+1} - P_i}{\Delta x}$$

21. What is the mathematical quantity that describes deformation?

$\underline{\underline{\Gamma}}$ - rate of strain tensor

or $\underline{p} \cdot \underline{\underline{\Gamma}} \cdot \underline{p}$ = deformation along $\underline{p}$

22. What is a streamline? How is it different than a streakline?

streamline - A line tangent to the velocity of the flow.

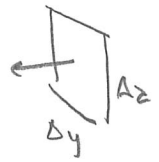
streakline - A path traced by a particle in a flow. stream lines & streak lines are equal when the flow is steady.

23. What does this equation mean physically & when is it valid?

$\nabla \cdot \underline{v} = 0$ ← conservation of mass of an incompressible fluid.

24. What is the mass flow rate through this surface?

$-\underline{e}_x = \underline{n}$



$\rho \underline{v} \cdot (-nA)$

$\rho \underline{v} \cdot (+\underline{e}_x \cdot \Delta z \Delta y)$

$\rho v_x \Delta z \Delta y$

25. Which of the following are surface forces?

- gravitation ← body force
- viscous forces } surface forces
- pressure

26. What must be true about v_x at the wall?

$v_x = u$

No-slip condition

27. What must be true about v_y at the wall?

$v_y = 0$

No-penetration condition

28. What is a more common name for Poiseuille flow?

Pipe-flow, pressure-driven.

29. Rank in order of the flatter velocity profile:

laminar shear-thinning — flattest

laminar Newtonian — next flattest

laminar shear-thickening — least flat

30. What is this equation ? when does it apply ?

$$\rho \frac{D\underline{v}}{Dt} = -\underline{\nabla} P$$

Euler's equation

Apply's @ high Re for all fluids

31. What does axisymmetric mean?

It means a flow is symmetric about θ in cylindrical coordinates and θ or ϕ in spherical coordinates.

32. Why do we need the pressure Poisson equation in CFD?

To find an explicit equation for the pressure.

33. How do you find the friction factor for turbulent flow?

- velocity profile
- Find u, avg velocity
- use $f = \frac{2\tau_w}{\frac{1}{2}\rho u^2}$ ← plug in u & re-arrange.

34. What is eddy diffusivity?

"Extra" momentum diffusion caused by turbulent motion beyond that caused by viscous stresses.